Dynamic Relationship between the Prices of Major Cereals in the Northern Region of Ghana

Abstract

This study employed a Vector Autoregressive (VAR) model to investigate the dynamic relationship between the returns of the cereals. VAR (2) and VAR (3) models were fitted to the data. Base on the Likelihood Ratio Test, VAR (3) model was the best for modeling the dynamic relationship between the returns of the cereals. The diagnostic checks revealed that VAR (3) model was adequate. The VAR (3) model was then used to make inference about the relationship between the returns of these cereals. The Granger causality test revealed a bilateral relationship between the returns of rice and that of millet whiles the returns of maize was independent of the returns of rice and millet. The Impulse Response Function (IRF) and Forecast Error Variance Decomposition (FEVD) analysis both affirm that there exists a dynamic relationship between the returns of the three cereals.

Keywords: Returns, Volatility, Maize, Rice, Millet, Granger-causality.

1. Introduction

Cereals are essential crops that serve the nutritional needs of millions of households worldwide. Both humans and livestock have depended on it extensively for their survival. For instance, the consumption of maize worldwide is more than 1177 million tons with Africa consuming 30% and Sub-Saharan Africa (SSA), 21% (IITA, 2022). Also, FAO, (2023) inferred that rice feeds more than half of the world's population.

Cereal market policies in Ghana may have undergone dramatic changes over the years, desirable outcomes in terms of intervention and prevention of market price volatility are still less satisfactory (Abokyi et al, 2018). As an agrarian economy, Ghana heavily relies on cereals such as maize, rice, millet, and sorghum, which serve as staple foods for a majority of the population. In the Northern region, these cereals are vital not only for household consumption but also as a source of income for smallholder farmers who form the backbone of the local economy (FAO, 2023).

However, Northern region is the second poorest region (61.1%) in the country (Tsiboe et al, 2023). This means that any occurrence of price volatility may not only affect food production, but also the demand for food by the people. Hence, it is important that measures including research efforts are triggered to offset any such occurrence.

Analyzing the price dynamics of cereals is essential for identifying patterns of market inefficiency and for implementing effective interventions (FAO, 2023). The government of Ghana, along with international organizations, has initiated programs to stabilize cereal prices by improving irrigation systems, offering subsidies on agricultural inputs, and enhancing market linkages (Glitse et al, 2017; Melagne and Ehuitche, 2022). Ironically, only little attention has been given to agricultural food price volatility and the interrelationships between major cereal markets in the Northern region and Ghana at large. Thus this study seeks to investigate the dynamic relationship between the prices of three major cereals in the Northern region of Ghana.

2. Materials and Methods

The study was carried using monthly prices of Rice, Maize and Millet obtained from the Ministry of Food and Agriculture, Northern Regional office. The data ranges from January 2000 to December 2023. The data for the cereals were transformed to obtain the returns of each of the cereals. The return for each cereal is given by;

$$\text{return} = \ln(P_t - P_{t-1}) \qquad (1)$$

Where P_t and P_{t-1} are the prices of each the cereal at time *t* and *t-1* respectively.

2.*1 Augmented Dickey Fuller (ADF) Test*

The order of integration of data was investigated using the Augmented Dickey-Fuller test. The regression model employed by Dickey and Fuller (1979) is given by;

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \cdots + \gamma_{p-1} \Delta Y_{t-p+1} + \varepsilon_t \quad (2)$$

Whereα is a constant, $\beta$is the coefficient on time trend series, $\gamma_1 \Delta Y_{t-1} + \cdots + \gamma_{p-1} \Delta Y_{t-p+1}$ is the sum of the lagged values of the dependent variable ΔY_t and *p* is the lag order of the AR process. Imposing the constraints $\alpha = 0$ and $\beta = 0$ corresponds to modelling a random walk and using constraint $\beta = 0$ corresponds to modelling a random walk with drift. By including lags of the order *p,* the ADF formulation allows for higher-order AR processes. The ADF test is concerned with the value of the parameter δ. If $\delta = 0$, it presupposes that the series contains unit root and hence non-stationary.

