

Application of Queueing Theory to Analyze the Performance Metrics of Manufacturing Systems

Abstract: Manufacturing is a processing medium used to transform raw materials into consumable finished goods. Manufacturing systems entail numerous interactions between personnel, data, supplies, and equipment. There is an urgent need for a queueing theory-based solution to handle the blocking and delay issues in production systems. The theory of queueing is a useful tool for analyzing and simulating the operation of production processes because it provides a variety of performance metrics, such as throughput, waiting time, and resource utilization. By considering the arrival pace of jobs and the service rate of different components, queueing models can offer valuable insights into the system's expected performance. The present study aims to analyze manufacturing systems within various frameworks, evaluate various performance metrics using the **M/M/1 queueing techniques**, and identify the optimal choice using typical optimization tools. **We have described various performance metrics like utilization, throughput, cycle time, and queue length. To validated, we have calculated these performance metrics in practical situations.** This project includes all intermediate steps required to create and include the product mechanism. **The findings of this work have the potential to inspire future research directions, encouraging interdisciplinary collaborations and the development of innovative solutions. This work serves as a vital resource for academics, practitioners, and policymakers aiming to expand and improve the knowledge in Queueing Theory.**

Keywords: Performance Analysis, Flexible Manufacturing System, Waiting Line, Queueing System, and Manufacturing Process.

2020 AMS Classifications: 60K30, 90B22, 90B25

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1. Introduction

Producing goods with manpower and tools for consumption or sale is known as manufacturing. Most commonly, people use the term to refer to industrial production, which involves the massive transformation of raw materials into finished goods. However, it may be applied to a broad variety of human pursuits, from handicrafts to cutting-edge technologies.

We can effectively complete the three main manufacturing tasks of "Procurement," "Production," and "Distribution" by using operations research tools. Warehouses hold finished products, which undergo conversion from raw materials to semi-finished or finished units during the production process, to meet client demand. We then distribute the finished products to various locations. Additional supporting roles include marketing, personal management, bookkeeping, planning, designing, and so on. The workshop, connected flow line, disconnected flow line, and continuous flow process are the four conventional forms of manufacturing systems. A wide range of machinery, some capable of handling a wide range of tasks, equips the job shop. Even though there are fewer distinguishable task kinds and routings in unconnected flow lines, inventory might nevertheless accumulate between workstations. The linked flow line, on the other hand, mandates that all jobs contact the machines and workstations in the same order. We frequently find connected flow lines within the automotive business. Products in continuous flow processes move automatically along a predetermined path.

Queueing theory helps identify bottlenecks and devise techniques for more efficient operations by looking at things like arrival rates, service times, and system capacity. Aalto and Scully's [1] study looked at general service time distributions and the occupancy distribution for an open network with an infinite number of server queues and batch arrivals that follow a non-homogeneous Poisson process. They proved that the predicted occupancy duration for each batch must be finite for ergodicity and also derived a probability-generating function for the network's transient occupancy distribution. Abourizk et al. [2] have presented an overview of the history of construction simulation theory, delving into the Cyclone modeling approach and its significant advancements. This study provides an overview of long-term simulation initiatives that will lead to the development of the next generation of construction computer modeling systems. Agrawal and Mohanty [3] have presented an innovative approach to multi-attribute decision-making in a linguistically ambiguous and hesitant setting. It comprises hesitant fuzzy linguistic term sets' non-uniform, non-regular, or arbitrarily specified linguistic terms. A group of researchers led by Amjath et al. [4] created a way to use finite queueing networks to improve performance and find the best topology. They focused on buffer allocations. The proposed framework includes a finite closed queueing network to represent the intra-logistics material transfer process, and a finite open queueing network to represent the outbound logistics process in a manufacturing setup. The generalized expansion method (GEM) considers the blocking issue when calculating the system's network performance metrics. We build discrete event simulation (DES) models using simulation software, incorporating optimization setups to find the best buffer allocations for maximizing system

throughput. The study's conclusions significantly influence decision-making and offer opportunities to enhance the effectiveness of industrial systems.

