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Modeling Mechanical Properties of Concrete with Bida Natural Stones as Coarse Aggregate

ABSTRACT

This paper presents the results of statistical models for the mechanical properties of concrete made using Bida natural stones (pebbles) as coarse aggregate. Fresh and hardened concrete properties were analyzed using response surface methodology (RSM). The research employed central composite design using three (3) independent variables with equivalent ranges as Water to Cement ratio (W/C)=0.4, 0.5, 0.6, Coarse Aggregate to Total Aggregate ratio (CA/TA)=0.55, 0.615, 0.68 and Total Aggregate to Cement ratio (TA/C)= 3.0, 4.5 and 6.0. Minitab 14 (2004) selected 20 possible mix proportions for the experiment. The concrete specimens that were used for this research work include 150mm x 150mm x 150mm concrete cube, Ø150mm x 300mm concrete cylinder and 100mm x 100mm x 500mm concrete prism for compressive strength, elastic modulus/splitting tensile strength and flexural strength respectively. For each mix, five numbers of specimens were cast and cured for 7, 14, 21 and 28days. The twenty-eight days measured responses were used to develop models for the mechanical properties. Statistical and experimental model validations were employed. The p-value for all individual terms were less than 0.05, p-values for lack of fit were also greater than 0.05 in all cases tested, coefficient of determination (R^2), adjusted coefficient of determination ($adjR^2$) were in the range of 89.7% to 100% for compressive strength, splitting tensile strength, elastic modulus, flexural strength, and slump respectively which indicated adequate model validity. The predicted responses closely agree with experimental values, this further confirms the validity of the models developed. The study therefore suggested interaction, pure quadratic, reduced full quadratic, interaction models for the mechanical properties and slump respectively for the concrete made using Bida natural stones as coarse aggregate. The concrete with mix constituent mentioned above can be used for structural application such as reinforced concrete slabs, beams, columns and foundations.

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Keywords: Bida natural coarse aggregates (Pebbles); Mechanical properties; models; Response surface methodology

1. INTRODUCTION

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Concrete is one of the world's most widely used construction materials, second only to water [1, 2]. It is a product that forms from the mixture of water, cement, aggregates and in some occasions, admixtures. This mixture hardens into a rock-like material through a chemical reaction (hydration) between the cement and water and continues to gain strength with age. Aggregates make up approximately 75% of concrete and significantly influence its properties. The aggregate must be of acceptable shape, free from dirt, strong and well graded. It should possess chemical stability and, in many cases, exhibit abrasion resistance and resistance to freezing and thawing [3, 4].

26 Compressive strength, tensile strength, flexural strength which define the degree of
27 resistance a structural element can offer to deformation remain the most important
28 properties of structural concrete, from an engineering point of view. Over time, researchers
29 have made efforts in understanding the relationship between concrete's mechanical
30 properties and composition of constituent materials. The strength of the concrete is
31 determined by the characteristics of the hardened mortar, coarse aggregate, and the
32 aggregate/mortar interfacial zone. Different strengths of concrete have been measured for
33 mixes for mortars are of the same quality but having different aggregate texture, shape and
34 mineralogy. However, the influence of the limitation on water to cement ratio especially as
35 regards high performance concrete has become apparent, with the nature of aggregates
36 used contributing largely to performance [5]. Thus, using a coarse aggregate of higher
37 strength, good mineralogical and textural characteristics may improve the performance and
38 other desirable properties of concrete.

39 The most important factors which influence a mix design are the water to cement ratio
40 (W/C), total aggregate to cement ratio (TA/C) and the coarse aggregate to total aggregate
41 ratio (CA/TA). Since those factors are interdependent, proper design and analysis of
42 experiments for studying their influences on the mix proportions is necessary. One traditional
43 method of studying the effects of various factors is to vary one factor at a time and keep all
44 others constant, respond readings are taken for different level of these factors one after the
45 other until the entire factors have been treated. This approach may not be satisfactory or
46 may involve a large number of unnecessary experiments if interactions between the factors
47 occur [6]. To overcome this difficulty recourse is made to Response surface methodology
48 (RSM).