The test statistic for the ADF test is given by

$$\mathrm{F}_\tau = \frac{\hat{\delta}}{\mathrm{SE}(\hat{\delta})} \qquad (3)$$

Where$\hat{\delta}$ is the least square estimate and $\mathrm{SE}(\hat{\delta})$ is the standard error estimate of $\hat{\delta}$. If the calculated value of the test statistic is greater than the critical value, we reject the null hypothesis of $\delta = 0$.

2.2 *Vector Autoregressive (VAR) Model*

A VAR process consists of a set of k –endogenous time series variables $R_t = (r_{1t},\ r_{2t}, \ldots., r_{kt})'\ for\ k = 1, \ldots \ldots, K$. A VAR model of order p denoted as VAR (p) is given by;

$$R_t = v + A_1 r_{t-1} + \cdots \ldots \ldots \ldots . + A_p r_{t-p} + u_t, \quad t = 0, 1, \ldots., T \quad (4)$$

where $R_t = (r_{1t}, \ldots., r_{kt})'$ is a $(k \times 1)$ random vector of the rates, $A_i, i = 1, \ldots \ldots, p$ is a fixed $(K \times K)$parameter (coefficient) matrice, $v = (v_1, \ldots.., v_k)'$ is a fixed $(K \times 1)$ vector of intercept allowing for the possibility of a zero mean and $u_t = (u_{1t}, \ldots., u_{kt})'$ is a K –dimensional white noise series or innovation process with time invariant positive definite covariance matrix and zero mean. It is assumed that u_t has a multivariate normal distribution. An important characteristic of a VAR (*p)* process is its stability, this implies that given sufficient starting values, the VAR (*p)* process generates stationary time series with time invariant means, variances and covariance's structure. The stability is determined by evaluating the reverse characteristic polynomial.
$\det(I_k - A_I Z - \cdots - A_p Z^p) \neq 0\ for\ |Z| \leq 1$. If the solution of the reverse characteristic polynomial has a root Z=1, the either some or all the variables in the VAR (*p*) process are integrated of order one. In practice, the stability of an empirical VAR (*p*) process can be analysed by calculating the eigenvalues of the coefficient matrix. If the moduli of the eigenvalues of A_i are less than one, the VAR (*p*) process is stable.

2.3 *VAR Lag Order Selection*

An important step in fitting a VAR (*p*) process is determining the optimum lag for the process. In this study, the the Akaike Information Criterion (AIC), the Schwarz Bayesian Information Criterion (SBIC) and the Hannan-Quinn Information Criterion (HQIC) were employed to determine the optimal lag length for VAR (*p)* process. The criteria are given by

$$AIC = ln\left|\widehat{\sum_{u}(p)}\right| + \frac{2}{T}pK^2 \quad (5)$$

$$HQIC = ln\left|\widehat{\sum_{u}(p)}\right| + \frac{2\,ln\{ln(T)\}}{T}pK^2 \quad (6)$$

$$SBIC = ln\left|\widehat{\sum_{u}(p)}\right| + \frac{ln(T)}{T}pK^2 \quad (7)$$

where T denotes the number of observations in the data, p assigns the lag order, $\widehat{\sum_u(p)} = T^{-1}\sum_{t=1}^{T}\widehat{u_t}\widehat{u_t'}$.

2.4 *Impulse Response Function*

The Impulse Response Function was used to investigate the dynamic interactions between the endogenous variables and is based upon the Wold representation of the VAR (*p*) process. The Wold representation is based on the orthogonal errors $\boldsymbol{\eta_t}$ and is given by;

$$\boldsymbol{R_t} = \boldsymbol{\mu} + \boldsymbol{\Theta_0}\boldsymbol{\eta_t} + \boldsymbol{\Theta_1}\boldsymbol{\eta_{t-1}} + \boldsymbol{\Theta_2}\boldsymbol{\eta_{t-2}} + \ldots \quad (8)$$

