Comparing ARFIMA and ARIMA Models in Forecasting Under Five Mortality Rate in Tanzania

ABSTRACT

Tanzania has been taking various measures to drop the Under-Five Mortality Rate (UFMR), but the pace to meet national and global UFMR targets has been slow. Nevertheless, the decline for the past years has continued to be low as compared to the Sustainable Development Goals (SDGs) target which is set at 25 deaths/1000 live births by 2030. The lack of statistical modeling-based forecast values of UFMR results into setting targets that are not SMART towards realization of national and international goals of the health sector. Thus, the current study uses both ARFIMA and ARIMA to make forecasts of UFMR in Tanzania from 2021 to 2030 by using data extracted from the World Databank - World Development Indicators (WDI). Also, an accuracy comparison between the ARFIMA and ARIMA best-fit models in forecasting UFMR was conducted. The forecasts from the best ARFIMA (1, 0.284243, 2) model indicate that by June 2026 the rate will on average be 41 deaths/1,000 live births as compared to the Tanzanian Five Year Development Plan Phase III (TFYDP-III) target of 40 deaths/1,000 live births; whereas the best fit ARIMA (1, 2, 0) model forecasts depict that the rate will be 40.1 deaths/1,000 live births as compared to the TFYDP-III target. In relation to the UN SDGs target of 25 deaths/1,000 live births by 2030, the ARFIMA (1, 0.284243, 2) model forecast values indicate that by 2030, Tanzania will experience a decrease in UFMR to 35.2 deaths/1,000 live births. The ARIMA (1, 2, 0) forecast values indicate that by 2030, Tanzania will experience a decrease in UFMR to 32.9 deaths/1,000 live births. By using RMSE and MAPE forecasting model accuracy measures, the results reveal that the ARFIMA (1, 0.284243, 2) model performs better than ARIMA (1, 2, 0) in forecasting UFMR.

Keywords: Forecasting, mortality rate, integrated moving average, Autoregressive, ARFIMA, public health.

1. INTRODUCTION

The Under Five Mortality Rate (UFMR) is the most common indicator of health sector performance as well as a basis for formulating public health policies and frameworks. In addition to monitoring the deaths of children under five resulting from various causes, UFMR can also reflect other social conditions such as unequal access to health care (Lutambi et al., 2010; Dlamini et al., 2024; Zhang et al., 2023; Mwanga et al., 2024).

All countries are concentrating on lowering the UFMR to 25 deaths/1,000 live births by 2030 to achieve SDG 3.2.1, which calls for the eradication of all cases of preventable deaths of children under the age of 5. According to the SDGs, 2020 report, 125 countries worldwide have already achieved the SDG objective, and 16 more are expected to do so if the present trend continues. If not, 54 countries will need to step up their efforts if they want to avoid not meeting the target by 2030 given the current trends. Without accounting for the additional challenges brought on by the COVID-19 pandemic, 25 of these nations will need to triple their current rate of UFMR reduction (UN, 2019). According to the UN (2019), the 54 off-track countries will reduce the number of children under five by 8 million between 2021 and 2030, bringing the overall number of children under five to 2.5 million by that time. Deliberate efforts are still required in sub-Saharan Africa and southern Asia to drop UFMR through the engagement of various health interventions. In 2020, the UFMR for sub-Saharan Africa stood at 74 deaths/1,000 live births, and to achieve the SDGs target of 25 deaths/1,000 live births by 2030, the rate must go down by 66%, which is equivalent to a 6% decrease per year.

According to the 2021 UNICEF Infant Mortality Report, UFMR worldwide has declined from 93 deaths/1000 live births in 1990 to 37 deaths/1,000 live births in 2020 and to achieve the target of SDG of 25 deaths/1,000 live births by 2030, the rate has to drop by 33%, which is equivalent to 3% decline annually (Salman & Aboudi, 2022). Table 1 shows that in 2020, UFMR in the region experienced 74 deaths/1,000 live births. This rate was 14 times the risk of children in Europe and North America and 19 times that of Australia. The UFMR for children born in low-income countries in 2020 was 66 deaths/1,000 live births one times higher at risk than children born in middle-income countries, while for higher income countries UFMR was 5 deaths/1,000 live births. At the nation level, UFMR in 2020 extended from 2 to 115 deaths/1,000 live births, with a child born within the highest-mortality nation suffering a 65-fold higher chance of a child passing away than a child born within the lowest-mortality nation (UNICEF et al., 2020).

The WHO site (https://www.who.int/news-room/fact-sheets/detail/levels-and-trends-in-child-under-5-mortality-in-2020) has highlighted that the driving causes of deaths in children aged below 5 are pre-mature birth complications, birth asphyxia/trauma, pneumonia, the runs and intestinal sickness, all of which can be avoided or treated with rational medication and health interventions. In responding to the problem, the WHO calls on all nations to improve social wellbeing to all children so that they can get to fundamental health services under minimum cost. Moreover, multisectoral endeavors are required to overcome imbalances and any negative impacts from social well-being determinants.