The work by Boer et al. [5] consists of six concise pieces that delve into the use and application of theory in management research, specifically addressing the question of what defines a valuable or significant addition to theory. The authors explore the current application of theory in operations management (OM) (Harry Boer), the types of theories they have applied to OM (Chris Voss), the role of theory in advancing a general understanding of OM problems (Roger Schmenner), whether they can draw from other disciplines' theories or develop their own (Matthias Holweg), how to contribute to theory in various ways (Martin Kilduff), and the process of formulating a theoretical argument (Mark Pagell). Chen et al. [6] have been engaged in Manufacturing units use automated guided vehicle-based flow production systems. Optimizing workstation layouts is the subject of the study. The approximate flow production system was modelled using an open queuing network (OQN). When used with an OQN model, simulated annealing (SA) is a powerful way to solve the facility layout problem in a stochastic flow manufacturing setting. Gongshan et al. [7] have described that process analysis is a strategy used to examine the current production process, identify issues with the entire process, and then take into account the company's development plan, operational costs, and service quality. Research indicates that high costs, low efficiency, and long service times can be addressed by combining process analysis and queuing theory.

Over the next decade, the manufacturing sector will compete primarily based on its ability to adapt quickly to shifting market conditions. The manufacturing industry requires methods for customer acquisition and order fulfilment that can precisely manage anticipated changes and simultaneously respond quickly and adaptably to unanticipated ones. Flexible manufacturing systems (FMS) help us achieve these goals. A production system with some degree of flexibility to adapt to changes, whether expected or unexpected, is known as a flexible manufacturing system, or FMS. The necessity to adapt the manufacturing process gave rise to flexible manufacturing. Customers began to place more value on delivery speed as the market grew more complex and cost and quality concerns increased. Businesses must be more flexible in their operations and production processes to keep up with fast-changing industrial trends. The main advantages of flexible manufacturing systems (FMS) stem from their remarkable flexibility in allocating production resources, such as labor and time, to generate novel products. FMS works well when creating small sets of goods, such as those produced through mass production.

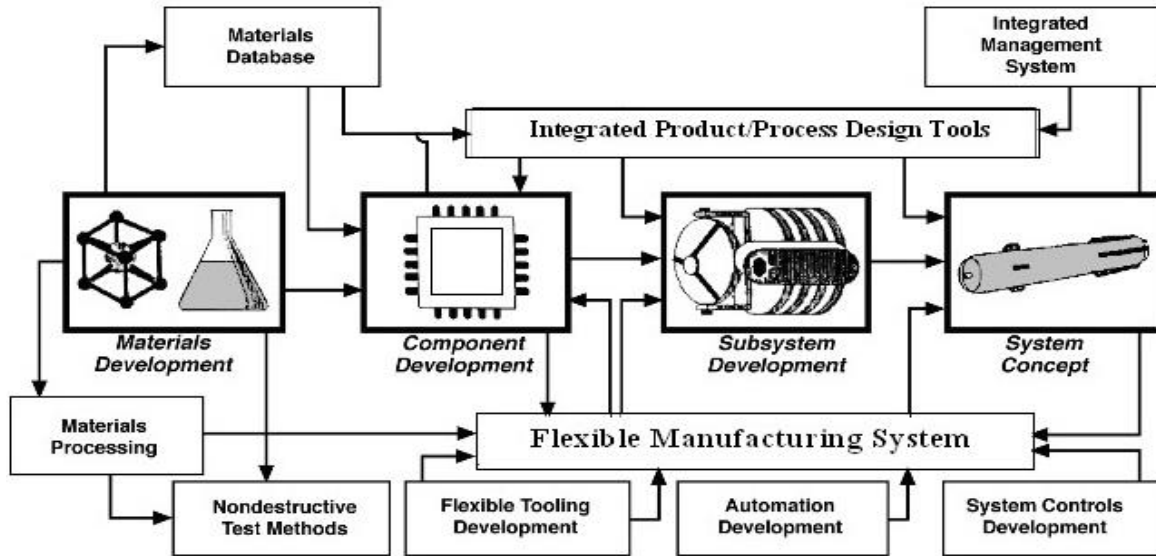


Figure 1.1. Flexible manufacturing system

The phenomenon of queueing arises in various real-world scenarios of congestion when resources (such as manufacturing machinery, elevators, phone lines, and traffic signals) are not able to promptly provide the quantity or quality of service that customers demand. Queueing systems provide valuable tools for the performance analysis of manufacturing systems. By considering the behavior of waiting lines and the interactions between different components, it provides a quantitative framework for evaluating system performance and identifying areas for improvement. By utilizing queueing theory, manufacturing system analysts can gain insights into system blockage, resource utilization, and potential areas for improvement. For example, by identifying the blockage in the system, managers can focus on optimizing the performance of that particular component to increase overall system throughput.