where $\boldsymbol{\Theta_0}$is a lower triangular matrix. The impulse responses to the orthogonal shocks $\eta_{jt}$are;

$$\frac{\partial R_{i,t+s}}{\partial \eta_{j,t}} = \frac{\partial R_{i,t}}{\partial \eta_{j,t-s}} = \Theta_{ij}^{s} \quad i,j = 1,2,\ldots,k, s > 0 \quad (9)$$

where $\boldsymbol{\Theta_{ij}^{s}}$ is the $(i,j)th$ element of Θ_0**.** For$k$ variables there are k^2 possible IRF.

2.5 *Forecast Error Variance Decomposition (FEVD) Analysis*

The FEVD was used to determine the contribution of the j^{th} variable to the *h*-step forecast error variance of the i^{th} variable. The FEVD is given as;

$$FEVD_{i,j}(h) = \frac{\sigma_{\eta_j}^2 \sum_{s=0}^{h-1}(\Theta_{ij}^{s})^2}{\sigma_{\eta_1}^2 \sum_{s=0}^{h-1}(\Theta_{i1}^{s})^2 + \ldots + \sigma_{\eta_k}^2 \sum_{s=0}^{h-1}(\Theta_{ik}^{s})^2} \quad i,j = 1,2,\ldots,k \quad (10)$$

where $\sigma_{\eta_j}^2$ is the variance of η_{jt}. A VAR (*p*) process with k variables will have $k^2 FEVD_{i,j}(h)$ values.

2.6 *Causality Test*

A stationary time series variable x_t is Granger causal for another stationary time series variable z_t, if past values of $x_t$have additional power in predicting z_t after controlling for past values of z_t(Gelper and Croux, 2007). Causality may be classified as unidirectional, bilateral or independent (Gujurati, 2003).

3.0 Results and Discussions

Table 1 shows the ADF test performed on the returns of the cereals. The test performed with constant only and with constant and trend revealed that the data was stationary. The stationarity in the returns of the cereals is affirmed by the time series plot of the data. Figure 1 displays the time series plots for the returns of the cereals. From the plot it was evident that the returns of the cereals fluctuates about a fixed point. This is an indication of stationarity in the returns of the cereals. This feature of the data provides a good justification for fitting the Vector Autoregressive model. The optimal lag order for the model was selected using the information criteria: from Table 2, the AIC selected lag three (3) but BIC and HQIC selected lag two (2). Both VAR (2) and VAR (3) models were fitted to the series, and the Likelihood Ratio Test (LRT) was used to select the best model for investigating the dynamic relationship. From Table 3, the significant likelihood ratio test statistic revealed that the VAR (3) was best for modeling the dynamic relationship.

Thus, VAR (3) was estimated for the returns as shown in Table 4. The results in Table 4 revealed that, lag 1and 2 of rice returns were useful predictors of itself at the 5% significance level. Also, lag 3 of millet returns was a useful predictor of rice returns. However, lag 3 of rice and lag 1, 2 and 3of maize were not statistically significant at the 5% significance level in predicting the returns of rice. Lag 1, 2 and 3 of maize returns were statistically useful predictors of itself. While lag 1, 2 and 3 of both rice and millet were not useful predictors of the returns of maize. It was also seen that, lag 1and 2 of millet returns were useful predictors of itself at the 5% significance level. Whereas lag 1, 2 and 3 of rice returns were statistically significant at the 5% significance level in predicting the returns of millet, lag 3 of millet and lag 1, 2 and 3 of maize were not statistically significant at the 5% significance level in predicting the returns of millet.

The stability of the VAR (3) model was investigated. The results revealed the model was stable as all the eigenvalues have modulus less than one as shown in Table 5. This affirms that all the series used are stationary as revealed by the ADF test. Also, the CUSUM plot in Figure 2 affirms that the model is stable as the recursive residuals for the individual equations are within the confidence limit.