49 Response Surface Methodology encompasses techniques that are mathematically and
50 statistically based to develop and optimize dependent variable expected to be derived from
51 independent variables [7]. RSM is particularly useful where performance characteristics or
52 responses (e.g compressive strength, slump, bulk density and cost of concrete) are
53 influenced by several design factors or variables. Response Surface Methodology can be
54 used to optimize one or more responses or to meet a given set of specifications for example
55 a minimum strength specification or an allowable range of slump values. RSM comprises of
56 three steps, namely; experimental design, modelling and optimization [8]. There are four
57 types of models, one linear while the other three are quadratic. The three quadratic models
58 are interaction, pure quadratic and full quadratic. The full quadratic model comprises of the
59 other models and it has the highest coefficients. These four different models were tried and
60 the model that offers the best fit for the response surface design format of the concrete mix
61 proportion was considered. Thus, this research developed models for the prediction of the
62 mechanical properties of concrete using Bida natural stones as coarse aggregate.
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64 **2. SIGNIFICANCE OF THE STUDY**

65
66 Crushed stones are available in some part of Nigeria, like Abuja and are transported to Bida
67 for production of concrete at high cost. This is because there is no quarry in Bida and its
68 environs thereby increasing the haulage distance. Bida natural coarse aggregate are
69 abundantly available in Bida area. This aggregate is cheaper than the crushed granite. This
70 aggregate has been used for concrete production, but extensive research work has not been
71 conducted on it as compared to the crushed granite. It is therefore appropriate to study the
72 engineering properties of the concrete produced using Bida natural coarse aggregate to
73 make available data that are useful in the mix proportioning, and also in design of mixes for
74 specified compressive strength of concrete and subsequent estimation of optimal quantity of
75 aggregate material to reduce the overall concrete production cost.
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77 **3. MATERIALS AND METHODS**

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79 **3.1 Materials**

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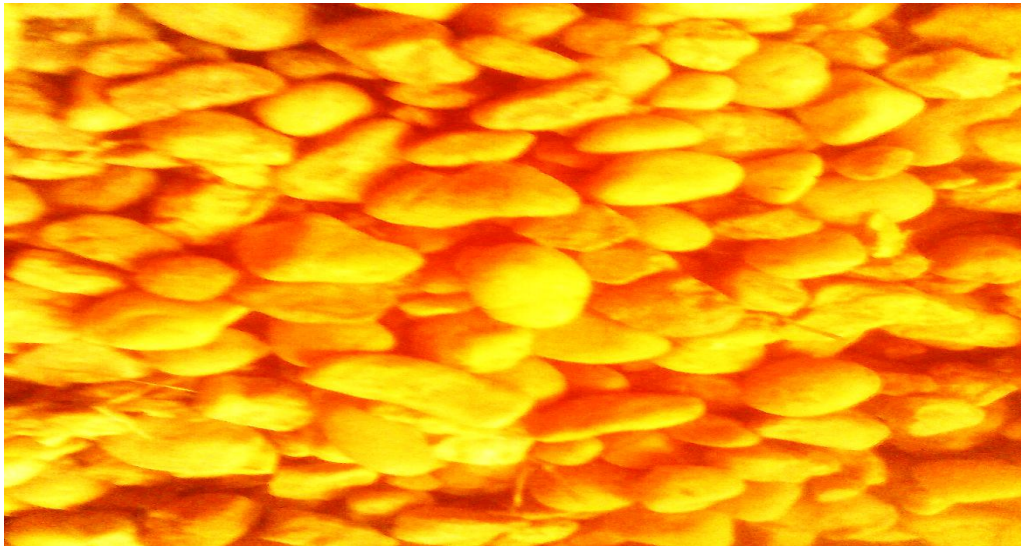
81 The materials utilized in this research work are as follows;

82 a) Cement; Ordinary Portland Cement (42.5N) being the most common binding
83 material used in Nigeria was used.

84 b) Fine aggregate; The sand was collected from Gidanmongoro, Minna, Niger state,
85 Nigeria. It conforms to the standard requirement of BS EN 12620 [9].

86 c) Water; Potable drinking water from Civil Engineering Laboratory, Federal University
87 of Technology Minna was used throughout this work. The water is clean, free from
88 deleterious materials and potable as specified by BS EN 1008 [10].

