

Review Form 3

Journal Name:	Asian Research Journal of Mathematics
Manuscript Number:	Ms_ARJOM_127792
Title of the Manuscript:	ON FUZZY SOBOLEV SPACES $\tilde{W}^{(1,p)}(\Omega)$: ANALYSIS OF DOT PRODUCT AND FUZZY NORM
Type of the Article	Original Research Article

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PART 1: Comments

	Reviewer's comment	Author's Feedback <i>(Please correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)</i>
Please write a few sentences regarding the importance of this manuscript for the scientific community. A minimum of 3-4 sentences may be required for this part.	The Sobolev spaces, particularly the weak ones, are mathematically fundamental as they provide a rigorous framework for studying functions with weak derivatives, extending the concept of differentiation to functions that may not be classically differentiable. They play a crucial role in embedding theorems, demonstrating that certain properties of integrability and regularity in a Sobolev space ensure inclusion in other functional spaces, thereby facilitating boundary problem analysis. In practical applications, Sobolev spaces are essential for solving partial differential equations (PDEs), allowing for the formulation and resolution of variational problems critical in physics and engineering. They are also used to model physical phenomena in fields such as fluid mechanics and heat theory, where solutions may be discontinuous or non-smooth but can still be analyzed through weak derivatives. Furthermore, in numerical analysis, Sobolev spaces are important for the convergence of numerical methods, such as the finite element method, which is employed to approximate solutions to complex problems in engineering and applied sciences. Fuzzy Sobolev spaces extend the classical Sobolev spaces by incorporating fuzzy logic, allowing for the representation of uncertainty and imprecision in functions. This is particularly important in applications where data is not precise, such as in engineering and decision-making processes, enabling more flexible modeling of real-world phenomena. Mathematically, fuzzy Sobolev spaces maintain the essential properties of regularity and integrability while accommodating fuzziness, thus providing a robust framework for analyzing problems with inherent uncertainty. They facilitate the study of partial differential equations in fuzzy environments, where traditional methods may fail, ensuring that solutions can still be derived and understood. Overall, fuzzy Sobolev spaces enhance both theoretical and practical approaches to complex problems, bridging the gap between classical analysis and real-world applications involving uncertainty.	
Is the title of the article suitable? (If not please suggest an alternative title)	The title of the article is appropriate and very precise, it responds to a general description of the topic and specifies the contribution.	
Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.	The abstract is well structured, as it starts with the objective and describes the content in a synthetic way. It could be structured as two paragraphs or as just one.	
Is the manuscript scientifically, correct? Please write here.	The article is well written. The definitions necessary to understand the contribution are used, as well as previous results that have been published. Mathematical rigor and quality in the presentation of the reasoning are appreciated.	
Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form.	References are appropriate and correspond to the logical sequence of the article. References from the last five years could be used, but not necessarily. For example, in the introduction to the work, a paragraph could be included with a brief overview of the research that requires Sobolev's diffuse spaces. Not only does this help to "refresh" the references, but it is also used to better understand the assessments made in the conclusions of the work. Reference [16] is truncated. Reference [21] is truncated and off-margin. Reference [18] needs to correct the initial capital letter of Wang. In general, minor details related to spacing before commas and full stops need to be corrected in all references.	
Is the language/English quality of the article suitable for scholarly communications?	The article is well written in English. In some cases, it is necessary to correct typographical problems such as not leaving a space between some mathematical equations and the text that follows.	
Optional/General comments	It would be useful to reflect in the conclusions of the article on new open problems that arise from the research. For example, what other types of properties can be investigated, what would happen in generalizations (as in the fuzzy Pythagorean approach), etc.	

PART 2:

	Reviewer's comment	Author's comment <i>(if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)</i>
Are there ethical issues in this manuscript?	<i>(If yes, Kindly please write down the ethical issues here in details)</i>	

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