The univariate Ljung-Box test and ARCH-LM test were used to diagnose the model and as shown in Table 6 and Table 7, the model residuals are free from serial correlation and conditional heteroscedasticity respectively; this indicates that the fitted model is adequate. The model was then used to investigate Granger causality among the cereals. Table 8 revealed that the returns of rice granger cause the returns of millet and vice versa, confirming the bilateral relationship between the rice returns and millet returns. Also, Maize does not Granger-cause Millet and Rice but Maize and Rice Granger-cause Millet: These implies that if the previous values of rice returns are known, then future values of millet returns can be predicted and vice versa. In addition, the returns of Maize alone cannot be used to predict the returns of the other cereals unless it is combined with that of Rice.

Furthermore, an impulse response analysis was employed to examine how the cereals in the VAR (3) model will interact following a shock in the VAR (3) model. When the response variable was rice returns, the rice returns showed a negative reaction in the first period and then a positive reaction after the second period until a stable response was obtained after period ten. The maize returns caused a negative shock in the first period, a positive shock in the second period, negative shock in the third period, a positive shock in the fourth period, a negative shock in period five and a positive shock in period seven until a stable response was obtained after the eleventh period. The rice returns reacted positively to a shock in the millet returns in the first period followed with a negative response in the second period and a positive shock from the third period to the sixth period and then a stable response for the rest of the periods.

For the returns of maize as a response variable, a shock in the returns of rice cause a positive reaction in the returns of maize in the first period, a negative reaction in second period, a positive reaction in the third period, a negative reaction in the fourth period, a positive reaction in the fifth period and a negative reaction from the sixth period to the ninth period and then a stable response after period eleven. In the first period, the maize returns showed a positive response to a shock in its own values, the second period showed a negative response up to the fifth period and then a stable response to its own shocks onwards. Also, the returns of maize showed a negative reaction at the first, fourth and seventh period and then a positive reaction at the third, sixth and tenth period and stabilizes after period twelfth to a shock in the returns of millet.

When the response variable was millet returns, a shock in the rice returns caused a negative reaction of millet returns in the first two periods, a positive reaction in the third period, a negative reaction in the fifth period, a positive reaction in the sixth period until a stable response was obtained after the seventh period. Maize returns showed a positive reaction in the first period, a negative response at the second period, a positive reaction between the third and fourth periods, a negative reaction at the fifth period and a continues positive reaction from period six to period fifteen with a stable response for the rest of the periods. Millet returns showed a negative response to a shock in its own values at the first period and both negative and positive reactions between period two and period seven and then a stable response to its own shocks onwards.

The Impulse Response analysis does not clearly show the magnitude of the relationship among the variables. The Variance Decomposition for the variables were therefore examined. Table 9 displays the Variance Decomposition for Rice. Aside Rice itself, Millet also contributes in forecasting the uncertainty of Rice. For instance at period ten, about 98.03% of the variance in Rice appears to have been explained by innovations in Rice, while 1.42% and 0.22% were explained by innovations in Millet and Maize respectively. Rice has the highest contribution in forecasting the uncertainty in Millet as shown in Table 10. At period ten about 70.51% of the variance in Millet appears to have been explained by innovations in Rice, while 29.17% and 0.68% were explained by innovations in Millet and Maize respectively. Finally, the returns of maize contributes most in forecasting the uncertainty of maize. At period ten, about 99.4% of the error variance in the returns of maize have been explained by innovations in the returns of maize, whiles 0.5% and 0.1% of the error variance explained by innovations in the returns of rice and millet respectively. As shown in Table 11. The results of maize variance decomposition also agrees with views of the Granger causality test and the estimated VAR (3) model which revealed that, the past prices of rice and millet are not the most influencing determinant of the price of maize.

4.0 conclusion

In this study, the relationship between the returns of three major cereals in Northern region of Ghana was investigated. The results revealed that there was bilateral causality between Rice and Millet. Also, Maize does not Granger-cause Millet and Rice but Maize and Rice Granger-cause Millet. The returns of Maize cannot be used in predicting the returns of the other cereals. The Forecast Error Variance Decomposition revealed that the returns of Millet explains an appreciable amount of the forecast uncertainty in Rice and Maize.

