

Research progress on the Schrödinger equation that can describe the Earth's revolution and its applications

Abstract

People believe that the Schrödinger equation cannot be used to describe macroscopic objects like the Earth, and Newtonian mechanics cannot be used to describe microscopic systems. The old concept of the relationship between the existing laws of quantum mechanics and classical mechanics undoubtedly has a serious impact on people's understanding of the natural world, the development of physics theories, and the application of existing physics theories. The continuous development of physics theory requires constant changes to some incorrect old concepts. The Schrödinger equation that can describe planetary motion was successfully obtained by replacing the potential energy in the Hamiltonian operator from electromagnetic interaction potential energy to gravitational interaction potential energy. If the distance between the sun and the earth is approximated as a constant, the energy eigenvalues obtained by solving the Schrödinger equation for the Earth's revolution are completely consistent with the results obtained directly using classical mechanics. The direct significance of establishing and applying such equations is that they can simultaneously use classical mechanics and wave dynamics to describe all objects (no longer limited by the mass of the objects), simplifying the calculation process of quantum mechanics. It has been proven that classical mechanics and wave dynamics are compatible. It has been proven that classical mechanics and wave dynamics are compatible, and there is no insurmountable gap between them. This result has a huge positive impact on the theoretical updates and applications of quantum mechanics.

Keywords: planetary model, Schrödinger equation, quantum mechanics, classical mechanics, compatibility, meaning of wave function.

1. Introduction

Can the revolution of the earth be described by Schrödinger equation? If it can, it will have a great impact on the existing theoretical physics. After this method is extended to all objects, there will be the Schrödinger equation of gravitational potential energy and the Schrödinger equation applicable to all objects. Previously, people were bound by the uncertainty of microscopic particles and the non-localized realism, and the Schrödinger equation of gravitational potential energy or the Schrödinger equation describing macroscopic objects never appeared in textbooks. Since people have never tried to establish such Schrödinger equation, it is of great significance for us to try it here. This article is an attempt, and it has been successful.

When people have to establish and apply wave mechanics or quantum mechanics, they all realize that the micro world is so different from the macro world. And recognized the notion that "the cognition, experience, rules, and theories of the macro world established by humans in theory and practice have largely failed in the micro world". So that people think that there is a huge gap between the micro world and the macro world. Specifically, the causality or determinism of Newtonian mechanics and classical electrodynamics, which are applicable to the macro world, are no longer applicable to the corresponding occasions in the micro world, but wave mechanics, natural randomness and uncertainty, which are applicable to the micro world, are not suitable for describing the movement changes of macro objects. Although there has been Ehrenfest Theorem [1] and it is the mission of condensed matter physics to

explain macroscopic phenomena with microscopic theory. But this has not fundamentally bridged the gap between the micro-world and the macro-world. Because, influenced by Ehrenfest theorem and condensed matter physics, the relationship between macroscopic objects and microscopic particles is the relationship between quantitative change and qualitative change. Only when the Schrödinger equation is applied to the macro world without being limited by mass can the gap between the macro world and the micro world be basically bridged (the difference between them is no longer dominant).

In fact, the form and application of Hamiltonian operator are not limited by the mass of the object, nor is it limited that the potential energy can only be electromagnetic interaction potential energy. Before this paper, people only used Schrödinger equation on microscopic objects (if it is a microscopic object in a bound state, the binding force is only the electromagnetic interaction force, and the potential energy is the potential energy of the electromagnetic interaction force). We can't find the mathematical and logical basis for doing this, and we have to say that this is a habit formed by the bondage of ideas. There is no theoretical obstacle or mathematical logic obstacle in using gravitational potential energy in Hamiltonian operator. In this way, according to Schrödinger's method, Schrödinger equation suitable for macro system can be established completely. The establishment of section 2 in this paper is applicable to the Schrödinger equation of gravitational potential energy of macro-system. In the third section of this paper, the newly established Schrödinger equation applicable to macroscopic objects (including gravitational potential bound state system) is verified by using the known data of the earth's revolution.