Apart from implementing the global health performance indicators, the United Republic of Tanzania (URT) has been setting targets for all health performance indicators including UFMR in its Five Years-Based Development Plans. For example, the implementation report of the Second Phase Five Year Development Plan II (FYDP II), shows that UFMR declined from 67 per 1,000 live births in the Financial Year 2015/16 to 48.9 deaths/1,000 live births in the Financial Year 2019/20. The implication of this rate in relation to SDGs is that, in order to achieve the target of 25 deaths/1,000 live births by 2030, the rate of UFMR has to go down by 49%, which is equivalent to 4% decrease annually. Moreover, the Third Phase Five-Year Development Plan 2021/22 – 2025/26 (FYDP III 2021/22 – 2025/26) has earmarked UFMR as one of the seven Key Indicators (KI) of the public health sector performance in Tanzania. Table 2 indicates the performance of each indicator as of June 2020 and the anticipated target by June 2026 (MoFP, 2021). Furthermore, the Tanzania Mortality and Health Monograph of the 2012 Census, indicates that the overall UFMR stood at 66 deaths/1,000 live births and the mortality level for male children under five years of age stood at 73 deaths/1,000 live births and for females was 60 deaths/1,000 live births (NBS, 2015).

Currently, the URT has been setting targets for health sector indicators including UFMR basing on available baseline data and interventions such as improved vaccinations, training more health care attendants and increasing the number of health centers all over the country. However, the current targets for health indicators including UFMR are not derived from statistical modeling and forecasting, therefore they lack statistical jurisdiction for planning and policy formulation in the health sector. In ensuring that the national and world targets are SMART, decent forecasts of health performance indicators including UFMR are needed. Due to that fact, this study deliberately intended to develop the best fit Autoregressive Fractionally Integrated Moving Average (ARFIMA) and Autoregressive Integrated Moving Average (ARIMA) models for forecasting UFMR in Tanzania and compare the forecasting accuracy between the two models (Wiri et al., 2022). The selection of the model is based on the fact that ARFIMA deals with both short and long-range memory time series data, while ARIMA deals with short-range memory only.

2. MATERIAL AND METHODS

2.1 Location of the Study

The study is based in the United Republic of Tanzania which includes all 26 regions of Tanzania main land and Zanzibar.

2.2 Research Design

Time Series Analysis (TSA) is a method reasonable for longitudinal designs of research. They contract with single subjects or units that are measured more than once at a standard interval of time (Ensor, 2002). Hence, the study utilizes longitudinal design since it is working with a big number of data points (61 data points of UFMR from 1960 to 2020 (Creswell, 2014).

2.3 Source of Data

Secondary data from 1960 to 2020 (61 data points) extracted from the World Databank - World Development Indicators (WDI) via link *https://data.worldbank.org/indicator/SH.DNY.MORT?loacations=TZ* and was used in the study. The World Databank is a reliable open source of data and other researchers like Eke and Ewere (2020a) used data from the source in modeling and forecasting UFMR in Nigeria using 59 UFMR data points (from 1960 to 2018) by employing the ARIMA model.

2.4 Data Analysis Plan

Thus, the entire data analysis process is facilitated by R Software (**Version 4.2.1**) commands from ARFIMA and ARIMA packages. The UFMR dataset is disintegrated into two portions, whereby the first 51 (1960 to 2010) data points were used in the course of ARFIMA and ARIMA models development and the last 10 (2011 to 2020) data points were used for checking the forecasting accuracy of the best-fit models.

The ARFIMA model is derived as follows:

If X_t is a stationary time series with zero mean and with long-range dependence, then the mathematical model for ARFIMA (p, d, q) is given by:

$$\left(1-\sum_{i=1}^{p}\Phi_i C^i\right)\left(1-C\right)^d X_t=\left(1+\sum_{i=1}^{q}\theta_i C^i\right)\varepsilon_t \qquad (1)$$

Whereby C = backshift operator, d = differencing parameter, $(1-C)^d$ = difference operator, Φ_i = Autoregressive (AR) parameters, θ_i = Moving Average (MA) parameters, $\left(1-\sum_{i=1}^{p}\Phi_i C^i\right)$ = AR polynomial of order p and $\left(1+\sum_{i=1}^{q}\theta_i C^i\right)$ = MA polynomial of order q. However, on expansion, the ARFIMA (p, d, q) can be generalized by the following mathematical equation:

$$\Phi(C)(1-C)^d X_t = \theta(C)\varepsilon_t \qquad (2)$$

In the course of developing the ARFIMA (p, d, q) models, the following sub-tasks are carried out:

a) Testing for Long Memory Dependency Existence in the UFRM Data

Erfani and Samimi (2009) make an emphasis on several procedures that have been established for testing the existence of long memory in the time series data. But for the purpose of this study, Rescaled Range Analysis (R/S) and ACF Plots were used for LMD in the data.

i) Rescaled Range Analysis Method

Rescaled Range Analysis aims at examining the existence of autocorrelations in time series (Erfani & Samimi, 2009). The theory behind R/S analysis is to study the behaviour of the rescaled cumulative deviations from the mean. The statistic for R/S is computed using the following formula:

$$R = Max_{k=1,2,...,n}\sum_{i=1}^{k}\left(X_i-\overline{X}\right) - Min_{k=1,2,....n}\sum_{i=1}^{k}\left(X_i-\overline{X}\right) \qquad (3)$$

Whereby R stands for ranges of accumulated deviations of a series X_t over a period of length n and $\overline{X}$ is the grand mean of the series.
We know that the ordinary standard deviation estimator is given by:

$$s = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{k}\left(X_i-\overline{X}\right)^2\right)} \qquad (4)$$

If the sample size enlarges and on simplicity, the following relationship is produced:

$$log\left[\frac{R}{s}\right] = l\log\alpha + H * l\log n \qquad (5)$$

From the above equation (3.3), it is obvious that the estimate of H, which is a gradient, gives the mean values $\left(\frac{R}{s}\right)$ of sample groups of the same size to the number of observations within each sample group.