2. Literature Review

A.K. Erlang invented the queueing theory in 1903 after deciding to apply it to solve the telephone traffic congestion issue. Erlang initially tried to identify the causes of a single operator's delay. We expanded the outcome of this experiment to determine the latency for several operators, and then effectively applied it to more general queue-related problems. Subsequently, several scholars provided their own definitions of queueing theory. Here we described some previous research work related to manufacturing systems.

Li [8] described that the coordination delay time of the coordination control optimization timing method can be decreased by establishing information contact between vehicles through complex wireless communication. This work first develops a pertinent mathematical model for smart automobile driving, a complicated signal system. We apply it to a comparison of three widely used mathematical models. The same-condition findings demonstrate that, in terms of transmission accuracy across all segments, the mathematical model performs better when handling the complicated signal system. Mehra and Taylor [9] looked into the general service time distributions and the occupancy distribution for an open network with an infinite number of server queues and batch arrivals that don't follow a Poisson process that is homogeneous. They proved that the predicted occupancy duration for each batch must be finite for ergodicity, and a probability-generating function for the transient occupancy distribution of the network. They also recover recurrence relations for the transient probability mass function. This function is expressed in terms of a distribution that is produced by multiplying the batch size by a multinomial distribution.

Murdapa et al. [10] have addressed the emission variable single-stage M/M/1 queueing model. Its primary goal is to ascertain how carbon emissions might fit into the fundamental queueing theory. As it turned out, the inclusion of emission variables in the model changed it from the conventional single-stage queue model to one that calculates the quantity of production lots permitted per period. Pasandideh et al. [11] have studied a facility location problem with stationary servers and stochastic customer demand. This problem has two goals in mind: (1) reducing the typical customer wait time and (2) lowering the typical facility idle-time percentage. They used queueing theory to describe this problem, and within the context of the desired function, they used a genetic algorithm to solve the model. Rece et al. [12] have introduced a novel approach that makes use of queueing theory models to guarantee the ideal size for the production department, reduce production costs, and optimize supply. To solve it, they used the Monte Carlo approach to estimate all the system factors that affect the cost.

Saini et al. [13] have delved into the application of queueing theory in the industrial sector, highlighting its potential to boost productivity, reduce wait times, and preserve resources. Selvamuthu and Kapoor [14] have studied that a simple probability technique is used to determine the time-dependent solution of a fluid model driven by an M/M/1 queue. Numerical findings serve as an example of the suggested methodology. Sharma et al. [15] have discussed the queueing model and queueing theory methods to describe everyday life. Ulku et al. [16] have examined the association between waiting

times and subsequent purchasing decisions. They investigated the impact of managerial strategies frequently used by businesses to enhance the waiting experience for clients. Vorhölter et al. [17] have presented an overview of the state of mathematical modeling in German-speaking nations. The authors provide a broad conceptualization of modeling competency and its definition from the German Educational Standards, following a brief overview of previous years' developments. Mathematical modeling can be implemented in classrooms, in regular classes, and in modeling projects.

3. Queueing System in Manufacturing Process

Research on queueing theory is a very active field. Its remarkable viability can be attributed, in part, to the fact that fresh and intriguing challenges arising from manufacturing and production processes frequently give birth to difficult and novel queueing difficulties. The existence of groups of units or customers who show up at random to acquire the service is indicative of a queueing problem. Upon arrival, the units or customers may receive immediate attention, or they may need to wait for the server to become available. Numerous industries and sectors, including business, manufacturing, government, transportation, and healthcare, can benefit from this strategy.

The main goal of queueing theory is to mathematically represent and analyze systems that fulfill random requests. In general, queueing models apply to service-oriented companies and offer strategies for enhancing service effectiveness. A queueing model can abstractly describe a manufacturing system. Generally, a queueing model indicates the number and type of servers that serve the units/customers, which describes the physical configuration of the system. It also indicates the variability in the arrival and service processes, which indicates the stochastic (i.e., statistical or probabilistic) nature of the demands.