Since the planetary model and Schrödinger equation can be used to describe an object at the same time, there is no obstacle to using the planetary model in the microscopic system to which Schrödinger equation applies. That is to say, both microscopic and macroscopic systems can use planetary model (or classical mechanics) and wave mechanics at the same time. This gives birth to the power that can make people change their existing ideas. Have readers heard of (or seen) the Schrödinger equation of the earth's revolution? If not, then the author's research work may lead a new trend. The successful establishment of Schrödinger equation of planetary motion shows that we have a theoretical basis for using wave mechanics and classical mechanics at the same time.

In reference [2-7], the author calculates atoms and molecules by using wave mechanics and planetary model at the same time, which shows that the viewpoint, theory and method that "wave mechanics and classical mechanics can be used to describe the motion system at the same time" have a wide application prospect. Wave mechanics and classical mechanics are used to describe a system at the same time, which is not limited by the mass of the system. The natural rules accumulated by the long experience in the past will not completely fail even in the micro world. This is a major conceptual revolution in the history of human development. It will also lead to a great revolution in the theory and method of basic physics.

The basic assumption in references [6-10] that "specific waves propagate along a small circle to form electrons" lays the foundation for the conclusion that "classical mechanics and quantum mechanics are compatible and can be used simultaneously on macroscopic and microscopic objects". Reference [10] derived the Schrödinger equation that can describe the Earth's orbital motion. This article is an expanded description of the content in section 3 of reference [10]. This article introduces the significance and application of the Schrödinger equation. It belongs to the category of macroscopic system Schrödinger equation and its application research progress.

The theoretical basis of this article is based on references [10,11]. After Bohr proposed the hydrogen atom planetary structure model, there was a certain connection between the old quantum theory and the classical planetary structure model. Unfortunately, it is widely believed that quantum mechanics cannot describe macroscopic systems, while classical mechanics cannot describe microscopic systems. Denying the compatibility between classical mechanics and quantum mechanics. And gradually formed this stubborn ideological concept. This closes the door to the idea of using the Schrödinger equation to describe the classical motion of macroscopic objects. Lost the opportunity to use quantum mechanics methods to describe

classical planetary motion. It was not until the author of this article published multiple academic research results demonstrating the compatibility between classical mechanics and quantum mechanics [4-6], and simultaneously calculated the physical and chemical parameters (such as ionization energy, dissociation energy, bond length, etc.) of multiple atoms (ions) and small molecules using both classical mechanics and quantum mechanics methods, that the issue of compatibility between classical mechanics and quantum mechanics was reconsidered. In June 2024, the author of this article finally derived the Schrödinger equation for planetary motion [7,8].

It is the wave element electronic structure model [9-12] that resurrects the planetary model in the microscopic field. The significance of establishing and using the Schrödinger equation to describe planetary motion is not that this method is more accurate than mechanical methods, but that the establishment of this method can change people's old concepts (i.e., the idea that classical mechanics and quantum mechanics are incompatible), leading to a revolution in the interpretation system of quantum mechanics and ultimately promoting the development of quantum mechanics and even the entire physics. This research achievement is comparable to de Broglie's previous work. At that time, no one realized that the wavelength momentum relationship in the wave law also applied to particles with non-zero rest mass (especially macroscopic moving objects). However, de Broglie believed that the wavelength formula also applies to particles with zero rest mass (even moving macroscopic objects). Now, people think that the Schrödinger equation cannot be used to describe macroscopic objects, but I think it can. Our similar research work can both change people's mindset. We can wait and see if it can trigger a revolution in physics.

2. Schrödinger equation that can describe the revolution of the earth

In mechanics, the potential energy of phase interactions takes negative values, while the kinetic energy takes positive values. Therefore, for the interaction potential energy, it is represented by the algebraic symbol V , without a negative sign before V . If expressed using a specific calculation formula, a negative sign needs to be added before the formula. The following four expressions of the Hamiltonian operator are correct. $\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V$, $\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} - \frac{Zee^2}{r}$, $\hat{H} = -\left[\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} - V\right]$, $\hat{H} = -\left[\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{GMm}{R}\right]$. It is recommended that readers focus on checking the absolute values of the calculation results when examining the expressions and calculations in this article. This can save a lot of energy.