5.0 References

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Table 1: ADF test for the returns of Rice, Maize and Millet

Cereal	**Constant**		**Constant + Trend**	
	Test Statistic	***P*-value**	**Test Statistic**	***P*-value**
Rice	-10.203	0.000	-10.172	0.000
Maize	-9.713	0.000	-9.693	0.000
Millet	-9.909	0.000	-9.883	0.000

Table 2: lag Order Selection for Fitting VAR Model

Lag	**AIC**	**BIC**	**HQIC**
1	2.995	3.172	3.067
2	2.759	3.112*	2.902*
3	2.717*	3.247	2.932
4	2.790	3.497	3.077
5	2.856	3.740	3.215
6	2.934	3.995	3.365
7	3.011	4.248	3.514
8	3.083	4.496	3.657
9	3.109	4.700	3.755
10	3.121	4.888	3.838

*: Means best based on model selection criteria

Table 3: Model Selection Criteria

Model	**AIC**	**BIC**	**HQIC**
VAR (2)	2.672	3.015*	2.811*

VAR (3)	2.632*	3.146	2.841
Likelihood Ratio Test Statistic = 24.48 *P*-value =0.004**			

*: Means best based on model selection criteria

**: significant at the 5% significance level

Table 4: Parameter estimates of VAR (3) Model

Equation	Variable	Coefficient	Std. Error	*t*- ratio	*P*-value
Rice	Rice.L1	-0.731	0.079	-9.274	0.000*
	Rice.L2	-0.498	0.146	-3.406	0.001*
	Rice.L3	-2.209	0.139	-1.504	0.135
	Maize.L1	0.056	0.082	0.683	0.496
	Maize.L2	0.069	0.086	0.801	0.424
	Maize.L3	0.098	0.077	1.286	0.201
	Millet.L1	-0.011	0.125	-0.088	0.930
	Millet.L2	-0.141	0.130	-1.088	0.278
	Millet.L3	-0.149	0.071	-2.087	0.039*
Maize	Rice.L1	-0.069	0.077	-0.905	0.367
	Rice.L2	-0.082	0.143	-0.578	0.564
	Rice.L3	-0.063	0.135	-0.466	0.642
	Maize.L1	0.442	0.080	-5.548	0.000*
	Maize.L2	0.314	0.084	-3.762	0.000*
	Maize.L3	0.180	0.075	-0.420	0.017*
	Millet.L1	0.032	0.122	0.261	0.794
	Millet.L2	0.006	0.126	0.046	0.963
	Millet.L3	-0.036	0.069	-0.516	0.607
Millet	Rice.L1	0.926	0.050	18.655	0.000*
	Rice.L2	0.579	0.092	6.285	0.000*
	Rice.L3	0.254	0.088	2.902	0.004*
	Maize.L1	0.053	0.052	1.033	0.303
	Maize.L2	0.013	0.054	-0.242	0.809
	Maize.L3	0.010	0.048	0.206	0.837
	Millet.L1	-0.597	0.079	-7.569	0.000*
	Millet.L2	-0.240	0.082	-2.935	0.003*
	Millet.L3	-0.031	0.045	-0.701	0.485
AIC = 2.632 BIC = 3.146 HQIC = 2.841 Log-Likelihood = -186.148					

*: Means significant at the 5% significance level

Table 5: VAR (3) Model Stability test

Eigen values	**Modulus**
0.1600388 + 0.6341251*i*	0.654008
0.1600388 - 0.6341251*i*	0.654008
-0.55128 + 0.2715227*i*	0.614519
-0.55128 - 0.2715227*i*	0.614519
0.3553569 + 0.6125116*i*	0.613542
0.3553568 - 0.6125116*i*	0.613542
-0.2651747 + 0.4600057*i*	0.530964
-0.2651747 – 0.4600057*i*	0.530964
-0.5278848	0.527885