The majority of queueing models make the following basic assumptions. An input source produces arrivals over time that require servicing. These customers log into the queueing system and then join a queue. A regulation known as the queue discipline selects a queue member for service at specific times. Once the service mechanism has completed the necessary service for the customer, the member exits the queueing system. Based on the depiction of this process, one can formulate several hypotheses about various aspects of the queueing process.

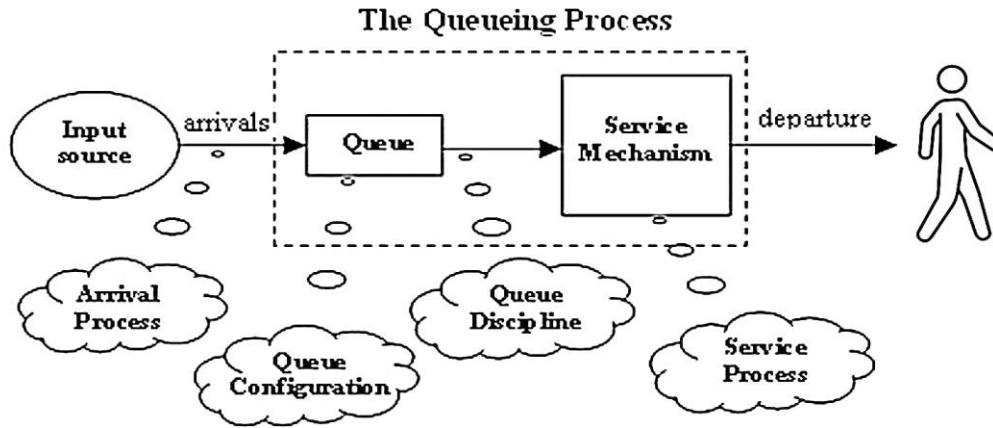


Figure 1.2. Components of a basic queueing process

If a queueing system is interpreted correctly, it is possible to characterize it as implicitly incorporated into a single-stage or multistage manufacturing process. The queueing system analysis, which can be explained as follows, can be used to analyze the manufacturing system. Several resources in a typical manufacturing system handle different kinds of products. If resources are unavailable, the pieces arriving at the various resources must wait in a queue. As a result, the system has many queues at different resource levels, and they interact with one another. The system is dynamic in that the patterns of product arrivals are contingent upon the stochastic nature of product demand.

3.1. Assumptions

The following assumptions are used in the proposed work:

- a. Characterize the demand
- b. Characterize the service rate and time
- c. Determine the performance of the system.

3.2. Notations and Terminology

Let's define the following notations:

- λ : Arrivals (the mean quantity of objects arriving in a given amount of time).
- μ : Rate of Service (the mean number of objects repaired in a given amount of time).
- ρ : Factor of utilization ($\rho = \lambda / \mu$).
- L : The mean quantity of items in the system
(including those in the line and those being serviced).
- L_q : The mean quantity of items (not including those being serviced) in the line.

W : The average amount of time an item spends in the system
(including service time and queue waiting).

W_q : The average amount of time an item is held in line.

4. The M/M/1 Queueing Model

A method for modeling and assessing the performance of a manufacturing system is necessary before we can analyze it. Based on queueing theory, there are numerous well accepted models for production systems. Let's consider a single manufacturing system with multiple processing stages. Each stage is modelled as a queue, and the arrival of items at each stage follows a Poisson process. The processing time for each item at a stage follows an exponential distribution. Here, we enumerate the manufacturing system's performance using the following metrics:

- **Arrival Rate (λ_i):** The arrival rate at each stage i is clarify as the mean number of items arriving at a fix interval of time. The inter-arrival time between consecutive items at each stage follows an exponential distribution with rate λ_i .
- **Service Rate (μ_i):** The service rate at each stage i is clarify as the mean number of items processed at a fix interval of time. The service time for each item at stage i follows an exponential distribution with rate μ_i .
- **Utilization Factor (ρ_i):** The utilization of each stage i represents the proportion of time the stage is busy processing items. It can be calculated as $\rho_i = \frac{\lambda_i}{\mu_i}$.
- **Little's Law:** Little's Law states that the average number of items (L_i) in a stable system is equal to the arrival rate (λ_i) multiplied by the average time a customer spends in the system (W_i), mathematically represented as $L_i = \lambda_i * W_i$.
- **Length of Queue:** The difference between the number of clients receiving service and the figure of clients wait for service to start determines the length of the queue.
- **Time of Waiting:** If we assume that mean service time is constant $1/\mu$ for all $n \geq 1$, and let W_q be the waiting time in the queue excluding service time for each individual client, then waiting time can be given by

