Research review on identification of time-varying bridge damage under moving loads

Abstract：With the rapid development of our country's economy, as the main hub of transportation, bridge undertakes increasingly heavy traffic tasks, and its safety requirements are getting higher and higher. In order to quickly and effectively judge the location and degree of damage of Bridges during service, effective measures can be taken to ensure the safe operation of Bridges, which makes the application of bridge structural damage identification technology more and more extensive in the field of bridge engineering. This paper reviews the research status, methods and challenges of time-varying bridge damage identification under moving loads, aiming to provide reference for relevant researchers.

Keyword：Moving load；Damage identification of bridge；Time-varying damage；Dynamic response；Strain energy；Wavelet packet analysis

1 Introduction

As an important infrastructure for transport, the safety of bridges is directly related to the safety of people's lives and property and the stable development of the national economy. During the operation of bridges, various damages, such as cracks and deformations, may occur in the bridge structure due to vehicle loads, natural environment and other factors [1-2]. Therefore, timely and accurate identification of the location and degree of damage of bridges is of great significance to ensure the safe operation of bridges [3].

In recent years, with the rapid development of sensor technology, signal processing technology and computer technology, the bridge structure damage identification technology has made significant progress. Wu et al [4] according to the continuous girder bridge long diameter strain influence line with the change of the position of the vehicle on the bridge of the change rule, put forward a spatially distributed long strain sensing based on the concrete continuous girder bridge damage identification of a new method. And the effectiveness of the proposed method is verified by numerical simulation and experiment. Yin [5] et al. proposed a physically guided deep neural network approach to address the challenge of modelling uncertainty or modelling error to improve the accuracy of damage identification of vehicle-induced bridge vibration response and minimize the impact of modelling uncertainty on damage identification. Faridi et al. [6] proposed a non-parametric damage detection method for truss bridges based on statistical analysis by using the stochastic The normalized acceleration response time course of the bridge under random excitation was used to calculate the covariance matrix coefficients of the truss bridge in the base and damage states, and to determine the location and relative severity of the damage by means of the covariance matrix and the histogram of the sum of the covariance matrix differences [16,17].

A vibration sensor-based method is an area that has received increasing attention in recent years due to the fact that indirect measurement of moving loads is more economical and convenient compared to direct methods [7-8]. One of the main loads that bridge structures are subjected to is vehicle loads and the signals generated by vehicle-excited bridges change with the service time, and damage identification of bridges through such changes not only saves resources, but also can be carried out without interrupting traffic. Indirect identification methods can effectively identify the moving loads in the structural response through the relationship between the moving loads and the response [9]. With the continuous development of big data, wireless IoT technology is increasingly being applied in structural monitoring, i.e., through the effective use of real-time structural test data obtained from various IoT sensors provided by cloud-based data storage systems [10,18-20].

The response data are obtained by real-time monitoring of the bridge, and this signal is used as the basis for identifying the vehicle loads. The identification of vehicle loads provides a new basis for structural health monitoring, and since there are changes in the vehicle signals before and after the damage of the bridge structure according to the changes in the mechanical properties of the bridge itself, many researchers have been invested in the field of the synergistic identification of mobile loads and damages. Pourzeynali et al [11] proposed a method for the simultaneous identification of moving load and structural damage based on the explicit form of the Newmark-β method. The study of structural condition assessment of bridges under the action of moving vehicles considering sensor placement, sampling frequency, damage type, measurement noise, vehicle speed, road roughness, etc. was numerically and experimentally validated. Hester et al [12] In this paper, a bridge damage detection method based on direct rotational measurements was proposed. The difference in rotational measurements between healthy and damaged states obtained by using a single point of moving load was proposed as an indicator of damage. The robustness of the proposed damage detection method was tested in a series of blind tests.

This paper reviews the current research status of identifying time-varying bridge damage under moving loads, introduces damage identification techniques based on methods such as dynamic response, strain energy and wavelet packet analysis, and discusses the advantages and disadvantages of these methods as well as future development trends.

2 Principle of bridge damage recognition under moving load

When a moving load is applied to a bridge structure, the structure generates dynamic responses, such as vibration and impact. If damage occurs in a certain part of the bridge, as the mobile load gradually approaches or moves away from the damage location, the collected dynamic response will inevitably change. For the simply supported beam under the action of moving mass, its equation of motion is

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}+c\frac{\partial y(x,t)}{\partial t}=\delta(x-Vt)m\left(g-\frac{\partial^2 y(x,t)}{\partial t^2}\right) \tag{1}$$

Where the parameters are shown in the figure 1, where the moving mass is m.