89 d) Coarse aggregate; The natural stones used for this work were taken from Bida,
90 Niger state in the middle belt region of Nigeria located about 250 Km North-East of
91 Bida in land from the Federal capital city of Abuja. The deposits aggregates lie in
92 zone 31, which falls within the latitude N 90 55'E and longitude N 50 52' E [11]. The
93 Bida natural deposits aggregates in character occur in middle Niger basin of Nigeria
94 in several million metric tons. These aggregates are characterized with round and
95 smooth surfaces, and are reddish-brown in colour. The aggregates used consist of
96 maximum size of 19mm (Fig. 1). It also conforms to the standard requirement of BS
97 EN 12620 [12].



98

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Fig. 1.Cluster of Bida natural coarse aggregate

100 **3.2 Methods**

101 The concrete specimen produced and tested were 150mmx150mmx150mm (cube),
102 100mmx100mmx500mm (prism), 300mmx150mm (cylinder) and also 300mmx150mm
103 (cylinder) for compressive strength, flexural strength, splitting tensile strength and elastic
104 modulus test respectively in accordance to specifications of BS EN 12390 [13].

105 Laboratory tests conducted on the sample of Bida natural coarse aggregate and fine
106 aggregate to determine their physical properties include the following: moisture content,
107 specific gravity, sieve analysis, aggregate impart value, aggregate crushing value, bulk
108 density (compacted and uncompacted), water absorption and silt test in accordance with BS
109 EN 12620 [9]. Also, the laboratory test conducted on the fresh and hardened concrete are:
110 slump test, compacting factor test (compacted and uncompacted) according to BS EN
111 12350-1 [14]. The mechanical property tests were carried out on accordance with relevant
112 parts of BS EN 12390 [13].

113 A total of 1600 samples were cast. Five samples per mix point for the 20 mix proportions
114 generated by Minitab to test the 7,14, 21 and 28 days strength of each of the mechanical
115 properties.

116 a) Production of Concrete for Compressive strength

117 The sample was manually mixed until achieving the required homogeneity; the standard iron
118 moulds of (150×150×150) mm³ were used. The moulds were lubricated with engine oil in
119 other to reduce friction and to ease the removal of cubes from the moulds, they were then
120 filled with fresh concrete in three layers and each layer was tamped 25 times. The same
121 procedure was carried out for other concrete specimen (that is prism and cylinder for flexural
122 strength, and splitting tensile and elastic modulus respectively).

123 b) Curing and crushing of the concrete

124 The curing method used was total immersion, the hardened concrete samples were
125 demoulded after 24 hours, cured and crushed for 7, 14, 21 and 28 days respectively. The
126 mechanical properties that were determined are the elastic modulus, compressive, splitting
127 tensile and flexural strengths.

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130 4. MODEL DEVELOPMENT

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132 The experimental result of the elastic modulus, compressive, splitting tensile and flexural
133 strengths were used for model development. All models: linear, linear +square, linear +
134 interaction and full quadratic were tried for each property. The best models were chosen for
135 each property, having satisfied the statistical validation parameters (Standard deviation S,
136 R^2 , $adjR^2$ and lack of fit value respectively). Table 1 shows the model summary and
137 summary of ANOVA for the chosen models.

138

139 a) **Model adequacy checking;** Summary of the coefficients and significant p-value for each
140 term in selected models are as follow

141

142 i. P- value (Level of significance)

143 The p-value of individual terms in all cases for the mechanical properties and slump are all
144 less than 0.05 which is an indication that, all terms are highly significant in the model. Hence,
145 this is a confirmation of the model adequacy.

146

147 ii. Standard deviation, S

148 The standard deviations of individual terms are very low in all cases of the properties as
149 shown in Table 1. This is also a proof that the models in all cases are adequate to explain
150 the data fully well.

151

152 iii. R^2 and $adjR^2$

153 The values of the R^2 and $adjR^2$ for the models are displayed in Table 1. These values range
 154 between of 93.7% - 100.0%, indication that the models are adequate to explain the data.
 155 This implies that about 93.7% -100% of the variability in all the properties, viz compressive
 156 strength, flexural strength, splitting tensile strength and slump is accounted for by the
 157 variables in the models developed.