The stationary Schrödinger equation for hydrogen atoms is

$$-\left[\frac{\hbar^2}{2m_e} \nabla^2 + \frac{Ze^2}{r}\right] \psi = E_e \psi. \quad (1)$$

Its one-dimensional form is

$$-\left[\frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial x^2} + \frac{Ze^2}{r}\right] \psi = E_e \psi. \quad (2)$$

Where ψ is often called wave function. One of its forms is

$$\psi = Ae^{-i2\pi(vt-x/\lambda)}. \quad (3)$$

In this paper, letters with subscript e represent the physical quantity of electrons. The existing mathematical formal system and interpretation system of quantum mechanics believe that there is an insurmountable gap between the micro-world and the macro-world in terms of the laws or performances that things follow. One of the concrete expressions is that the macroscopic object cannot use wave mechanics (including Schrödinger equation), while the microscopic object does not conform to the classical mechanical theory and rules. However, as long as we analyze it

carefully, it is not difficult to see that the first term of Hamiltonian operator used to establish Schrödinger equation is the kinetic energy operator, and the second term is the potential energy operator (also the potential energy function itself). In the solar system, the bound earth also has kinetic energy and potential energy, and it conforms to Virial theorem. Nowadays, quantum mechanics does not limit the mass of moving objects that conform to de Broglie wave formula. We have no reason to say that we can't use Hamiltonian operator to describe macroscopic objects. The reason why Schrödinger equation uses wave function is unknown, but it is very useful to use wave function in reality. Particles such as electrons and macroscopic objects are entities with static mass. We have no reason to say that wave functions can only be used to describe microscopic objects, but not to describe macroscopic objects. Because, for de Broglie wave, only its wavelength is related to the quality, but there is no upper limit of the quality (that is, the macroscopic object with great mass also has the corresponding de Broglie wave).

The known de Broglie relationship is $\lambda=h/(m v)$ The group velocity of moving physical particles and/or macroscopic objects is also the velocity of the center of motion of particles and macroscopic objects. Let's redefine $v=\lambda \nu$. Among them are the frequencies of de Broglie waves of macroscopic objects or physical particles in motion. According to these two formulas, we know $h\nu=mv^2$. Considering that $E_k=\frac{1}{2}mv^2$, we have $h\nu=2E_k$.

In Hamiltonian, the V is not limited in the range of electromagnetic interaction potential energy. We also have no reason to think that the first term in Hamiltonian operator can only be applied to microscopic objects. It can be seen that we have no reason to exclude the following choices: the first term in Hamiltonian operator is also applicable to macroscopic objects, and the second term can be both electromagnetic interaction potential energy function and gravitational potential energy function. Under this premise, we can completely describe the macroscopic object by Schrödinger equation, and the Schrödinger equation used to describe the revolution of the planet is formula (4). The scope of application of Schrödinger equation, which was originally only applicable to the interaction between microscopic objects and electromagnetism, has been extended to the macro system, and it is also applicable to the four basic interaction bound state systems. This paper does not discuss the basic interaction system other than electromagnetic interaction system and gravitational interaction system.

$$-i\frac{\hbar}{2}\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{GMm}{R}\psi = E\psi. \quad (4)$$

Where r is the distance between the earth and the sun, and e is the energy of the revolution of the earth. When the potential energy $V=0$, equation (4) is applicable to unbound macroscopic systems. Generally speaking, the logical idea of establishing equation (4) is that since there is no reason why we can't use Hamiltonian and wave function in the macro system, we might as well try to use the wave equation (Schrödinger equation) to describe the macro system in which the bound state is maintained by gravity. In the equation, E is the energy eigenvalue of a classical system bound by gravity. (4) The formula is not applicable to particles with zero rest mass, but to macroscopic systems in bound states. When $V=0$, equation (4) still applies to unconstrained macroscopic systems [see equation (8) for details]. Simply put, the logical idea for establishing equation (4) is that since there is no reason why we cannot use Hamiltonian operators and wave functions in macroscopic systems, we may try to use the wave equation (Schrödinger equation) to describe macroscopic systems where bound states are maintained by gravity (the description of unbound state systems is of course simpler). For the stationary Schrödinger equation of the Earth's revolution, there is no denominator 2 in the leftmost term of equation (4). Readers can verify it themselves. If there must be a denominator of 2, it indicates that the original Schrödinger equation is incorrect. If the denominator does not have that 2, then the table indicates that equation (8) is incorrect. This is a very serious issue that must be taken seriously.