$$W = W_q + \frac{1}{\mu}$$

- **Average Length of Queue:** The average length of queue is shown by the amounts of clients in the line per unit of time. The average length of queue is determined by the probability P_n that there are precisely n consumers in the queuing framework:

$$L = \sum_{n=0}^{\infty} nP_n$$

- **Busy and Idle Phase:** A server's busy phase is when he is working on serving customers. It is therefore the amount of time that passes between the customer's start of service and the last client in line's conclusion of service. The server's idle time starts when all of the customers in the queue are served and lasts until the client arrives. A server's idle period is the time when he is available because there isn't a customer using the system.

For an **M/M/1** queuing model, we have the following mathematical equations:

Utilization Factor (ρ): $\rho = \frac{\lambda}{\mu}$

The mean quantity of items in the system (L): $L = \frac{\lambda}{(\mu - \lambda)}$

The mean quantity of items in the line (L_q): $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$

The mean amount of time an item spends in the system (W): $W = \frac{1}{(\mu - \lambda)}$

The mean amount of time an item is held in line (W_q): $W_q = \frac{\lambda}{\mu(\mu - \lambda)}$

Keep in mind that these equations are specific to the M/M/1 queue model, which is just one of many possible queueing models used in manufacturing processes.

5. Performance Measures of Manufacturing System

In order to analyze and compare production systems, queuing theory develops mathematical models to investigate congestion conditions. The question is, what standards do we use to assess and contrast production systems? Queue analysis is an essential resource for efficiently building queueing systems in various frameworks. Manufacturing systems frequently use the M/M/1 model, a queueing

model that assumes a single-server system with exponentially distributed task arrivals and service times. A service system's performance can be defined using a number of general performance metrics. We used the following performance metrics to analyze the queueing models in this study:

- **Throughput (X_i):**

The throughput at each stage i represents the average number of items leaving the stage per unit of time. It can be calculated as

$$X_i = \lambda_i * (1 - P_i),$$

where P_i is the probability of the queue being empty, given by

$$P_i = (1 - \rho_i).$$

- **Cycle Time (C_i):**

The cycle time at each stage i represents the mean time of a single item that spends in the system, with both waiting and processing time. It can be calculated by

$$C_i = 1 / (\mu_i - \lambda_i), \text{ assuming the system is stable.}$$

- **Length of Queue (Q_i):**

The average amount of items waiting in the line at each stage i can be numerate using the Little's Law as

$$Q_i = \lambda_i * W_i.$$

The derivation of the proposed model is based on fundamental principles of queueing theory, including the Poisson arrival process and the exponential service time distribution. By employing Little's Law and considering the stability conditions, we can prove the accuracy and validity of the model.

6. Numerical Analysis

To illustrate the performance analysis of manufacturing systems using queueing theory, let's consider an example of a simple manufacturing system with two processing stages: Stage A and Stage B.

- **System Description:**

Stage A: This is the first processing stage, where items arrive to be processed. Let the arrival rate at Stage A is $\lambda_A = 10$ items per hour and the service rate at Stage A is $\mu_A = 12$ items per hour.

Stage B: After processing at Stage A, the items move to Stage B. Let the arrival rate at Stage B is $\lambda_B = 9$ items per hour, and the service rate at Stage B is $\mu_B = 10$ items per hour.

- **Utilization Calculation:**

Utilization (ρ) is the proportion of time a resource is busy processing items. For each stage, we can calculate the utilization as follows:

Stage A: $\rho_A = \frac{\lambda_A}{\mu_A} = \frac{10}{12} \approx 0.8333$ (83.33%)

Stage B: $\rho_B = \frac{\lambda_B}{\mu_B} = \frac{9}{10} \approx 0.9$ (90%)

- **Throughput Calculation:**

Throughput (X) is the average number of items leaving a stage per unit of time. For each stage, we can calculate the throughput as follows:

Stage A: $X_A = \lambda_A * (1 - P_A)$, where P_A is the probability of the queue being empty at Stage A.