The Hurst Exponent Statistics for UFMR data are executed by R software using the function "**hurstexp ()**" and the interpretation of results is based on theoretical Hurst Exponent Statistic.

ii) ACF Plots Method

When a time series is non-stationary with ACF Plots decreasing not exponentially and gradually, such kind of series is termed as long memory series (Safitri et al., 2019). In this study the ACF plots of the original UFMR data were plotted by using "**acf ()**" function in R software and visualized whether there was a declining trend in UFMR.

b) Checking for Stationarity of UFMR Data

A time series is said to be stationary if the mean, variability, and covariance between two time periods depend solely on the interval between the two time periods and on the time at which the covariance is computed (Gujarati & Poter, 2009). In this study, the stationarity of UFMR data was studied by Time Plots; ACF, and PACF Plots; and verified by the commonly used Unit Root test known as the Augmented Dickey-Fuller (ADF) test.

i) Time Plots and ACF Plots

A time plot is the most used graphical method whereby the data points are plotted over time. It directly reveals any trends over time, any regular seasonal behavior, and other systematic features of the data. All these need to be identified so that they can be incorporated into statistical modeling.

On the other hand, a plot of the ACF is a standard tool in exploring a time series before modeling and forecasting and it offers a useful check for seasonality, cycles, and other time series shapes by examining the autocorrelations of the series with itself, lagged one period, two periods, and so forth. In this regard, the study used time plots and ACF plots as primary tools in examining the stationarity in the UFMR data.

ii) ADF Test

The stationarity of UFMR data for both general and sex-wise was checked by ADF test with the following hypotheses:

H_o: The UFMR data are non-stationary (unit root exists), versus

H_1: The UFMR data are stationary (unit root does not exist)

The decision rule is that, if the computed p-value is greater than 0.05 level of significance, then we fail to reject the null hypothesis, hence concluding that the UFMR data is non-stationary and vice-versa.

c) Estimating Fractional Differencing Parameter (d)

A time series is considered to have long-range memory or dependence if the differencing parameter (d) lies in the range of $-0.5 < d < 0.5$ whereby if $0 < d < 0.5$ then the series is stationary and the autocorrelations are positive and hyperbolically dropping to zero, implying existence of long memory; $0.5 < d < 1$ for a non-stationary time series; and if $-0.5 < d < 0$ (when d is negative) the process is said to be anti-persistent or has intermediate memory (Ensor, 2002)**.**

During the study, the differencing parameters (d) were estimated by Geweke and Porter-Hudak (GPH) method and the computation was aided by R software through "fdGPH ()" function.

2.5 ARFIMA Model Selection

The best-fit models are chosen based on the smallest values of AIC.

2.6 Residual Analysis for the Best Fit ARFIMA (p, d, q) Model

Residual analysis helps in checking if the fitted model is adequately capturing the information in a given dataset. The best-fit forecasting model must possess the following properties:

i) The residuals should be uncorrelated. If residuals are correlated, then there is information being left in the residuals that should be used in capturing forecasts.
ii) Residuals should have zero mean, that is $E(\varepsilon)=0$; if residuals have a mean different from zero, then the forecasts shall be biased.
iii) Residuals should have constant variance.
iv) Residuals should be normally distributed.

The above assumptions were checked using time plots ACF of residuals to ensure that the selected models are adequate.

Also, Portmanteau tests designed to test whether a set of autocorrelations (r_k) values is significantly different from a zero set were performed. These tests include Box-Pierce and Ljung-Box tests, but since the Ljung-Box test is an advancement of the Box-Pierce test, the recent study employed the Ljung-Box test for analyzing the residuals of UFMR fitted models (Shalalfeh et al., 2019).

The Ljung-Box test is a diagnostic measure of white noise for a time series by assessing whether there are patterns in a group of autocorrelations. The test statistic is given by:

$$Q^{*}=n(n+2)\sum_{k=1}^{h}\frac{r_k^2}{n-k}\sim\chi_{h-m}^{2} \qquad (6)$$

Where n stands for the number of observations in a time series; h is the maximum lag being considered and m is the number of parameters in the fitted model.

Hypotheses:
H_0: ACFs = 0 (ACF patterns are white noise)
H_1: ACFs ≠ 0 (ACF patterns are not white noise)
The decision rule is that**,** if the p-value is greater than 0.05 level of significance, the null hypothesis is not rejected and it is concluded that the ACF patterns are white noise and vice-versa. The Ljung-Box test is executed using the function "**box. test ()**" in R software.

2.7 The Best Fit Autoregressive Integrated Moving Average (ARIMA) Model for Forecasting UFMR

According to Gujarati and Poter (2009), in building the ARIMA (p, d, q) model for forecasting UFMR, the Box-Jenkins modeling steps namely; data preparation, model selection, estimation of model parameters; performing diagnostics for adequacy of the model and using the model forecasting future values, were followed by the researcher. ARFIMA is an extension of the ARIMA model and applies the same steps in its modeling as in ARIMA except for the differencing parameter, whereby the differencing parameter for ARIMA is the number of differences taken to the original data to become stable. The steps that are accomplished in constructing the ARIMA model are an estimation of model parameters and diagnostics or checking for adequacy of the fitted model.