When R is the distance between the earth and the sun and E is the energy of the earth's revolution state, equation (4) is the Schrödinger equation of the earth's revolution observed on a curved surface. When R is the distance between

the earth and the sun and E is the energy of the earth's revolution state, equation (4) is the Schrödinger equation of the earth's revolution observed on a curved surface. In this way, R is also known. Equation (4) need not be solved, but only the known constants can be substituted into Equation (4) to calculate the kinetic energy and potential energy of the earth's revolution and the total energy E of the earth's revolution. Using the three-dimensional form of Equation (4) to calculate the energy eigenvalues of the earth is unnecessary.

3. Verification of the Schrödinger Equation of the Earth's Revolution

Considering de Broglie relation $\lambda=h/p=h/mv$, classical momentum formula $p=mv$, $v=\lambda\nu$, $\hbar=h/2\pi$ and Eq. (2), the value of the first term on the left of the equal sign of Eq. (4) is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = \frac{\hbar^2}{2m}\left(\frac{2\pi}{\lambda}\right)^2\psi = \frac{mv^2}{2}\psi. \quad (5)$$

The coefficient $\frac{mv^2}{2}$ is the sum of kinetic energy, when the earth revolves, and v is the revolution speed of the earth.

According to the relationship that the centripetal force of the earth's revolution is equal to the attraction of the sun to the earth, $m=rv/g$. Therefore, the value of the second item on the left of equation (4) is

$$-\frac{GMm}{R}\psi = -mv^2\psi. \quad (6)$$

Substituting the expressions (5) and (6) into the expression (3) and eliminating ψ , we get the energy eigenvalue of the system

$$E = \frac{mv^2}{2} - mv^2 = -\frac{mv^2}{2}. \quad (7)$$

Among them, $-mv^2$ is the potential energy of the earth's bound motion, and it is exactly twice the kinetic energy of the earth's revolution (which shows that the Schrödinger equation of the earth's revolution guarantees the establishment of the Virial theorem). This is exactly the same as the result calculated according to classical mechanics.

In this way, as shown in the second equation of equation (7), the earth energy e in equation (3) is equal to $-\frac{mv^2}{2}$.

This is the classical expression of the energy of the earth's revolution, and it is the energy eigenvalue solution of equation (4). Obviously, we have proved that equation (3) holds, and we can use Schrödinger equation to describe the revolution of the earth.

4. Schrödinger equation of planetary model of hydrogen atom and its verification

Since the physics of macroscopic objects such as Earth can be described using both the Schrödinger equation and the planetary model, there is no reason to restrict the simultaneous use of the Schrödinger equation and classical mechanical models (of which the planetary model is one) to describe microscopic systems such as hydrogen atoms. We can use both the planetary model and the Schrödinger equation to describe the hydrogen atom. We still choose to observe hydrogen atoms on a curved surface (using a planetary model, the orbital motion of electrons is similar to the orbital motion of planets. We can observe hydrogen atoms in Riemannian space). Use equation (2) to describe the orbital motion of electrons in hydrogen atoms, where the first term on the left side of the equation corresponds to the kinetic energy E_k of the electron orbital motion. The Schrödinger equation in atoms also conforms to equation (5). No way, m_e is the mass of the electron, and $mv^2/2$ is the kinetic energy of the electron in the hydrogen atom (where v is the group velocity of the electron, i.e. the group velocity of the electron's de Broglie wave). When $V=0$, equation (4) becomes

$$i\frac{\hbar}{2}\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = \frac{1}{2}mv^2\psi = E_k\psi. \quad (8)$$

In the equation, $\frac{m_e v^2}{2}$ is the kinetic energy of the electrons in the hydrogen atom (where v is the group velocity, i.e.