158
 159 iv. ANOVA

160 The results of the analysis of variance for all models are also displayed in Table 1. The lack
 161 of fit test is regarded as testing the lack of fit for a model under consideration. If the p-value
 162 of this parameter is high (That is greater than 0.05), it implies that there is no lack of fit and
 163 the model is regarded to be adequate. It is against this background that suggestion is made
 164 for the adequacy of the models developed in each case having satisfied the specification.
 165 Thus, p-values of lack of fit should be greater than 0.05 [15].

166 The p-values for linear, interaction and square models as shown in Table 1 for all the
 167 properties are less than 0.05 suggesting that the models are adequate to explain the
 168 variability in the data, thus the terms contribute significantly to the model. In addition, the
 169 values of residual error are considerably very close to that of pure error in all cases (that is in
 170 all models) as shown in Table 1. This also indicates that there is no lack of fit. Thus, the
 171 ANOVA test on model in all cases further confirms the adequacy of the models mentioned
 172 above. This is in agreement with the result obtained by Abdullahi [4]; Senthil and Baskar
 173 [16].

174

175 **Table 1. The coefficients and significant p-value of each term in the selected models**

Terms	Compressive Strength (Interaction)		Flexural Strength (Pure Quadratic)		Splitting Strength (Reduce Full Quadratic)		Elastic Modulus (Interaction)		Slump (Reduced Full Quadratic)	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-val	Coefficient	p-val	Coefficient	p-val
Constant	-121.71	0.00	18.67	0.001	-24.16	0.001	117.2	0.000	-201.4	0.007
x_1	172.04	0.00	30.37	0.001	44.89	0.001	121.6	0.000	1181.6	0.001
x_2	435.64	0.00	-76.14	0.000	37.88	0.001	-95.24	0.000	376.63	0.000
x_3	-10.01	0.00	2.98	0.000	1.55	0.000	-36.93	0.000	-120.7	0.019
x_1^2	-	-	-36.54	0.000	-	-	-	-	-232.1	0.000
x_2^2	-	-	64.03	0.000	-	-	-	-	-	-
x_3^2	-	-	-0.43	0.000	-0.19	0.000	-	-	15.09	0.000
$x_1 x_2$	-630.17	0.00	-	-	-72.92	0.001	-320.1	0.000	-	-
$x_1 x_3$	50.83	0.00	-	-	-	-	14.49	0.000	-129.1	0.000
$x_2 x_3$	-29.67	0.00	-	-	-	-	46.21	0.000	-	-

S	0.2619	0.0810	0.1643	0.07981	2.480
R-sq %	99.8	99.8	93.7	100.0	93.7
adjRsqr%	99.6	99.7	89.8	100.0	89.8
Lack of fit	0.572	0.661	0.612	0.790	0.612

176

177 Regression equation for each response is as follows;

178

179 $Y_1 = -121.71 + 172.04x_1 + 435.64x_2 - 10.01x_3 - 630.17x_1x_2 + 50.83x_1x_3 - 21.67x_2x_3$ (1)

180 $Y_2 = 117.21 + 121.66x_1 - 95.24x_2 - 36.93x_3 - 320.05x_1x_2 + 14.49x_1x_3 + 46.21x_2x_3$ (2)

181 $Y_3 = -24.164 + 44.89x_1 + 37.875x_2 + 1.546x_3 - 0.193x_1x_3 - 72.92x_1x_2$ (3)

182 $Y_4 = 18.666 + 30.372x_1 - 76.137x_2 + 2.978x_3 - 36.543x_1^2 + 64.029x_2^2 - 0.426x_3^2$ (4)

183 $Y_5 = -201.37 + 1181.602x_1 + 376.63x_2 - 120.67x_3 - 232.05x_1^2 + 15.09x_2^2 - 129.06x_1x_3$ (5)