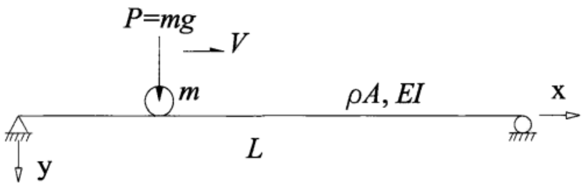


Fig. 1 Simply supported beam with moving mass

Similar to solving the equations of motion for moving loads, the displacement $y(x,t)$ is written in generalised coordinate form according to the vibration mode decomposition method and integrated to get

$$\begin{aligned}&\left[\rho A \int_0^L \phi_n^2(x)\mathrm{d}x\right]\frac{\mathrm{d}^2 q_n(t)}{\mathrm{d}t^2} + \left[c \int_0^L \phi_n^2(x)\mathrm{d}x\right]\frac{\mathrm{d}q_n(t)}{\mathrm{d}t} + \\ &\left[EI \int_0^L \phi_n(x)\frac{\mathrm{d}^4\phi_n(x)}{\mathrm{d}x^4}\mathrm{d}x\right] q_n(t) = mg\phi_n(x) - m\sum_{i=1}^{\infty} \frac{\mathrm{d}^2 q_i(t)}{\mathrm{d}t^2}\phi_i(x)\phi_n(x)\end{aligned} \tag{2}$$

Since the damage beam with cracks is studied here, the modes should be calculated in terms of the modal parameters with damage rather than simple trigonometric functions. Therefore, Eq. (2) is changed to

$$\begin{aligned}&\left[\left(\rho A \int_0^t \phi_n^2(x)\mathrm{d}x\right)\ddot{q}_n(t) + m\sum_{i=1}^{\infty} \ddot{q}_i(t)\phi_i(x)\phi_n(x)\right] + \left[2\xi_n \rho A \omega_n \int_0^t \phi_n^2(x)\mathrm{d}x\right]\dot{q}_n(t) + \\ &\left[EI \int_0^L \phi_n(x)\frac{\mathrm{d}^4\phi_n(x)}{\mathrm{d}x^4}\mathrm{d}x\right] q_n(t) = mg\phi_n(x)\end{aligned} \tag{3}$$

Under the condition that $\phi_n(x)$ has been obtained, this is a differential equation for $q_n(t)$, which can be obtained using the Newmark-β method to obtain $q_n(t)$ and hence the response of the beam.

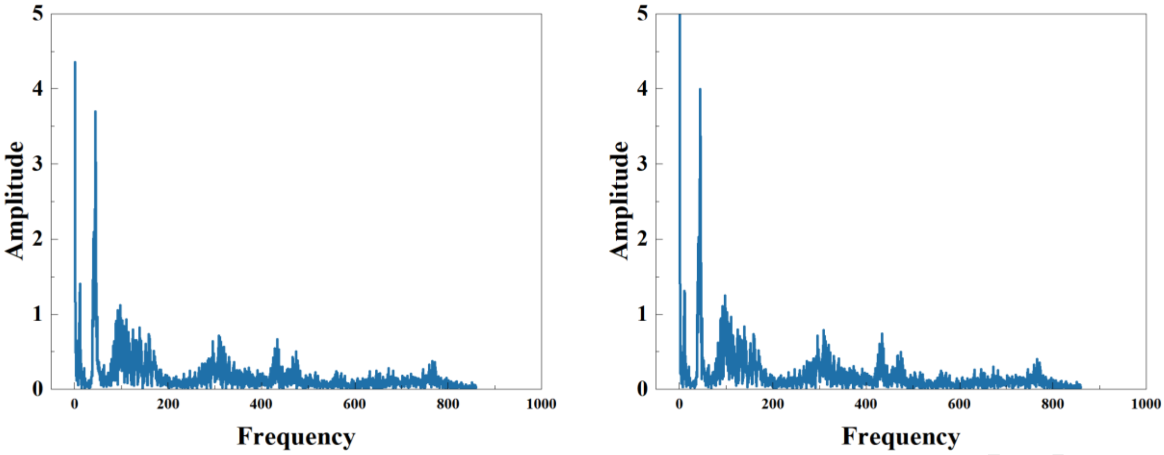
At present, the research of damage identification is still in the basic stage, and various theories and methods are developing day by day. Classified according to the source of test data, structural damage identification methods can be divided into static identification, dynamic identification and distributed fiber identification. Among them, static identification is the use of structural displacement, strain and other data measured under static conditions for damage identification; dynamic I dentification is the use of the dynamic response of the structure, such as acceleration, dynamic strain, to carry out damage identification research; dynamic identification according to the different signal response function can be divided into the frequency domain method, the time domain method, and the time-frequency domain method.