the group velocity of the electron's de Broglie wave). The potential energy of an electron is $-\frac{Ze^2}{r}$. When r is constant value a_0 , When the potential energy of the system is zero, it is not simply to eliminate the potential energy in equation (4), but to consider the properties of the calculated energy, that is, to take into account the sign of the calculated result. The result calculated according to equation (8) is kinetic energy, while equation (4) calculates the energy eigenvalue of the system. This is the reason why the first term in equation (8) does not have a negative sign. The ground state energy eigenvalue of the hydrogen atom solved according to equation (2) is $-\frac{Ze^2}{a_0}$. According to the force law of uniform circular motion of bound state

$$m v^2 = \frac{Z e^2}{a_0}, \quad v = \sqrt{\frac{Z e^2}{a_0 m_e}}. \quad (9)$$

According to the two relationships of $a_0 = \frac{h}{m_e c \alpha}$ and $\alpha = \frac{e^2}{h c}$, by eliminating a_0 and e^2 in equation (9), we can obtain $v = \alpha c$. Here, $v = \alpha c$ is not only the planetary orbital velocity of ground-state hydrogen atom, but also the group velocity of de Broglie wave of electrons in ground-state hydrogen atom. Although equation (9) contains the solution of wave mechanics equation, it conforms to classical mechanics theory. This proves once again that classical mechanics and quantum mechanics can be compatible (The previous proof is that they are compatible in the macro field, and this section proves that they are compatible in the micro field).

5. Exploration on the essence of wave function ψ

Schrödinger used the wave function (4) in those days, but he didn't know its true meaning, just regarded it as a part of mathematical tools. After him, no one clarified the essence of wave function ψ . There is still an unsolved mystery about the nature of matter wave. The first formula in the Eq.(9) accords with the classical mechanical theory.

Macroscopic objects moving in a straight line can also be described by Schrödinger equation. For example, the Schrödinger equation of a train moving in a straight line at the speed v_t is

$$-\frac{\hbar^2}{2m_t} \frac{\partial^2}{\partial x^2} \psi = E_t \psi. \quad (10)$$

Solving this equation is very simple, just need to find the partial derivative, and use the formula (4) and the de Broglie relation: $\lambda_t = h/p_t = h/(m_t v_t)$. The solution of equation (10) is $E_t = E_k = \frac{1}{2} m_t v_t^2$. This solution shows that the energy eigenvalue solution of an object in linear inertial motion is the non-relativistic kinetic energy of this object. Comparing this solution with the velocity-frequency relation $v_t = \lambda_t \nu_t$ of monochromatic wave, we can get

$$E_t = \frac{1}{2} h \nu_t. \quad (11)$$

Equation (11) shows that the energy of the object's matter wave is twice its kinetic energy. This does not conform to the law of real monochromatic waves. It shows that the matter wave of an object cannot be a complete wave. This result is obviously beneficial to explore the essence of matter wave.

Since the major obstacle of "describing macroscopic objects by wave mechanics" has been removed by establishing the available Schrödinger equation of planetary motion, we can calculate how many deuterium-like atoms are contained in the earth by using the wave mechanics method of microscopic system (deuterium atoms are calculated as simulated cells that make up macroscopic objects). The result must be very interesting.

The angular momentum of an object moving in a circle is $L = r \times p$. Replacing P with the momentum operator $\vec{L} = \vec{R} \times \vec{p}$, we get the operator of the orbital angular momentum of the electrons in the hydrogen atom.

$$\hat{L}_N = -i N \hbar R \frac{\partial}{\partial x}. \quad (12)$$

Adding footnote N only emphasizes the description of N unit objects. By applying it to Apollo function ψ [see formula 3], we can get

$$-i\hbar NR \frac{\partial}{\partial x} \psi = \hbar N \frac{R}{r} \psi. \quad (13)$$

Eliminate ψ in the above formula, and the angular momentum formula of the macroscopic object or the simulated composite particle with hydrogen atom as the unit to do the circular motion of the bound state.