Table 6: Univariate Ljung-Box Test and ARCH-LM Test

	Ljung Box-Test			**ARCH-LM Test**	
Equation	**Lag**	**Test-Statistic**	***P*-value**	**Test-Statistic**	*P*-value
Rice	12	7.257	0.840	0.819	1.000
	24	14.856	0.925	0.828	1.000
	36	18.511	0.993	0.936	1.000
	48	21.512	1.000	1.205	1.000
Maize	12	7.914	0.792	5.808	0.925
	24	11.061	0.989	5.514	1.000
	36	19.565	0.988	6.992	1.000
	48	24.322	0.998	5.609	1.000
Millet	12	3.674	0.989	2.625	0.998
	24	7.353	1.000	4.779	1.000
	36	12.886	1.000	6.574	1.000
	48	18.391	1.000	10.528	1.000

Table 7: Multivariate Ljung-Box Test and ARCH-LM Test of VAR (3) Model

	Ljung Box-Test			**ARCH-LM Test**	
Equation	**Lag**	**Test-Statistic**	***P*-value**	**Test-Statistic**	***P*-value**
	12	55.963	0.985	389.722	0.929
VAR (3)	24	159.345	0.943	828.000	0.806

36	209.767	1.000	756.000	1.000
48	33.873	0.999	684.000	1.000

Table 8: Granger Causality Test

Equations	Excluded	Chi-Squared	Df	*P*-value
Rice	Maize	1.980	3	0.577
	Millet	5.033	3	0.016**
	All	6.965	6	0.324
Maize	Rice	0.987	3	0.804
	Millet	0.374	3	0.945
	All	1.141	6	0.980
Millet	Rice	368.840	3	0.000**
	Maize	1.662	3	0.645
	All	369.090	6	0.000**

**Means significant at 5% significance level.

Table 9: Forecast Error Variance Decomposition for rice

Period	Std. Error	Rice	Maize	Millet
1	0.419	100.000	0.000	0.000
2	0.519	99.810	0.186	0.003
3	0.521	99.414	0.188	0.398
4	0.521	99.345	0.231	0.424
5	0.524	98.633	0.614	0.753
6	0.525	98.389	0.695	0.916
7	0.525	98.389	0.697	0.915
8	0.525	98.385	0.701	0.915
9	0.525	98.370	0.715	0.915
10	0.525	98.369	0.215	1.415

Table 10: Forecast Error Variance Decomposition for maize

Period	Std. Error	Rice	Maize	Millet

1	0.408	0.130	99.870	0.000
2	0.446	0.366	99.599	0.035
3	0.449	0.456	99.486	0.058
4	0.449	0.466	99.455	0.079
5	0.452	0.471	99.392	0.137
6	0.452	0.475	99.383	0.143
7	0.452	0.484	99.373	0.143
8	0.452	0.484	99.373	0.144
9	0.452	0.484	99.372	0.145
10	0.452	0.486	99.369	0.145

Table 11: Forecast Error Variance Decomposition for millet

Period	Std. Error	Rice	Maize	Millet
1	0.264	0.006	1.240	98.754
2	0.495	61.571	0.359	38.070
3	0.566	70.404	0.279	29.3173
4	0.567	70.277	0.344	29.380
5	0.567	70.308	0.352	29.340
6	0.572	70.221	0.607	29.172
7	0.572	70.123	0.654	29.223
8	0.572	70.170	0.655	29.175
9	0.572	70.164	0.663	29.173
10	0.572	70.152	0.678	29.170