2.8 Comparing the Accuracy of ARFIMA (p, d, q) and ARIMA (p, d, q) Models in Forecasting UFMR

To compare the performance of ARIMA (p, d, q) and ARFIMA (p, d, q) in Forecasting UFMR in Tanzania, the study employed various validation measures to both best-fitted models. The two commonly used measures which are Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) were employed as comparison tools.

a) Root Mean Square Error (RMSE)

The smaller the value of RMSE, the better the model for forecasting future values. The theoretical formula for RMSE is given by:

$$RMSE = \sqrt{\left(\frac{1}{n}\sum_{t=1}^{n}\varepsilon_t^2\right)} \qquad (11)$$

Where $\varepsilon_t = X_t - X_t$, X_t = observed value, X_t = forecast and n = number of observations.

b) Mean Absolute Percentage Error (MAPE)

MAPE is one of the frequently used measure for forecasting performance of models. Its coefficient measures the mean absolute percentage error of prediction (Chu, 2008). It is desirable that for a good forecast the obtained MAPE should be small. The mathematical formula for MAPE is given by:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|PE_t\right| \qquad (12)$$

Where $PE_t = \left(\frac{X_{t-}\hat{X}_t}{X_t}\right)*100$, X_t **=** observed value, $\hat{X}_t$ = forecast value and n = number of observations.

3. RESULTS AND DISCUSSION

3.1 Results

Best Fit ARFIMA Model for Forecasting UFMR

Test for Long Memory Dependency Existence in the UFRM Data

Using R Software with “hurstexp ()” command, the theoretical Hurst Exponent statistic was 0.51488. Thus, it is obvious that the theoretical Hurst exponent Statistic is within the range $0.5 < H < 1$ (Figure 1).

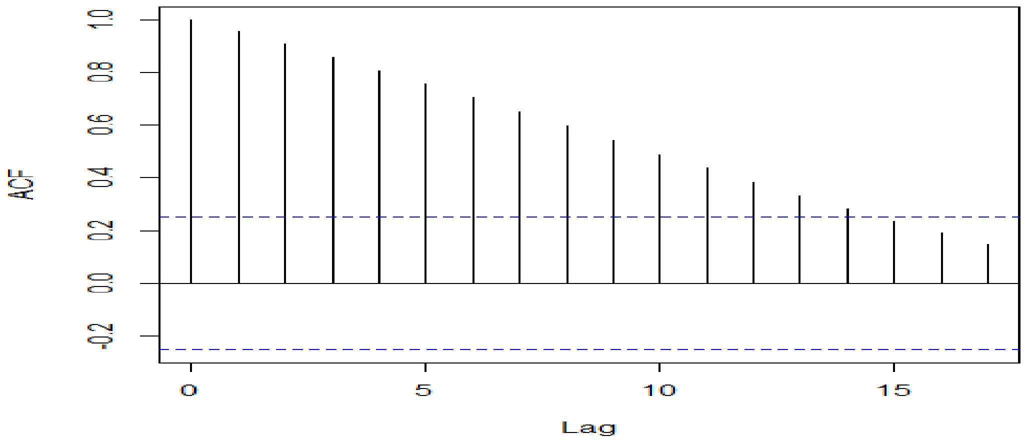

Figure 1: ACF Plot Approach for Testing LMD

Checking for Stationarity of UFMR Data (Figure 2)

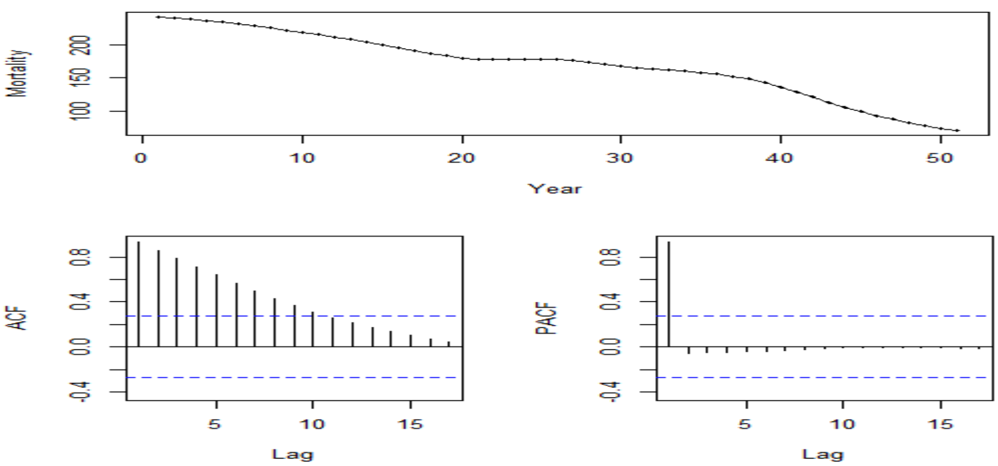

Figure 2: Time Plots, ACF, and PACF for UFMR Data

Unit Root Test for Stationarity of Original UFMR Data (Figure 3)

ADF Test for Original UFMR Data

Table 1: ADF Test Output after the Second Difference of UFMR Data

Description	Value
Dickey-Fuller Test Statistic	-3.8722
Lag order	3

P-value	0.02247

The output of the ADF test in Table 1 proves that after the second differencing, the UFMR data have developed a stationarity state, with a p-value of 0.02247 being less than 0.05 level of significance.