184

185 Where:

186 Y_1 = Compressive strength

187 Y_2 = Elastic modulus

188 Y_3 = Splitting tensile strength

189 Y_4 = Flexural strength

190 Y_5 = Slump

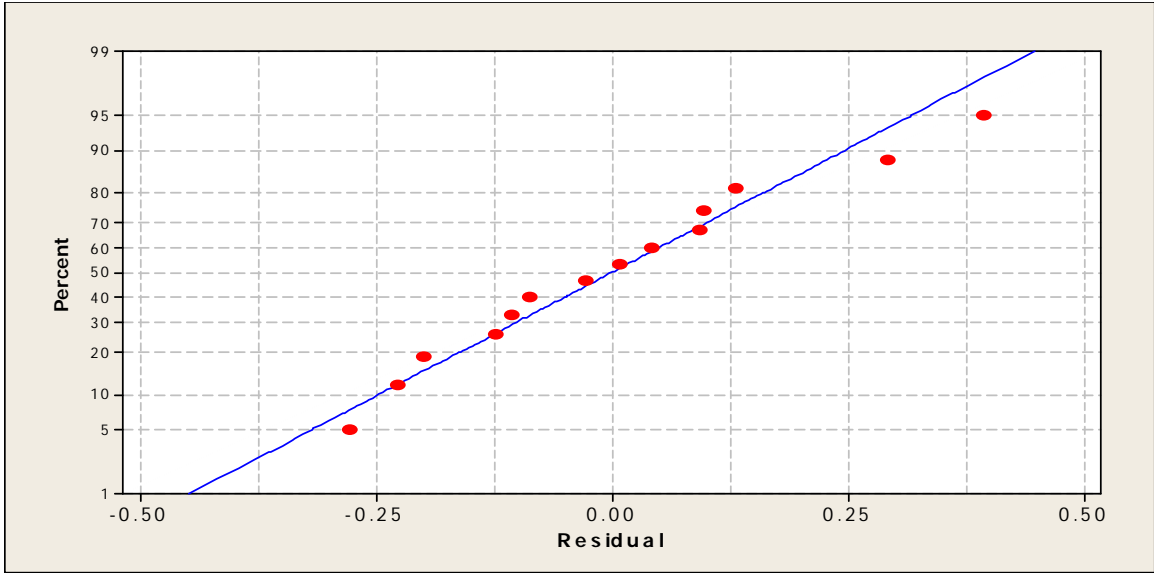
191 x_1, x_2 and x_3 are as previously defined

192

193 **b) Residual plot**

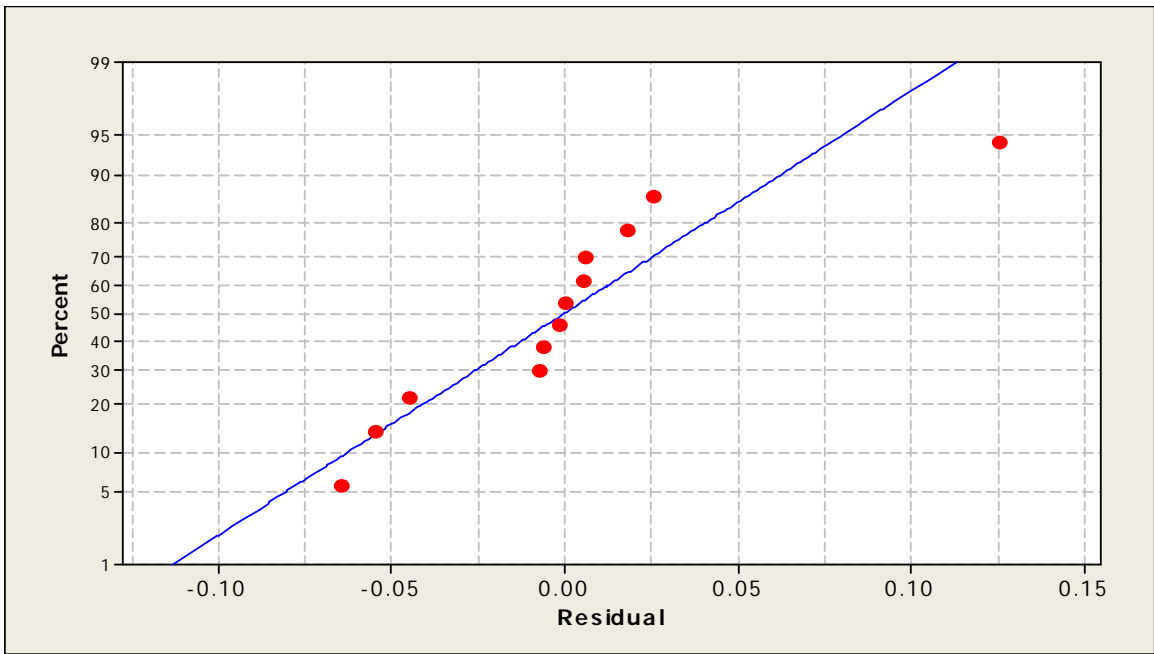
194 It is mandatory that the developed models provide an adequate similarity to the true system,
 195 proving that none of the least square regression assumptions are violated. Proceeding with
 196 exploration and optimization of a fitted response surface will likely give a poor or misleading
 197 result unless the model is adequately fitted [15].