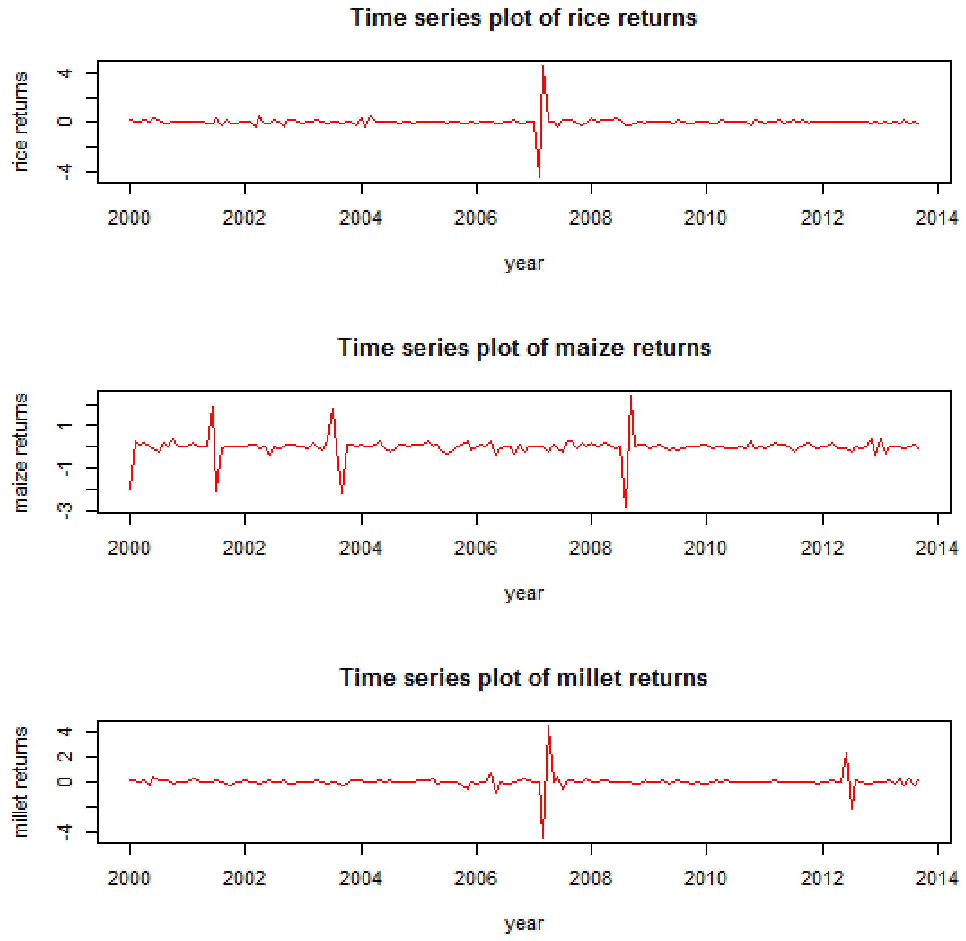

Figure 1: Time series plot of the returns of Rice, Maize and Millet

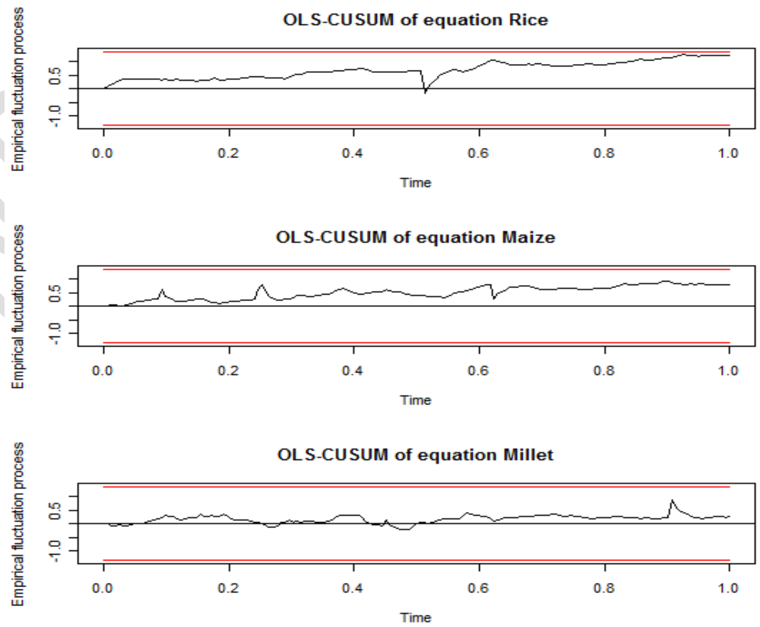