3 Damage Identification Method Based on Dynamic Response

3.1 Frequency domain method

The frequency domain method is used to determine the damage by analyzing the frequency response of a bridge structure under moving loads. When damage occurs to a bridge structure, its intrinsic frequency will change. Therefore, the frequency response of the bridge structure can be measured and compared with the frequency response in the healthy state to judge the damage location and degree of the bridge structure. The frequency domain method has the advantages of high recognition accuracy and strong noise resistance, but the calculation volume is large and the arrangement of sensors is demanding.

Spectrum estimation is an extremely common method of spectrum analysis, and the power spectrum estimation method is mainly divided into the periodogram method, the Bartlett method and the Welch method as shown in Figure 2. The Bartlett method is an improvement of the periodogram method, i.e., the signal is segmented first, and then processed by the periodogram method, and the periodogram of each segment is averaged to obtain a new type of power spectrum; one of the outstanding features of the Welch method improved on the basis of the Bartlett method is that it allows the use of different window functions to slide the intercepted data over the analyzed data string and interleave them. This allows the number of intercepted segments to be increased, and the averaging can represent the signal characteristics more smoothly, which can effectively reduce the spectral estimation variance of the data, and reduce the damage to the resolution of the signal, so that the information contained in the signal can be effectively preserved.



Welch method signal processing

Bartlett signal processing

Fig. 2 Comparison of signal processing

3.2 Time domain method

The time-domain method is to judge the damage by analyzing the time-domain response of the bridge structure under moving loads. When the bridge structure is damaged, its vibration pattern will change, such as amplitude and waveform. Therefore, the damage location and degree of the bridge structure can be judged by measuring the time domain response of the bridge structure and comparing it with the time domain response under the healthy state. The time-domain method has the advantages of real-time and easy operation, but it is weak in noise resistance and easily interfered by environmental noise. The commonly used time domain methods are as follows:

(1) ITD method (brahim time-domain method): proposed by brahim in the 1970s, the modal parameters are identified using the displacement, velocity or acceleration time-domain signals of the free vibration response of the structure. The modal parameters of the system are obtained by constructing the augmentation and generalization matrices of the free response sampling data, establishing the mathematical model of the eigenmatrix and solving the eigenvalue problem.

(2) Time Series Analysis: A modal parameter identification method based on discrete autoregressive models, which identifies the modal parameters of the structure through the time series of the output response of the structure under test. It is suitable for linear or non-linear parameter identification under white noise excitation, including autoregressive model (AR model), sliding average model (MA model) and autoregressive sliding average model (ARMA model) identification methods.

3.3 Time-frequency domain method

The time-frequency domain method combines the advantages of the frequency domain method and the time domain method, and is able to obtain the time domain and frequency domain information of the bridge structure under the action of moving load at the same time. By analysing the time-frequency domain response of the bridge structure, the damage location and degree of the bridge structure can be judged more accurately. The time-frequency domain method has the advantages of high recognition accuracy and strong noise resistance, but the calculation volume is large, and the requirements for the arrangement of sensors and signal processing technology are high.

In practical applications, time domain analysis and frequency domain analysis often need to be used in combination to obtain more comprehensive analysis results. For example, in the field of signal processing, the time domain characteristics of the signal can be observed through the time domain analysis to understand the transient response and stability of the signal; then, the spectral characteristics of the signal can be analyzed through the frequency domain analysis to understand the frequency components and bandwidth of the signal. In this way, a more comprehensive understanding of the characteristics of the signal can be achieved, providing strong support for signal processing and optimization.

4 Damage Identification Method Based on Dynamic Response

4.1 Basic Principles of Strain Energy

Strain energy-based damage identification methods require the construction of suitable damage indicators to quantify the extent of damage. Commonly used damage indicators include modal strain energy base indicators, strain modal curvature difference indicators, and so on.