198 The residual from the least square fit plays an important role in judging model adequacy. Fig.
 199 2 to 6 present normal probability plots of residual for the mechanical properties and slump.
 200 The normal probability plot shows that the points lie approximately along the straight line as
 201 shown in these Figures, hence no apparent problem with the normality in the plot, this
 202 implies that the normality assumption is satisfied. However, the straight line in this normal
 203 probability plot was determined by inspection concentrating on the central portion of the data
 204 [15].
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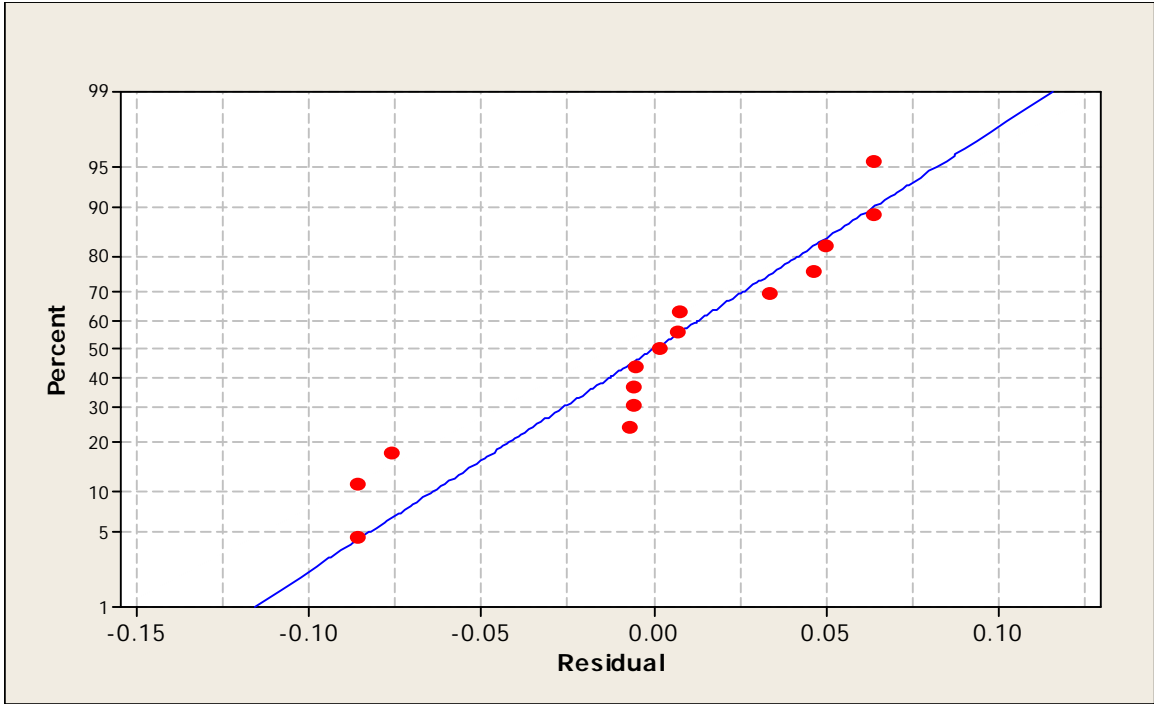
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Fig. 2. Normal probability plot of the residuals for compressive strength