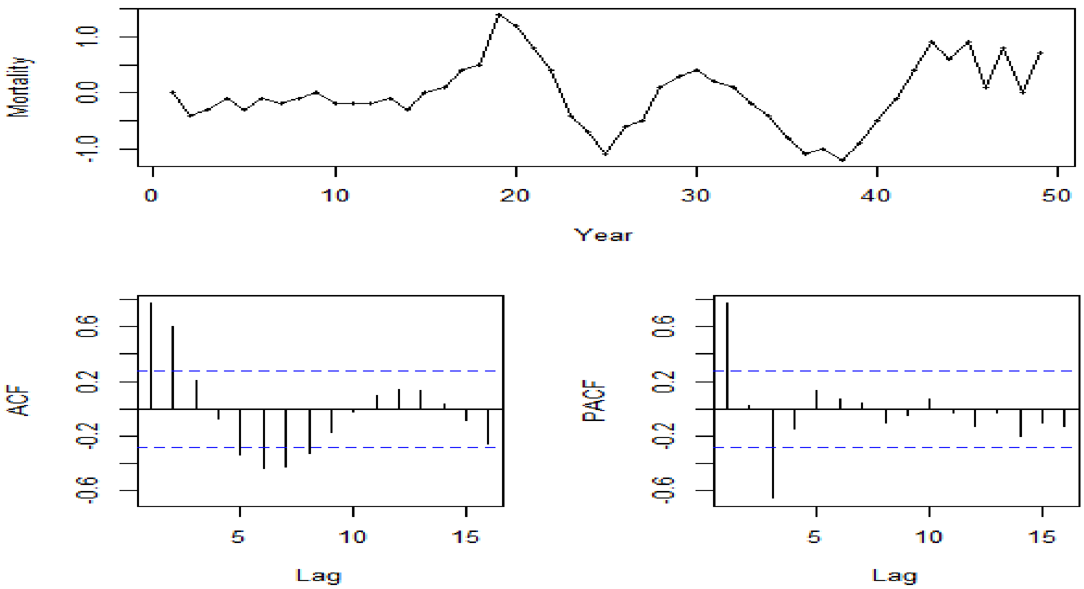

Figure 3: Time Plot, ACF, and PACF of Stationarized UFMR Data

a) Estimates of Fractional Differencing Parameters (d)

From the R function fdGPH (), the fractional differencing parameter was 0.284243, which is within the range $0 < d < 0.5$. The value d suggests that the process is stationary and its autocorrelations are positive and hyperbolically decaying to zero.

b) Estimates of AR and MA Parameters (p and q) and ARFIMA (p, d, q) Models

Table 2: Fitted ARFIMA (p, d, q) Models and the Selected Best Fit Model

ARFIMA Model	AIC
(1, 0.284243, 0)	-88.82931
(0, 0.284243, 1)	-76.77929
(1, 0.284243, 1)	-87.61196
(2, 0.284243, 0)	-89.24496
(0, 0.284243, 2)	-98.45682
(2, 0.284243, 1)	-94.53553
(1, 0.284243, 2)	**-106.776**
(2, 0.284243, 2)	-105.9662

There were eight identified contending ARFIMA (p, d, q) models as indicated in Table 2. Each model was fitted by setting the fractional differencing parameter (d = 0.284243) constant and thereafter the best model was identified based on the minimum value of AIC. The best-fit model was found to be ARFIMA (1, 0.284243, 2) (see Table 3) with the corresponding AIC minimum value of -106.776.

Table 3: Parameter Estimates for the ARFIMA (1, 0.284243, 2) Best Fit Model

Parameters	Estimates	Std. Error	Pr(>|z|)
ϕ_1	0.5899613	0.1253395	2.5151e-06 ***
θ_1	0.1767494	0.1023719	0.08425.
θ_2	-0.7630929	0.1110405	6.3219e-12 ***

σ^2 = 0.0912208; Log-likelihood = 59.388; AIC = -106.776.

From equation (2), the following mathematical model for ARFIMA (1, 0.284243, 2) is formulated as follows.

$$\left(1-0.59C\right)\left(1-C\right)^{0.284243} X_t = \left(1-0.1767C+0.7631C^2\right)\varepsilon_t$$

(13)

c) Residual Analysis for the ARFIMA (1, 0.284243, 2) Best Fit Model

Residuals of the fitted model were analysed using time plots, ACF plots, and the portmanteau test. This was meant to ensure that the selected model was adequate for forecasting.

Residual Analysis of ARFIMA (1, 0.284243, 2) Model by Time Plots, ACFs and PACFs Plots

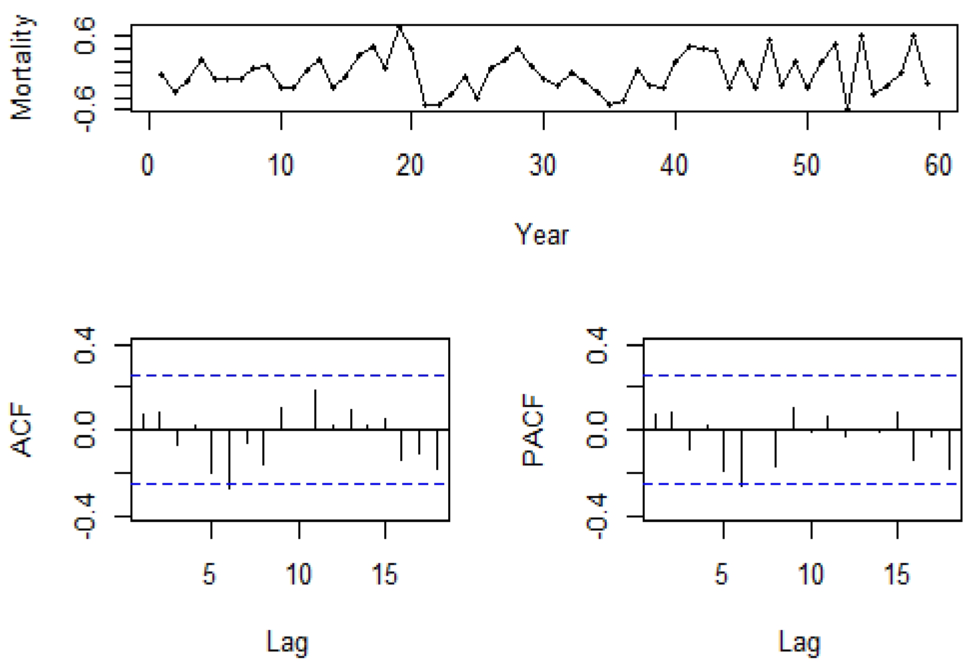

Figure 4: Time Plots, ACF, and PACF for Residuals of the Best Fit ARFIMA (1, 0.284243, 2)

The plots of ACF and PACF for residuals of the fitted ARFIMA (1, 0.284243, 2) model in Figure 4 indicate that all autocorrelations are almost within 95% lower and upper limits, which implies that the residuals are white noise or randomly distributed. The selected model is fit and used for forecasting UFMR in Tanzania.