(1) Modal Strain Energy Base Indicator

The damage identification method based on strain energy is an effective structural damage detection technique that utilizes the change in strain energy produced by the structure during the stressing process to identify the damage. When a structure is subjected to external forces, it deforms and releases strain energy. Structural damage leads to changes in its physical properties such as stiffness and mass, which in turn affects the distribution of strain energy. Therefore, by monitoring the change in strain energy, the damage of the structure can be indirectly inferred.

It is generally believed that damage will cause a decrease in the physical parameters (stiffness, mass, damping, etc.) of the structure, which further leads to changes in the modal parameters of the structure. Domestic and foreign scholars constructed damage factors such as curvature modulus, flexibility matrix, modal strain energy, etc. as indicators of structural damage identification based on this feature [13-14]. Since then, some studies have compared and analyzed the damage identification performance of Modal Strain Energy Change Rate, Modal Strain Energy Dissipation Rate and Modal Strain Energy Based Index, and the results show that the Modal Strain Energy Based Index can identify structural damages more accurately. The expression of modal strain energy basis index is as follows

$$MSE_i^e = \sum_{e=1}^{N} MSE_i^e \qquad i = 1,2,\cdots,N \tag{4}$$

Where MSE_i denotes the total strain energy of the i-th order modes， N is the total degrees of freedom of the structure。

Then the strain energy normalized with respect to the unit of mode e at order i is:

$$MSE_{ni}^e = \frac{MSE_{ni}^e}{MSE_i} \tag{5}$$

When the first m-step modes of the structure are considered, the average strain energy of the first m-step modes corresponding to element e is shown in equation (3):

$$MSE_m^e = \frac{\sum_{i=1}^{m} MSE_{ni}^e}{m} \tag{6}$$

Then the modal strain energy basis index of cell i is

$$MSE_B^e = max[0, \frac{MSE_{dm}^e - MSE_m^e}{MSE_m^e}] \tag{7}$$

where MSE_{dm}^e and MSE_m^e represent the average strain energy before and after damage of element e, respectively.

The study has shown that the unit modal strain energy is more sensitive to the damage after such a treatment. The unit modal strain energy before and after damage and the rate of change of unit modal strain energy can be expressed as

$$MSE_{ij} = \frac{1}{2}\{\varphi_{ij}\}^T[K_j]\{\varphi_{ij}\} \tag{8}$$

$$MSE_{ij} = \frac{1}{2}\{\varphi_{ij}d\}^T[K_j]\{\varphi_{ij}d\} \tag{9}$$

Considering that at higher order modes, the vibration frequency of the structure will deviate from the actual situation due to the influence of noise, therefore, the average unit modal strain rate of change of the unit is used as a damage indicator, which will reduce the influence of noise, and combining with Eq. (7) to obtain Eq

$$\overline{MSECR}_i^e = \frac{1}{m}\sum_{e=1}^{m} \frac{|MSE_i^{ed} - MSE_i^e|}{MSE_i^e} \tag{10}$$

MSE_i^{ed}、MSE_i^e denote the ith order unit modal strain energy of the eth unit before and after damage, respectively，$\overline{MSECR}_i^e$ denotes the rate of change of the ith order unitary modal strain energy of the eth element，Combining Eqs. (4), (8), and (9), it can be seen that the rate of change of unitary modal strain energy of the structure is affected by the displacement column vectors of the unitary nodes. By analyzing the strain energy change of each unit of the structure, the approximate location of the damage can be determined.

(2) Strain Modal Curvature Difference Indicator

The theory of modal analysis is described in detail in [15], from a mathematical point of view, the strain modal curvature reflects the rate of change of the slope of the strain modes with respect to the position, and the strain modal curvature can be obtained by performing a center difference calculation on the strain modes.

$$\Psi_{im}^{\varepsilon} = \frac{\Psi_{i(m-1)}^{\varepsilon} - 2\Psi_{i(m)}^{\varepsilon} + \Psi_{i(m+1)}^{\varepsilon}}{\Delta^2} \tag{11}$$

Where Ψ_{im}^{ε} is the rth order strain mode shape, m is the calculation point, and Δ is the spacing of the calculation nodes.

From Eq. (5), the expression for the difference in curvature of the strain mode shapes is obtained as

$$\Delta\Psi_{im}^{\varepsilon''} = \Psi_{id}^{\varepsilon''} - \Psi_{iu}^{\varepsilon''} \tag{12}$$

Where $\Psi_{iu}^{\varepsilon''}$ is the value of strain modal curvature of the structure before damage and $\Psi_{iu}^{\varepsilon''}$ is the value of strain modal curvature of the structure after damage.