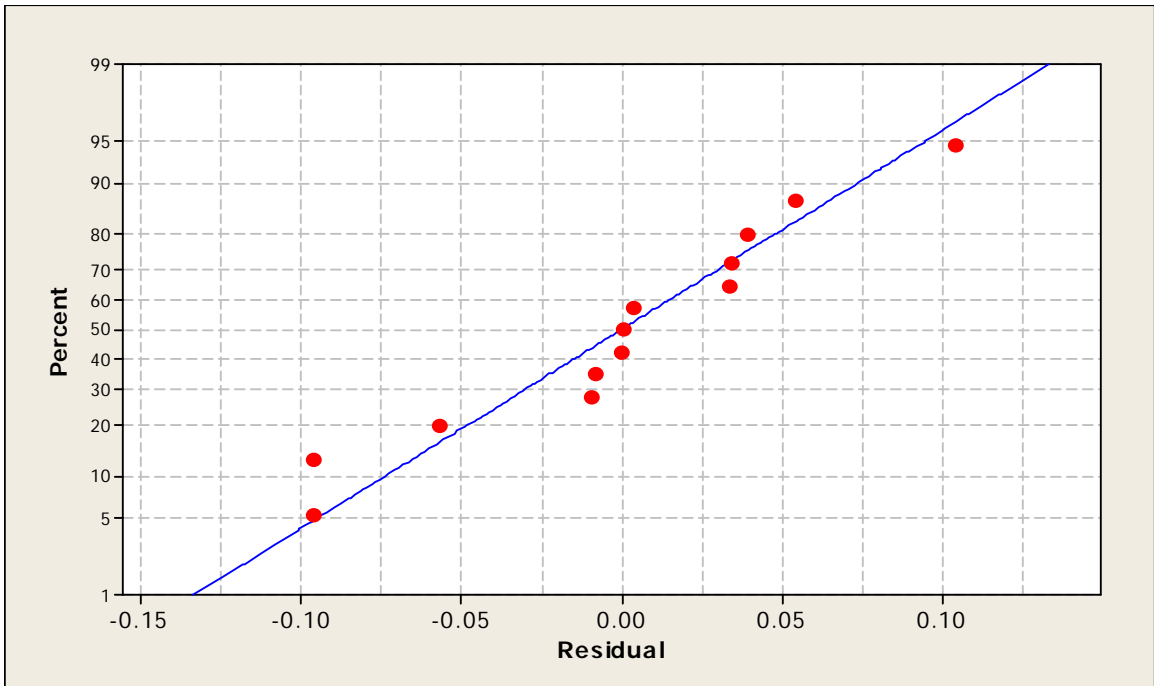
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Fig. 3. Normal probability plot of the residuals for elastic modulus



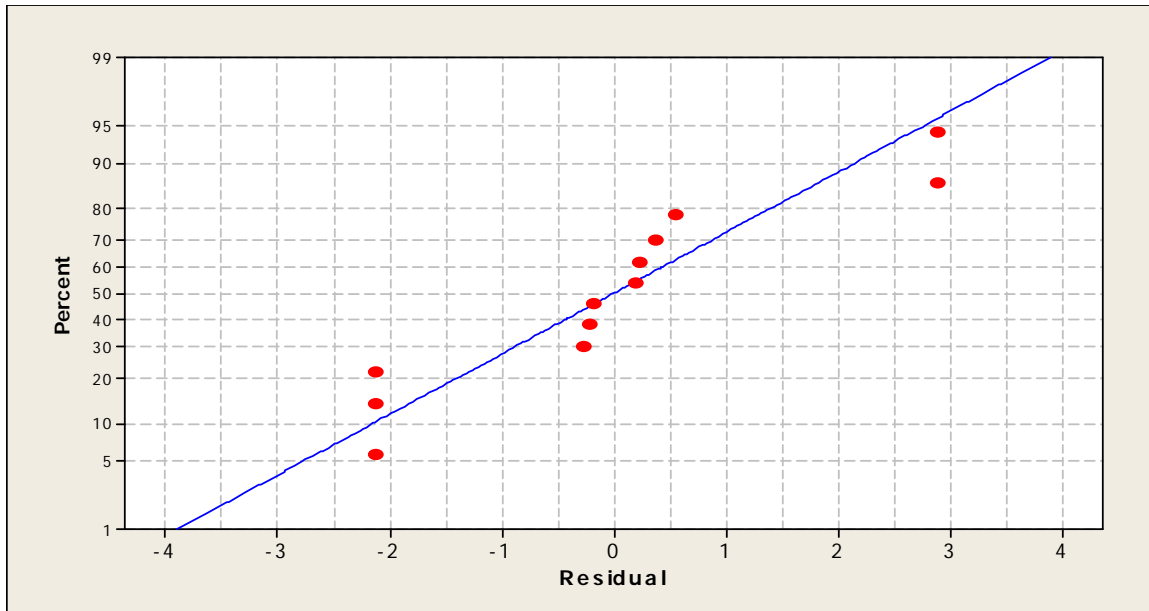
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Fig. 4. Normal probability plot of the residuals for splitting tensile strength