Figure 2: CUSUM Plots of the Individual Equations of the VAR (3) Model

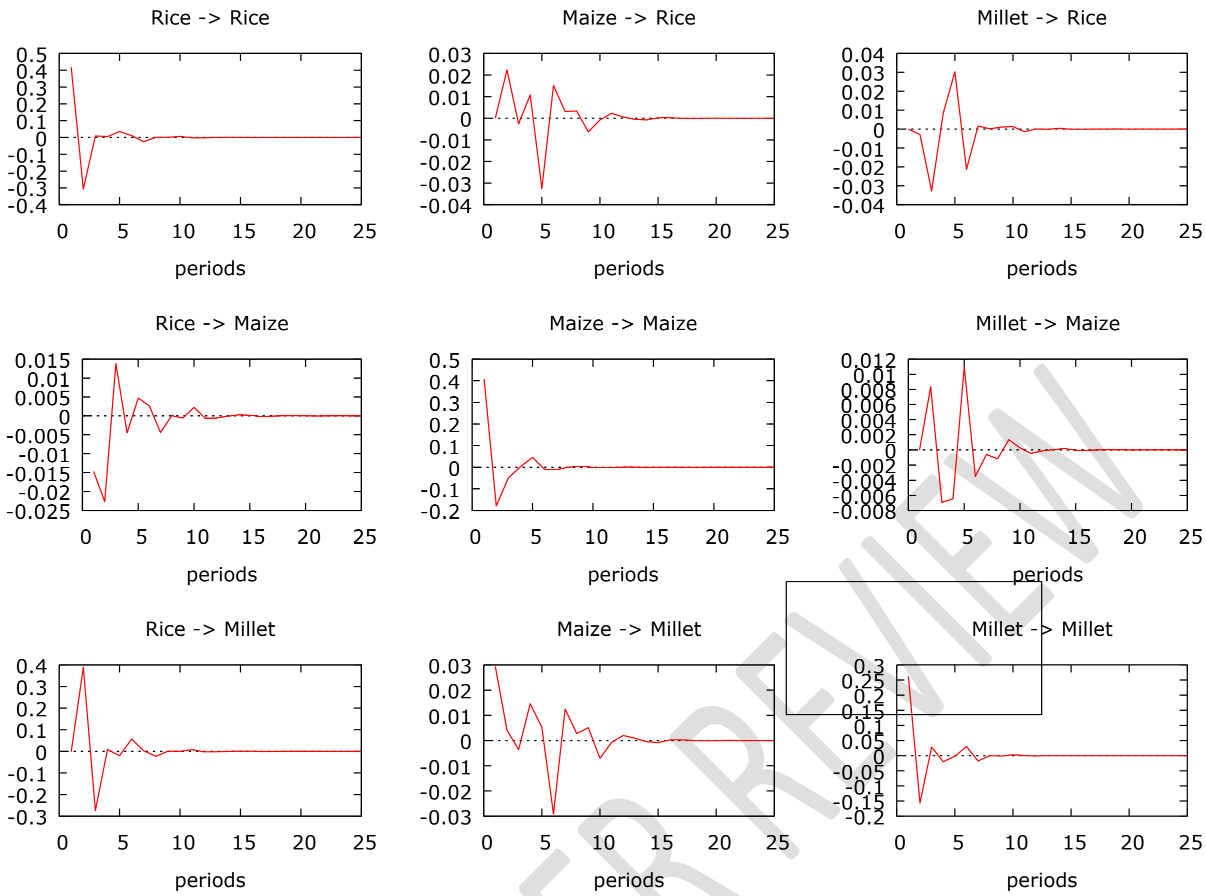

Figure 3: Plot of Impulse Response Analysis