4.2 Damage identification method based on wavelet packet analysis

In bridge structure damage identification, wavelet packet analysis can be used to decompose and reconstruct the vibration signal of the bridge structure under the action of moving load, and extract the feature information related to the damage, such as the wavelet packet total energy rate of change indicator (DSI indicator). The flowchart of wavelet packet total energy rate of change indicator to identify structural damage is shown in Figure 2.

A wavelet packet is a canonical orthogonal basis library of $L^2(R)$ constructed from a family of functions from which many sets of canonical orthogonal bases of $L^2(R)$ are selected, e.g., the commonly used wavelet orthogonal bases are selected. This family of functions is the wavelet packet family, which has a small width product in the time and frequency domains and is tightly branched. The generalized two-scale equation defining the wavelet packet decomposition is

$$\omega_{2n}(t) = \sqrt{2}\sum_{k\in z} h(k)\omega_n(2t-k)\omega_{2n+1}(t) = \sqrt{2}\sum_{k\in z} g(k)\omega_n(2t-k) \tag{12}$$

Where $h(k)$ is the scale vector; $g(k)$ is the wavelet vector; and when $n = 0$, $\omega_0(t) = \phi(t)$ and

$\omega_1(t) = \varphi(t)$, defining the set of functions $\{\omega_n(t)\}_{n\in z}$ the wavelet packet determined by $\omega_0(t) = \phi(t)$.

The wavelet packet reconstruction algorithm is

$$d_l^{j,2n+1} = \sum_k (h_{k-2} d_k^{j,2n} + g_{k-2l} d_k^{j,2n+2}) \tag{13}$$

The wavelet packet decomposition of the preprocessed signal $S(t)$ is to project $S(t)$ onto the wavelet packet basis to obtain a series of wavelet packet coefficients, and by analysing these wavelet packet coefficients, the features reflecting the defective signal are obtained. The expression of $S(t)$ is given by

$$S(t) = \sum_{j=0}^{2^i-1} f_{i,j}(t_j) = f_{i,0}(t_0) + f_{i,1}(t_1) + \cdots + f_{i,j}(t_j) \tag{14}$$

Where $f_{i,j}(t_j)$ is the reconstructed signal of the wavelet packet decomposition of the laser ultrasound signal onto node (i,j).

Since the wavelet basis functions have orthogonality, the wavelet packet transform can be regarded as energy conservation, and the energy of the signal is unchanged after the wavelet packet transform, and from Bashwa's theorem, the energy of the signal component at layer i is defined as

$$E_{i,j}(t_j) = \int \left|f_{i,j}(t_j)\right|^2 \mathrm{d}t = \sum_{k=1}^{m} \left|x_{j,k}\right|^2 \tag{15}$$

Where $E_{i,j}(t_j)$ is the band energy of the wavelet packet decomposition of the laser ultrasound signal to the jth node of the i-th layer; m is the number of sampling points of the laser ultrasound signal; $x_{j,k}$ is the amplitude of the reconstructed signal $f_{i,j}(t_j)$ at the discrete sampling points.