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Fig. 5. Normal probability plot of the residuals for flexural strength



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Fig. 6. Normal probability plot of the residuals for flexural strength

226 **c) Experimental Model Validation**

227 To validate each of the regression model five (5) numbers of specimens for five (5) different
228 mixes were cast and tested at 28days for compressive strength, flexural strength, splitting
229 tensile strength and elastic modulus. The independent variables as shown in Table 2 were
230 fixed into the regression model equations to arrive at the predicted values (responses) for
231 the samples [16].

232
233

Table 2. The Variables Used for Experimental Validation

Validation	W/C	CA/TA	TA/C
Mix order	X1	X2	X3
1	0.45	0.55	4.5
2	0.55	0.615	4.5
3	0.6	0.55	6.0
4	0.4	0.68	4.5
5	0.5	0.55	6.0

234

235 **d) Percentage errors of experimental and predicted values for the validated**
236 **sample**

237 In addition to the statistical validation of the developed models, the experimental model
238 validations were also carried out. Table 3 shows the percentage errors of experimental and
239 predicted values. It is obvious that the percentage errors were negligible, that is, the
240 predicted responses are very closed to the experimental results which further confirms the

241 validity of the developed models in each case of the compressive strength, flexural strength
 242 and elastic modulus. Hence the predicted models are reliable.

243

244 **Table 3. Percentage Errors of Experimental and Predicted Values for the Validated**
 245 **Samples**

Mix order	Compressive strength (N/mm ²)			Flexural strength (N/mm ²)			Splitting tensile strength (N/mm ²)			Elastic modulus (N/mm ²)		
	Exp	Pre	% error	Exp	Pre	% error	Exp	Pre	% error	Exp	Pre	% error
M1	24.1	23.8	1.263	6.8	7.2	-5.88	5.5	5.39	2	17750	17890	-0.79
M2	26.04	26.32	-1.09	6.6	6.48	1.82	5.6	5.63	-0.54	14940	14860	0.54
M3	35.91	38.18	-6.31	3.9	3.76	3.59	8	8.12	-1.5	15090	15280	-1.26
M4	27.73	27.59	0.491	7.3	7.57	-3.70	6.5	6.32	2.77	15160	15360	-1.32
M5	25.07	25.13	-0.25	4.7	4.74	-0.85	7.2	7.76	-7.78	12910	12030	6.82

246 % error= (Exp-Pre)/Expx100

247

248

249 5. CONCLUSION

250

251 It can be concluded, based on the findings of this research that;

252 1. Central composite design has been proven to be a robust tool in evaluating
 253 parameters effect of mixture and the interaction between the parameters of concrete
 254 made using Bida natural stones and can reduce the number of trials to achieve
 255 balance among mix variables.

256 2. The statistical models developed for compressive strength, flexural strength, splitting
 257 tensile strength, elastic modulus and slump are interaction, pure quadratic, reduced
 258 full quadratic, interaction and reduced full quadratic model respectively.

259 3. Graphical analysis of the results shows the deviation between the measured and
 260 fitted data could be effective methods to test the adequacy of the regression model.

261 Recommendation

262

263 The concrete mix described in this study is suitable for structural applications, including
 264 slabs, beams, columns, and foundations.

265

266 Disclaimer (Artificial intelligence)

267 Option 1:

268 Author(s) hereby declare that NO generative AI technologies such as Large Language
 269 Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the
 270 writing or editing of this manuscript.

271 Option 2:

272 Author(s) hereby declare that generative AI technologies such as Large Language Models,
273 etc. have been used during the writing or editing of manuscripts. This explanation will
274 include the name, version, model, and source of the generative AI technology and as well as
275 all input prompts provided to the generative AI technology

276 Details of the AI usage are given below:

277 1.

278 2.

279 3.

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