Stage B: $X_B = \lambda_B * (1 - P_B)$, where P_B is the probability of the queue being empty at Stage B.

To calculate the probabilities (P_A and P_B), we use the formula for an M/M/1 queue:

$$P_A = \frac{1}{1 + \rho_A} = \frac{1}{1 + 0.8333} \approx 0.5455$$
 (54.55%)

$$P_B = \frac{1}{1 + \rho_B} = \frac{1}{1 + 0.9} \approx 0.5263$$
 (52.63%)

Now, we can calculate the throughputs:

Stage A: $X_A = 10 * (1 - 0.5455) \approx 4.545$ items per hour

Stage B: $X_B = 9 * (1 - 0.5263) \approx 4.263$ items per hour

- **Cycle Time Calculation:**

Cycle time (C) is the average time of a single item spends in the system, including both waiting and processing time. For each stage, we can calculate the cycle time as follows:

Stage A: $C_A = \frac{1}{\mu_A - \lambda_A} = \frac{1}{12 - 10} = 0.5$ hours / item (30 minutes/item)

Stage B: $C_B = \frac{1}{\mu_B - \lambda_B} = \frac{1}{10 - 9} = 1$ hours / item

- **Queue Length Calculation:**

Queue length (Q) is the average number of items waiting in the queue at a stage. For each stage, we can calculate the queue length as follows:

Stage A: $Q_A = \lambda_A * W_A$, where W_A is the average time, a customer held in the queue at Stage A.

Stage B: $Q_B = \lambda_B * W_B$, where W_B is the average time, a customer held in the queue at Stage B.

Using **Little's Law** ($L = \lambda * W$), we can find the mean waiting times:

$$W_A = \frac{L_A}{\lambda_A}, \text{ where } L_A \text{ is the average amount of items in the line at Stage A.}$$

$$W_B = \frac{L_B}{\lambda_B}, \text{ where } L_B \text{ is the average amount of items in the line at Stage B.}$$

The average amount of items in the line at each stage can be calculated as follows:

$$\text{Stage A: } L_A = \frac{\rho_A^2}{1 - \rho_A} = \frac{(0.8333)^2}{(1 - 0.8333)} \approx 3.9984 \text{ items}$$

$$\text{Stage B: } L_B = \frac{\rho_B^2}{1 - \rho_B} = \frac{(0.9)^2}{(1 - 0.9)} \approx 8.9999 \text{ items}$$

Now, we can calculate the queue lengths:

$$\text{Stage A: } Q_A = 10 * \left(\frac{3.9984}{10} \right) \approx 3.9984 \text{ items}$$

$$\text{Stage B: } Q_B = 9 * \left(\frac{8.9999}{9} \right) \approx 8.9999 \text{ items}$$

The performance measurements would make it easier for us to calculate and research different system metrics that would aid in raising the manufacturing system's efficiency. Here we evaluate the above data by tabulation as follows:

Utilization Table:

Stage	Arrival Rate (λ)	Service Rate (μ)	Utilization ($\rho = \lambda/\mu$)
A	10 items per hour	12 items per hour	0.8333
B	9 items per hour	10 items per hour	0.9

Throughput Table:

Stage	Arrival Rate (λ)	Service Rate (μ)	Utilization (ρ)	Probability of Queue Being Empty (P)	Throughput [$X = \lambda * (1 - P)$]
A	10 items per hour	12 items per hour	0.8333	0.5455	4.545 items per hour
B	9 items per hour	10 items per hour	0.9	0.5263	4.263 items per hour

Cycle Time Table:

Stage	Arrival Rate (λ)	Service Rate (μ)	Cycle Time ($C = 1 / (\mu - \lambda)$)
A	10 items per hour	12 items per hour	0.5 hours per item
B	9 items per hour	10 items per hour	1 hours per item

Queue Length Table:

Stage	Arrival Rate (λ)	Average waiting time (W)	Queue Length ($Q = \lambda * W$)
A	10 items per hour	0.39984 hour	3.9984 items
B	9 items per hour	0.99999 hour	8.9999 items

In this example, we have populated the data tables with sample values for a manufacturing system with two stages. The Utilization table provides the value of utilization factor for both the stages. The Throughput table shows the amount of material or items passing through a system or process, which are for the Stage A: 4.545 items per hour, and for the Stage B: 4.263 items per hour. The Cycle time table describes the cycle time of items, which are for the Stage A: 0.5 hours per item, and for the Stage B: 1 hours per item. The Queue length table provides the information about the length of queue, which depends on the arrival rate of items and average waiting time in the system. Here for the Stage A: 3.9984 items, and for the Stage B: 8.9999 items. The data can be further analysed to optimize the system's performance and make informed decisions in a real-world manufacturing scenario.

