

### Review Form 3

Journal Name:	<a href="#">Journal of Advances in Mathematics and Computer Science</a>
Manuscript Number:	Ms_JAMCS_127395
Title of the Manuscript:	Hierarchic control for a two-stroke linear system with missing data
Type of the Article	

#### **PART 1: Review Comments**

<b>Compulsory</b> REVISION comments	Reviewer's comment	Author's Feedback <i>(Please correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)</i>
Please write a few sentences regarding the importance of this manuscript for the scientific community. Why do you like (or dislike) this manuscript? A minimum of 3-4 sentences may be required for this part.		
Is the title of the article suitable? (If not please suggest an alternative title)		
Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.		
Are subsections and structure of the manuscript appropriate?		
Please write a few sentences regarding the scientific correctness of this manuscript. Why do you think that this manuscript is scientifically robust and technically sound? A minimum of 3-4 sentences may be required for this part.		
Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form. =		

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<p>Minor REVISION comments</p> <p><b>Is the language/English quality of the article suitable for scholarly communications?</b></p>		
<p>Optional/General comments</p>	<p><b>Referee Report</b></p> <p><b>Hierarchic control for a two-stroke linear system with missing data</b></p> <p>This is an interesting study of a new hierarchical control problem for a linear two-stroke missing data problem, adjoint to an age and space structured single species population dynamics problem. The Authors consider a population with age dependence and spatial structure, and they assume that the population lives in a bounded domain.</p> <p>The Authors show that there are two controls such that the first control, called the follower, solves an optimal control problem which consists in bringing the state of the two-stroke linear system to a desired state, and the second control, called the leader, solves a null controllability problem. The results are obtained by means of an observability inequality associated with a homogeneous Dirichlet boundary condition.</p> <p>They propose a Stackelberg control problem with the supply of the invasive species as unknown, believe that such a Stackelberg control problem has not yet been considered. This problem is well-posed, they can not directly solve the associated optimal control problem but use of the low-regret control.</p> <p>The main difficulty lies in obtaining the Carleman-type observability inequality associated with the adjoint system, where we have sometimes had to resort to the Poincare inequality.</p> <p>The system of this problem is the adjoint of a population dynamics problem which, based on environmental sciences, can model an optimization process aimed, for example, at eradicating a harmful population such as armyworms in a cotton field. The presence of these pests can have devastating effects on crops, leading to significant losses in terms of yield and quality. This model plays a crucial role in biodiversity management system, as it seeks to maximize the protection of agricultural crops while minimizing the damage caused by these pests. The region represents the cultivated cotton field where the goal is to control and eventually eradicate the armyworms. The variable <math>u = u(t, a, x)</math> represent the natural mortality rate of individuals as a function of their age <math>a</math>, time <math>t</math>, and position <math>x</math> in a given domain. Models the natural mortality rate of the population, meaning it acts as a diminishing factor for the population. Mortality depends on three variables time, age, and position allowing for the representation of a population with spatial structure and dependencies on time and age. It is decomposed into <math>u_0(a)</math>, which depends solely on age <math>a</math>, and a function <math>u_1(t, a, x)</math>, which depends on all three variables. This decomposition distinguishes an intrinsic mortality component related to age from the temporal and spatial components. <math>u_1(t, a, x) &gt; 0</math>, ensuring that natural mortality does not negatively reduce the <math>u_0(a) &gt; 0</math>, so that intrinsic age-based mortality is positive.</p> <ul style="list-style-type: none"> <li>➤ <b>Originality of the approach to implementation:</b> The authors have developed some known methods to get new results for the considered problems.</li> <li>➤ <b>Keywords</b> Optimal control, Carleman inequality, Null controllability, Missing data, Population dynamics, Low regret control, Euler- Lagrange formula.</li> <li>➤ <b>Generalizability of the methods and results described:</b> Through hierarchical control, they have demonstrated the existence and uniqueness of two controls. This problem was ill-posed. This is why they have used the least regret control developed by Lions and adapted to problems with missing data. After establishing a Carleman observability inequality, they were able to solve the null controllability problem associated with the system. In perspective they will apply the Stackelberg-Nash strategy to this problem.</li> </ul>	

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	<p>➤ <b>Quality of the contribution to the implementation community:</b> I think the result of this paper will have a positive impact on the study of the Conservation Biology, Epidemiology, Demography Mathematical and Statistical Modeling, and Ecology.</p> <p>➤ <b>Clarity of presentation:</b> The paper is well presented and it can be understood easily and clearly. Moreover, it is well typed and organized correctly.</p> <ul style="list-style-type: none"> <li>- The mathematical formulae, symbols, abbreviations, and units are correctly defined and used. The over all presentation is well structured and clear generally.</li> <li>- The number and quality of references is appropriate and gives proper credit to related work.</li> </ul> <p>➤ <b>Scientific Quality (Rigour): Good</b></p> <ul style="list-style-type: none"> <li>- The paper does present the information in a straightforward and understandable manner. The scientific methods and assumptions are valid and clearly outlined.</li> <li>- The description of data and the calculations are sufficiently complete to be followed and would allow their reproduction by fellow scientists. This is the strength of the paper.</li> <li>- The paper is generally well written and structured. In my opinion the paper would be useful to a vast community of theoreticians working in the mechanical and engineering problems with energy functionals.</li> </ul> <p>➤ <b>Decision</b> I highly recommend this article <b>Ms_JAMCS_127395</b> to be published in the journal <a href="#">Journal of Advances in Mathematics and Computer Science</a></p>	
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**PART 2:**

	<b>Reviewer's comment</b>	<b>Author's comment</b> (if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)
<b>Are there ethical issues in this manuscript?</b>	<i>(If yes. Kindly please write down the ethical issues here in details)</i>	

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