Portmanteau Test (Box-Ljung Test) for Residuals of ARFIMA (1, 0.284243, 2) (see Table 4)

Table 4: Box-Ljung Test Output for Residuals of ARFIMA (1, 0.284243, 2) Best Fit Model

Description	**UFMR**
Chi-Square	0.56253
Degrees of freedom (df)	1
P-value	0.4532

Since the p-value is greater than 0.05 significance level, we fail to reject the null hypothesis hence verifying that the best fit ARFIMA (1, 0.284243, 2) model assumes the condition of white noise residuals.

d) Forecasts of UFMR from the ARFIMA (1, 0.284243, 2) Best Fit Model (see Table 5 and Figure 5)

Table 5: Forecasts for UFMR from the ARFIMA (1, 0.284243, 2) Best Fit Model

Year	**Point Forecast**
2021	47.203896
2022	45.98131
2023	44.52824
2024	43.08496
2025	41.67229
2026	40.29543
2027	38.95560
2028	37.65261
2029	36.38565
2030	35.1537

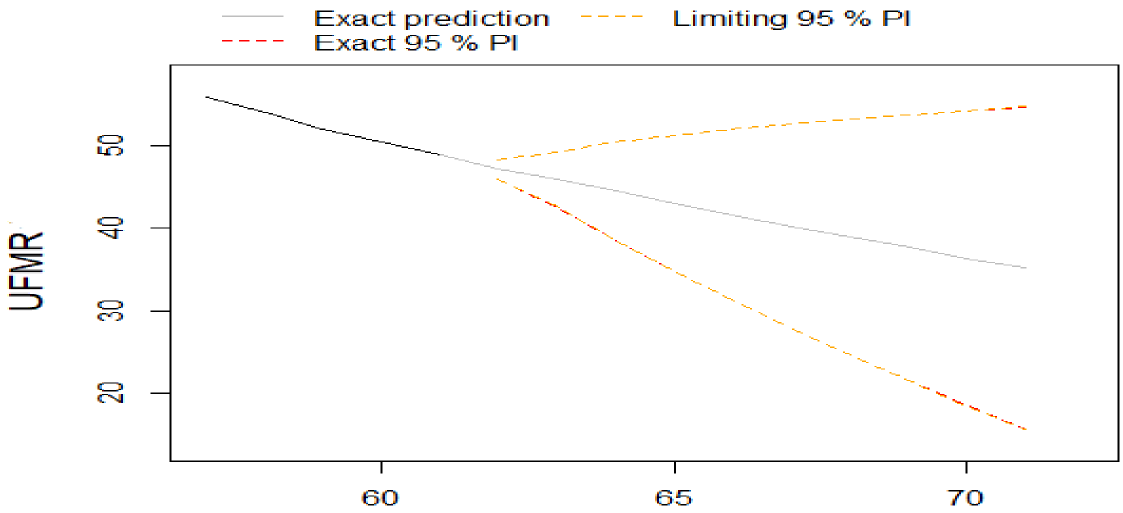

Figure 5: A Plot for Forecasts of UFMR from the ARFIMA (1, 0.284243, 2) Best Fit Model

3.2 The Best Fit Autoregressive Integrated Moving Average (ARIMA) Model for Forecasting UFMR

a) Fitted ARIMA (p, d, q) Candidate Models

Table 6: Fitted ARIMA (p, d, q) Models and the Selected Best Fit Model

ARIMA Model	AIC
(1, 2, 0)	**43.21612**
(0, 2, 1)	52.81308
(1, 2, 1)	44.41176
(2, 2, 0)	44.18472
(0, 2, 2)	48.20545
(2, 2, 1)	46.11667
(1, 2, 2)	45.17138
(2, 2, 2)	47.0442

Each model in Table 6 is fitted by setting the differencing parameter (d) constant; subsequently, the best model is chosen based on the minimum value of AIC. In this case, the best-fit model is ARIMA (1, 2, 0) with 43.21612 as the corresponding minimum value of AIC.

b) Parameter Estimates for the ARIMA (1, 2, 0) Best Fit Model (Table 7)

Table 7: Parameter Estimates for the ARIMA (1, 2, 0) Best Fit Model

Parameter	Estimate	Std. Error

ϕ_1	-0.8097	0.0971

Other parameter estimates are σ^2 = 0.1318; Log-likelihood = -19.61 and AIC = 43.22.

The following mathematical model for the best fit ARIMA (1, 2, 0) model looks as follows:

$$(1+0.8097C)(1-C)^2 X_t = \varepsilon_t \tag{14}$$

c) Residual Analysis for the ARIMA (1, 2, 0) Best Fit Model

Residuals of the fitted models have been analysed using time plots, ACF plots, and the portmanteau test. This is meant to ensure that the selected model is adequate for forecasting UFMR (Figure 6).