7. Importance of the Study

The proposed work investigates the use of queueing theory to study industrial systems, with an emphasis on important performance measures including throughput, cycle times, queue length, and resource use. Its significance rests in the following areas: Innovative Application, Comprehensive Analysis and Theoretical Contributions, Practical Implications and Resilience and Adaptability.

The research findings have the potential to inspire future research directions, encouraging interdisciplinary relationships and the development of innovative solutions. This work is an invaluable resource for academics, practitioners, and policymakers seeking to enhance and improve their knowledge of Queueing Theory.

8. Limitations of the Study

Despite the importance, the research work has some limitations. One limitation is that the work assumes a Poisson arrival process and Exponential service times, which is a restriction. These assumptions could not always hold in real-world situations, and reducing findings accuracy. Another limitation is that the paper focuses on a specific type of manufacturing system, which might restrict its applicability to other kinds of systems. Furthermore, the study takes a lot of data, such as arrival rates, service duration, and buffer capacity, to apply queueing theory in manufacturing systems. Collecting and analyzing this data can be expensive and time-consuming. These factors can greatly affect the performance of the manufacturing system. Future research can solve these constraints and build on the findings of this research to better understand queueing theory in manufacturing systems.

9. Conclusion

Queueing theory enables us to study complex manufacturing systems with multiple processing stages and resources, offering a systematic approach to evaluate their performance. The performance analysis of manufacturing systems using queueing theory provides valuable insights into system behavior and helps in optimization and decision-making processes. By employing mathematical models and simulations, this analysis allows us to predict key performance metrics, including throughput, cycle time, and queue lengths. Understanding the system's utilization and resilience during normal operations and under the impact of unpredictable disasters is crucial for improving efficiency and productivity. The utilization analysis helps identify potential bottlenecks, while the throughput calculations guide resource allocation and capacity planning. Additionally, cycle time and queue

length analysis help minimize waiting times and optimize production processes. The resilience analysis of manufacturing systems with disasters provides valuable insights into how the system responds to disruptive events and its ability to recover efficiently. This understanding can lead to the development of mitigation strategies and adaptive policies, making the system more robust and resilient.

Performance modeling is now a crucial component of industrial system design and is also necessary to keep the system operating at its best. With the introduction of advanced robots and computer control to improve production, organizations' manufacturing practices have seen a radical transition in recent years, resulting in lower costs and higher-quality products. Performance modeling is the most effective way to improve the production lines using this modeling and investment.

Declarations:

Ethics approval and consent to participate: The submitted work is original and has not been published anywhere else in any form or language. No data, text, or theories by others are presented as if they were the author's own (plagiarism). All authors agree to be accountable for all aspects of the work.

Consent for publication: All authors have given their consent for publication of this work. All authors have agreed to share all details upon publication of the article.

Availability of data and material: This study uses qualitative research techniques to analyze the performance metrics of manufacturing system. For this study, we obtain data from secondary sources, specifically Govt. reports, magazines, newspapers, and online sources. Numerical analysis of the data will be used in this study depending on the purpose of the study.

Competing Interest: The authors have no competing interests to declare that are relevant to the content of this article.