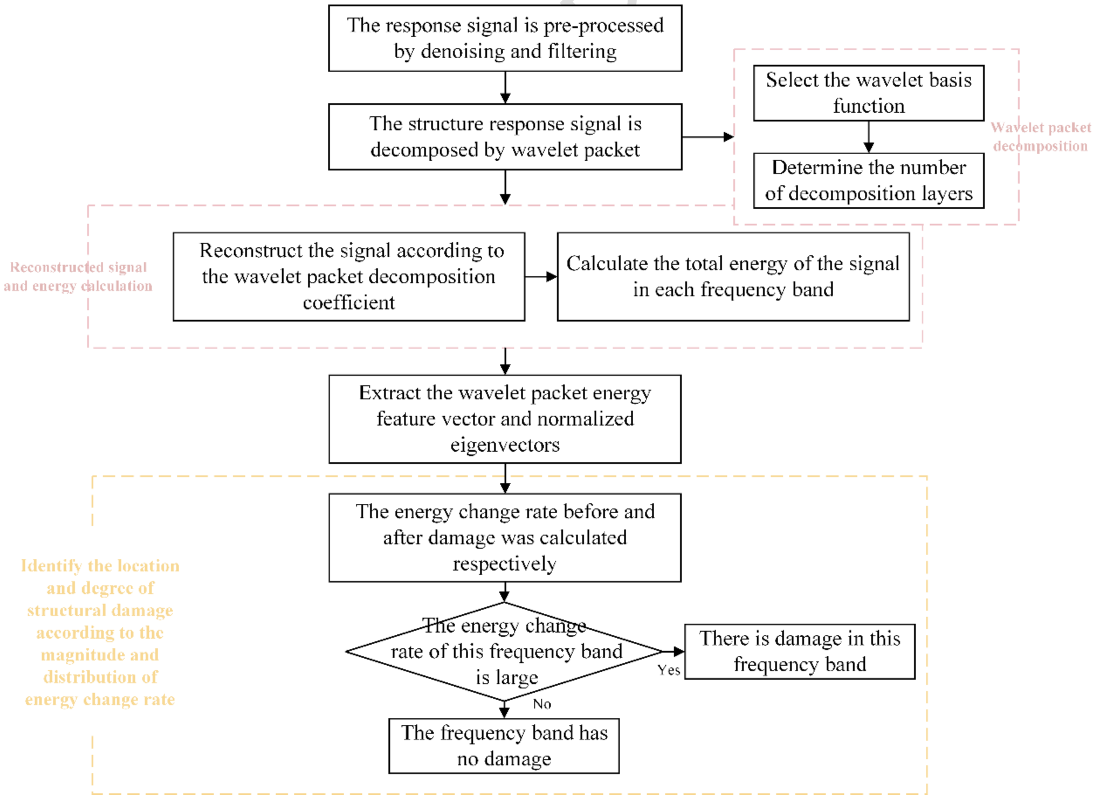


Fig. 3 Flowchart of wavelet packet energy rate of change identification of structural damage

The band energy $E_{i,j}(t_j)$ is the energy of the signal in the frequency band determined by the wavelet function, and the total energy of the signal is the sum of the wavelet packet group energies corresponding to the different frequency bands. Since the frequency band energy is sensitive to changes in signal characteristics and can be used to reveal the inherent characteristics of the signal, the wavelet packet energy rate of change metric is proposed for the characterizationof signals with different defect depths. Let the band energy of the lossless signal be $(E_{i,j})_\mathrm{h}$ and the band energy of the lossy signal be $(E_{i,j})_\mathrm{d}$, and define the wavelet packet energy change rate as

$$\Delta E_{i,j} = \frac{|(E_{i,j})_\mathrm{d} - (E_{i,j})_\mathrm{h}|}{(E_{i,j})_\mathrm{h}} \tag{16}$$

Wavelet packet analysis has the advantages of strong noise resistance and high recognition accuracy. In bridge structural damage identification, this method can effectively reduce the influence of environmental noise on the identification results and improve the accuracy and reliability of identification.

5 Challenges and Prospects

Despite significant progress in research on identifying time-varying bridge damage under moving loads, a number of challenges and problems remain.

(1) Uncertainty of Traffic Load: Traffic load is an important influencing factor in the identification of bridge damage under moving loads. However, traffic loads are characterized by randomness, diversity and uncertainty, which are difficult to accurately predict and simulate. Therefore, how to consider the uncertainty of traffic loading to improve the accuracy and reliability of damage identification is one of the problems that need to be solved in current research.

(2) Interference of environmental noise: In bridge health monitoring, environmental noise is a factor that cannot be ignored. Environmental noise will interfere with the vibration signals of bridge structures and affect the accuracy and reliability of damage identification. Therefore, how to effectively reduce the impact of environmental noise on the identification results is an important direction that current research needs to focus on.

(3) Multi-source information fusion: Combine multiple sensor technologies, signal processing technologies and machine learning algorithms to achieve the fusion and complementarity of multi-source information and improve the accuracy and reliability of damage identification. For example, acceleration sensors, displacement sensors and strain sensors can be combined to obtain more comprehensive bridge structure information; at the same time, machine learning algorithms are used to fuse and analyze the multi-source information to improve the intelligent level of damage identification.

(4) Intelligent materials and sensor technologies: Develop new intelligent materials and sensor technologies to improve the precision and reliability of bridge health monitoring. For example, new sensor technologies such as optical fiber sensors and piezoelectric sensors can be used to achieve high-precision monitoring of bridge structures; at the same time, smart materials such as shape memory alloys and carbon fiber composites can be used to improve the self-diagnostic and self-repairing capabilities of bridge structures.

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