Residual Analysis for the ARIMA (1, 2, 0) Best Fit Model by Time Plots, ACF and PACF Plots

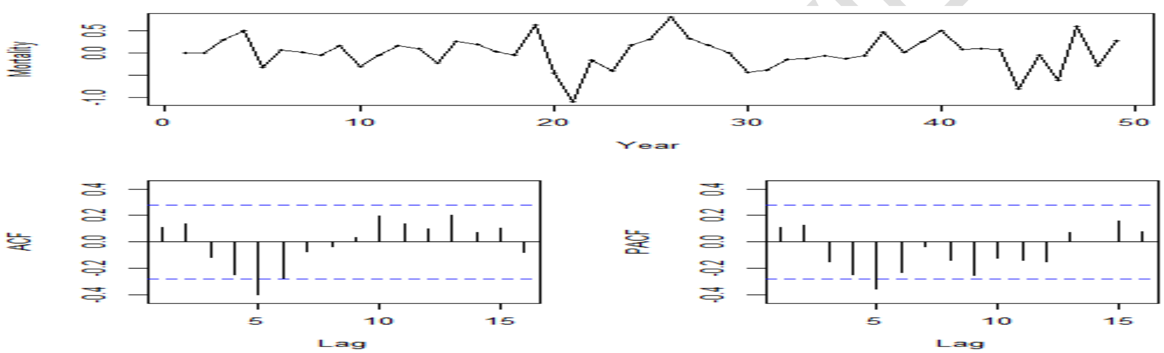

Figure 6: Time Plots, ACF, and PACF for the Best Fit ARIMA (1, 2, 0) Model Residuals Analysis

Box-Ljung Test for Residuals of the (1, 2, 0) Best Fit ARIMA Model (See Table 8 and Figure 7)

Table 8: Box-Ljung Test Output for Residuals of the ARIMA (1, 2, 0) Best Fit Model

Description	Value
Chi-Square	0.64842
Degrees of freedom (df)	1
P-value	0.4207

d) Forecasts from the ARIMA (1, 2, 0) Best Fit Model (Table 9)

Table 9: Forecasts of UFMR from the ARIMA (1, 2, 0) Best Fit Model

Year	Point Forecast
2021	47.3
2022	45.7
2023	44.1
2024	42.5
2025	40.9
2026	39.3
2027	37.7
2028	36.1
2029	34.5
2030	32.9

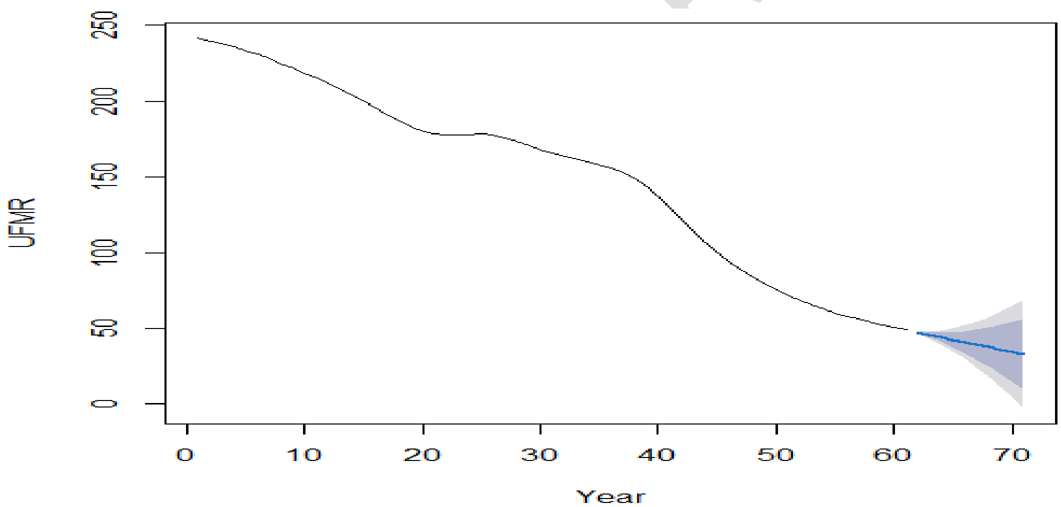

Figure 7: A Plot for Forecasts from the ARIMA (1, 2, 0) Best Fit Model

Accuracy Comparison between the ARFIMA (1, 0.284243, 2) and ARIMA (1, 2, 0) Best Fit Models in Forecasting UFMR (Table 10)

Table 10: RMSE and MAPE Results for Comparing the ARFIMA (1, 0.284243, 2) and ARIMA (1, 2, 0) Best Fit Models

Model Accuracy Measure	Models	
	ARIMA (1, 2, 0)	ARFIMA (1, 0.284243, 2)
RMSE	3.0717928	0.6753077
MAPE	5.0955665	1.041106

2. Discussion

The existence of LMD in UFMR data is studied by using the Rescaled Range Analysis and ACF plots approaches in order to examine whether the ARFIMA is appropriate for modeling and forecasting UFMR data. The Theoretical Hurst Exponent Statistic of 0.515 dwells within the range $0.5 < H < 1$. This result attests that UFMR data exhibit LMD and is therefore valid to be modeled and forecasted using ARFIMA. Also, it is revealed from ACF plots that the data for UFMR portray LMD. This conclusion is supported by Guskova (2017) and Zhang et al. (2023). Both methods, Rescaled Range Analysis and ACF Plots demonstrate that the ARFIMA model is appropriate for modeling and forecasting UFMR in Tanzania.