$$L_N = N\hbar \frac{R}{r}. \quad (14)$$

$L_N = Rmv = 2.658 \times 10^{37}$ (m·kg·s). The mass m of the earth is 5.965×10^{24} kg. The distance r between the sun and the earth is 149597870 kilometers (1.496 million kilometers). The average revolution linear velocity of the earth v is: 29.783 km/s (107,220 km/h). For describing the earth's revolution with wave function, r in wave function ψ and (14) is equivalent to the radius a_0 of hydrogen atom ($a_0 = 5.2918 \times 10^{-11}$ meters). $\hbar = 1.054571726 \times 10^{-34}$ J·s. We substitute these numbers into equation (14) and we can get

$$N = \frac{La_0}{\hbar R} = 8.734 \times 10^{50}. \quad (15)$$

It is not difficult to see that the product of the mass of N and the deuterium atom as the mass cell of macroscopic substances (including simulated composite particles with deuterium atom as the smallest unit and other compounds) is the mass of the earth. The mass of deuterium atom is 3.3688×10^{-27} kg. In this way, the mass of the earth calculated by wave mechanics method is

$$m_{\text{earth}} = N \times 3.3688 \times 10^{-27} \text{kg} = 2.942 \times 10^{24} \text{kg}. \quad (16)$$

This calculated value is of the same order of magnitude as the known Earth mass value of 5.965×10^{24} kg. There are two main sources of error: first, the molecular structure of the earth is complex, and deuterium atoms cannot be accurately used as simulated mass cells of the earth; Second, other energies contained in the earth and the sum of binding energies in molecules are not considered.

The mass of the earth is calculated by wave mechanics, which proves that wave mechanics is effective in dealing with planetary motion. The results of this analysis can at least remind us why we can use the wave function ψ when describing macroscopic and microscopic objects. It also supports the theory of wave element material structure proposed in references [3-6].

6. Summary

The revolution of the earth is absolutely in a plane, and in a certain period of time, R is a certain value. If we assume that the ball moves randomly in three-dimensional space (it is an uncertain motion without orbit), and solve the equation similar to Eq. (1), there will definitely be extraneous root. If the state of the electrons in the hydrogen atom is certain, the calculation by using the three-dimensional Schrödinger equation will also lead to unrealistic root growth. The research results of this paper increase this possibility.

A conclusion of Section 4 is that the classical mechanical method and the quantum mechanical method are compatible and can be used at the same time, whether describing macroscopic objects or microscopic objects. For convenience, we call this conclusion conclusion 1. The use of classical mechanical methods means that the described object is deterministic, realistic and localized, and conforms to determinism. It can be seen that conclusion 1 cannot absolutely deny that micro-objects are also deterministic, localized and causal (only in a narrow range or under certain conditions can people show uncertainty, non-localization, unreality and indecision). This is an important inference according to conclusion 1.

Conclusion 1 and its inference show that the gap between macro-system and micro-system can be eliminated or reduced. Wave mechanics and classical mechanics can be used to describe objects from micro to macro at the same time, which can simplify the calculation process. Determinism, localization, realism and determinism cannot be

completely denied in the microscopic system. This obviously has great influence on the interpretation system of quantum mechanics. The obstacles to establishing localized real quantum mechanics are also much smaller.

At that time, Schrödinger did not explain the reason why he used the wave function of formula (4) in the Schrödinger equation of hydrogen atom [that is, he did not specify the meaning of formula (4), but only used it as a tool]. We can be sure that the revolution of the earth is definitely not a wave like Eq. (4). This paper proves that "the correct result can be obtained by using equation (4) when describing the revolution of the earth". This result further strengthens the concept that "wave function ψ is a tool in wave mechanics". Unless both microscopic and macroscopic objects are made of waves. If there are no particles but waves in the constituent elements of matter, we can start to establish the theory of wave element material structure.

It is recognized that "human beings have not yet loved the combination of relativity (or gravity theory) and quantum mechanics." This paper proves that we can use Newton's mechanics and quantum mechanics at the same time (that is, Schrödinger equation, the basic equation of Newton's gravitational interaction potential energy and wave mechanics) to describe the motion and microscopic system of celestial bodies. This is the compatibility and combination of Newton's gravity theory and quantum mechanics to a certain extent (although gravity is not quantized, it is combined in another way).

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