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Disclaimer (Artificial Intelligence)

All the authors hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

REFERENCES

1. Aalto, S. & Scully, Z. (2023). Minimizing the mean slowdown in the M/G/1 queue. *Queueing Systems*, 104, 187–210. <https://doi.org/10.1007/s11134-023-09888-6>.
2. Abourizk, S. M., Halpin, D. W., Mohamed, Y. & Hermann, U. (2011). Research in modeling and simulation for improving construction engineering operations. *Journal of Construction Engineering and Management*, 137(10), 843-852. [http://dx.doi.org/10.1061/\(ASCE\)CO.1943-7862.0000288](http://dx.doi.org/10.1061/(ASCE)CO.1943-7862.0000288).
3. Aggarwal, E. & Mohanty, B. K. (2023). Hesitant fuzzy sets with non-uniform linguistic terms: an application in multi-attribute decision making. *International Journal of Mathematics in Operational Research*, 24(1) 1–28. DOI: 10.1504/IJMOR.2021.10044478.
4. Amjath, M., Kerbache, L., Smith, J. M. & Elomri, A. (2023). Optimisation of Buffer Allocations in Manufacturing Systems: A Study on Intra and Outbound Logistics Systems Using Finite Queueing Networks. *Applied Science*, 13, 9525. Doi:10.3390/app13179525.
5. Boer, H., Holweg, M., Kilduff, M., Pagell, M., Schmenner, R. & Voss, C. (2015). Making a meaningful contribution to theory. *International Journal of Operations & Production Management*, 35(9) 1231-1252. DOI:10.1108/IJOPM-03-2015-0119.
6. Chen, C. & Tiong, L. K. (2019). Using queueing theory and simulated annealing to design the facility layout in an AGV-based modular manufacturing system. *International Journal of Production Research*, 57(17), 5538-5555. DOI: 10.1080/00207543.2018.1533654.
7. Gongshan, C., Yuan, N., Lu, Y. & Yudong, G. (2020). On Production Process Optimization Based on Queueing Theory-Take Enterprise A as an Example. *Advances in Social Science, Education and Humanities Research*, 435. ICHSSR 2020.

8. Li, Yanjun (2022). Mathematical Modeling Methods and Their Application in the Analysis of Complex Signal Systems. *Hindawi Advances in Mathematical Physics* 2022, 1816814. DOI:10.1155/2022/1816814.
9. Mehra, S. & Taylor, P.G. (2023). Open networks of infinite server queues with non-homogeneous multivariate batch Poisson arrivals. *Queueing Systems*, 105, 171–187. <https://doi.org/10.1007/s11134-023-09891-x>.
10. Murdapa, P. S., Pujawan, I. N., Karningsih, P. D. & Nasution, A. H. (2018). Single stage queueing/manufacturing system model that involves emission variable. *IOP Conf. Series: Materials Science and Engineering*, 337, 012008. Doi:10.1088/1757-899X/337/1/012008.
11. Pasandideh, S. H. R. & Niaki, S. T. A. (2012). Genetic application in a facility location problem with random demand within queueing framework. *Journal of Intelligent Manufacturing*, 23, 651-659. <https://doi.org/10.1007/s10845-010-0416-1>.
12. Rece, L., Vlase, S., Ciuiu, D., Neculoiu, G., Mocanu S. & Modrea, A. (2022). Queueing Theory-Based Mathematical Models Applied to Enterprise Organization and Industrial Production Optimization. *MDPI*, 10(14), 2520. <https://doi.org/10.3390/math10142520>.
13. Saini, B., Singh, D. & Sharma, K. C. (2024). Exploring the Role of Queueing Theory in Manufacturing: An Analytical Study. *International Research Journal on Advanced Engineering and Management*, 2, 256-266. <https://doi.org/10.47392/IRJAEM.2024.0039>.
14. Selvamuthu, D. & Kapoor, S. (2023). On the time-dependent solution of fluid models driven by an M/M/1 queue using a probabilistic approach. *International Journal of Operational Research*, 46(1), 65–72. DOI:10.1504/IJOR.2023.128580.
15. Sharma, A. K. & Sharma, G. K. (2013). Queueing theory approach with queueing model: a study. *International Journal of Engineering Science Invention*, 2(2), 1-11. www.ijesi.org.
16. Ulku, S., Hydock, C. & Cui, S. (2019). Making the wait worthwhile: Experiments on the effect of queueing on consumption. *Management Science*, 66(3), 1149-1171. <https://doi.org/10.1287/mnsc.2018.3277>.
17. Vorholter, K., Greefrath, G., Borromeo, F. R., Leib, D. & Schukajlow, S.: Mathematical modelling, Traditions in German-speaking mathematics education research, ICME-13 Monographs. Springer, Cham. (2019). DOI:10.1007/978-3-030-11069-7_4.