The stationarity of UFMR data is studied by using time plots; ACFs and PACFs; and the commonly used Unit Root Test, the so-called Augmented Dickey-Fuller (ADF). The results from time plots for original data reveal that downward movements are indicating a decline of under-five mortality rates over time. Also, the shapes of ACF are examined and the findings show that the autocorrelations of the series with themselves lagged one period, two periods, and so forth decline as the number of lags increases. The downward trends suggest that the data are not stationary and stabilization has been a necessary action. The stationarity is attained after differencing the original data twice, being evidenced by the p-value of 0.02247 in Table 1 of ADF outputs and the ACF and PACF plots in Figure 3. These findings are in line with (Wang et al., 2007).

The computed value of the fractional differencing parameter is 0.284243 which is actually less than zero but within the range $0 < d < 0.5$. The value d suggests that the process is stationary and the autocorrelations are positive and hyperbolically decaying to zero. This finding is similar to a study conducted by Salman and Aboudi (2022).

There are eight ARFIMA (p, d, q) identified competing models, which are (1, 0.284243, 0); (0, 0.284243, 1); (1, 0.284243, 1); (2, 0.284243, 0); (0, 0.284243, 2); (2, 0.284243, 1); (1, 0.284243, 2) and (2, 0.284243, 2). Each of these models was fitted by setting the fractional differencing parameter (d) constant and thereafter the best model was identified basing on the minimum value of AIC (in bold) as shown in Table 2. Before applying the best-fit model for forecasting, it was subjected to residual analysis. The time plots, ACF, and PACF for residuals of the fitted ARFIMA (1, 0.284243, 2) model in Figure 4 indicate that the autocorrelations are within the 95% lower and upper limits, thus implying that the residuals are white noise or randomly distributed. Also, the Ljung-Box test findings in Table 4 reveal that the p-value is greater than the 0.05 significance level, hence verifying that the fitted ARFIMA (1, 0.284243, 2) model has white noise residuals. The forecasts from ARFIMA (1, 0.284243, 2) indicate that the Tanzanian FYDP Phase III (2021/2022 -2025/2026) target of 40 deaths/1,000 live births by June 2026 will nearly be achieved as on average the rate of death will decrease to 41 deaths /1,000 live births (MoFP, 2021).

The forecast values indicate that by 2030, Tanzania will experience a decrease in UFMR to 35.2 deaths/1,000 live births compared to 48.9 deaths/1,000 live births in 2020. Moreover, the results of UFMR forecast values indicate that there would be a 28% drop in the incidence of deaths to children aged less than five years from 2020 to 2030 which is equivalent to a 2% drop annually. While the results signify that there would be a success in reducing the UFMR, but still the country would have not achieved the UN SDGs target of reducing UFMR to 25 deaths per 1,000 live births for each country by 2030. According to Eke and Ewere (2020b), to achieve the target for UFMR in Tanzania as set by UN SDG by 2030, the country would have to experience a drop of 49% which is equivalent to 4% per annum.

On the other hand, the ARIMA (1, 2, 0) is found to be the best-fit model and used to forecast future values of UFMR from 2021 to 2030. The forecast trends indicate that by 2030, Tanzania will experience a decrease in UFMR to 32.9 deaths per 1,000 live births compared to 48.9 deaths per 1,000 live births in 2020. Furthermore, forecast values indicate that there would be a 33% drop in the incidence of deaths to children aged less than five years from 2020 to 2030 which is equivalent to 3% drop annually. The forecasts from ARIMA (1, 2, 0) indicate that the Tanzanian FYDP Phase III (2021/2022 -2025/2026) target of 40 deaths/1,000 live births by June 2026 is likely to be achieved as on average the rate of death will decrease to 40.1 deaths/1,000 live births (MoFP, 2021). While the results signify that there would be a success in reducing the UFMR, but still the country would have not achieved the UN SDGs target of reducing UFMR to 25 deaths/1,000 live births by 2030. According to Eke and Ewere (2020b), in order to achieve the target for UFMR in Tanzania as set by UN SDG by 2030, the country would have to experience a drop by 49% which is equivalent to 4% annually.

The results of the comparison between the forecasting accuracy of ARFIMA and ARIMA best by using RMSE and MAPE model forecasting accuracy measures, the results reveal that the ARFIMA (1, 0.284243, 2) model performs better than ARIMA (1, 2, 0). This conclusion is supported by both RMSE and MAPE values for the ARFIMA model being smaller than those of the ARIMA model. These findings are the same as those of a study by Dingari et al. (2019). Moreover, the results are related to a study by Lim et al. (2008) that focused on the comparison between ARFIMA and ARIMA in forecasting air pollution index in Malaysia, where the ARFIMA model emerged to have at least accurate forecasts according to MAE, RMSE and MAPE model accuracy forecasting techniques.

4. CONCLUSION

Under this study, the ARFIMA (1, 0.284243, 2) and ARIMA (1, 2, 0) are found to be the best-fit models. The comparison between the two models in terms of forecasting accuracy by using RMSE and MAPE measures is conducted and the results reveal that the ARFIMA model performs better than the ARIMA model, since both RMSE and MAPE values for the ARIMA are smaller than those of ARFIMA model. So, it suggested that the forecasts from ARFIMA model can be utilized by planners and policy makers in formulating frameworks and policies as well as setting UFMR targets for Tanzanian Five Year Development Plans.

CONSENT (WHERE EVER APPLICABLE)

This study was based on published data, so ethical approval was not required for published data.

DATA AVAILABILITY: Secondary data from 1960 to 2020 is drawn from the World Databank-World Development Indicators (WDI) via the link https://data.worldbank.org/indicator/SH.DYN.MORT?locations=TZ

Competing interests: No competing interests were disclosed.

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