

**Review Form 3**

Journal Name:	<a href="#">Asian Research Journal of Mathematics</a>
Manuscript Number:	Ms_ARJOM_129384
Title of the Manuscript:	STOCHASTIC SELF- STABILIZATION OF A NON-LINEAR OPTIMAL CONTROL DELAY DIFFERENTIAL EQUATIONS IN AN ALMOST SURE EXPONENTIAL SENSE BY - ITO TYPE BROWNIAN WHITE NOISE .
Type of the Article	Research Article

**General guidelines for the Peer Review process:**

This journal's peer review policy states that **NO** manuscript should be rejected only on the basis of '**lack of Novelty**', provided the manuscript is scientifically robust and technically sound. To know the complete guidelines for the Peer Review process, reviewers are requested to visit this link: <https://r1.reviewerhub.org/general-editorial-policy/>

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**PART 1: Comments**

	<b>Reviewer's comment</b>	<b>Author's Feedback</b> <i>(Please correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)</i>
<b>Please write a few sentences regarding the importance of this manuscript for the scientific community. A minimum of 3-4 sentences may be required for this part.</b>	The field addresses the open problem of how to guarantee stability in systems with both delay and noise, which are inherently destabilizing factors. This advances the broader understanding of how stochastic systems behave under real-world conditions. In summary, this topic contributes to advancing the theoretical understanding and practical capability to manage complex, noisy, and delayed systems, making it a cornerstone of modern scientific and engineering efforts. This topic is significant for the scientific community due to its relevance in various theoretical and practical fields.	
<b>Is the title of the article suitable? (If not please suggest an alternative title)</b>	The title of the topic is very cumbersome and not having very good readability and understanding. To maintains the technical rigor while streamlining the phrasing for better readability and academic clarity, this title can be replaced as <b>""Exponential Almost-Sure Stabilization of Nonlinear Delay Differential Systems Under Stochastic Optimal Control Driven by Ito Brownian Noise"</b>	

**Review Form 3**

<p><b>Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.</b></p>	<p>The abstract provides a concise summary of the study, its methodology, and its findings, but it could benefit from further refinement to enhance clarity, impact, and readability for publication. Below are some strengths and areas for improvement:</p> <p><b>Strengths of the abstract:</b></p> <ol style="list-style-type: none"> <li><b>Key Focus Identified:</b> The abstract highlights the primary goal: stabilizing deterministic nonlinear optimal control delay differential equations using Brownian white noise.</li> <li><b>Methodology Mentioned:</b> It references the use of the Lyapunov sample exponent and Mao's partial differential equation, which adds credibility.</li> <li><b>Key Findings:</b> The abstract clearly states the main result—stochastic perturbation enables self-stabilization in an almost sure exponential sense, contrasting it with the deterministic case.</li> </ol> <p><b>Suggestions for Improvement:</b></p> <p><b>1. Clarity and Grammar:</b> Certain sentences are difficult to follow due to grammatical issues or awkward phrasing. For instance, "were noise is absence" should be "where noise is absent." Use consistent and precise terminology, e.g., "Brownian white noise" might simply be "Brownian motion" if appropriate.</p> <p><b>2. Background Context:</b> The abstract could benefit from a brief mention of why the topic is significant or its practical applications.</p> <p><b>3. Omission of Superfluous Words:</b> The phrase "This study is focused on" can be shortened to "This study explores." Similarly, "It was established that" can be rephrased as "The study demonstrates that."</p> <p><b>4. Results and Implications:</b> The implications of the findings should be stated more clearly to highlight the novelty or potential applications of the work.</p> <p><b>5. Technical Precision:</b> If Mao's work (1967) is a crucial reference, it might be beneficial to specify its significance briefly or rephrase to avoid confusion over its relevance.</p> <p><b>The revised abstract can be written as follows:</b> "This study investigates the role of Brownian white noise in stabilizing nonlinear optimal control delay differential equations that are typically unstable in their deterministic form. Using the Lyapunov sample exponent and a specialized partial differential equation suggested by Mao (1967), it is demonstrated that stochastic perturbations with finite noise scaling parameters induce self-stabilization in an almost sure exponential sense. This phenomenon, which is absent in deterministic systems without noise, highlights the potential of stochastic approaches for enhancing stability in delay differential systems."</p>	
<p><b>Is the manuscript scientifically, correct? Please write here.</b></p>	<p>Technically and scientifically, by observing the details of the proofs and references, there is no doubt about the correctness of the article. The contents along with the proof of lemma and theorem are explained step by step in detail. The introduction covers delay differential equations (DDEs) extensively, highlighting their applications across various disciplines. The inclusion of stability concepts and references to key works in the field demonstrates a solid theoretical basis.</p> <p><b>But still there are few issues in this manuscript that should be improved as follows:</b></p>	

Review Form 3

	<p><b>Clarity and Organization:</b></p> <ol style="list-style-type: none"><li>1. The text is verbose and lacks focus. For instance, the phrase "delay differential equations occur everywhere in our daily real-life situations" is too broad and informal for a scientific paper.</li><li>2. The references to various researchers and their contributions (e.g., Mao, Shen, Wei) need better integration. Currently, it reads more like a literature review than a structured introduction.</li></ol> <p><b>Scientific Correctness:</b></p> <ol style="list-style-type: none"><li>1. Some statements are vague or overly generalized, such as "Solutions of linear and nonlinear ODEs and ODDEs usually display certain qualitative properties."</li><li>2. Not all cited references are contextualized or connected to the specific problem addressed in the manuscript.</li><li>3. The central premise of the study—using stochastic perturbations (Brownian noise) to stabilize otherwise unstable nonlinear DDEs—is scientifically valid and aligns with established research.</li><li>4. The use of tools like Lyapunov exponents and Ito calculus is appropriate for analyzing stochastic systems.</li><li>5. The manuscript references several relevant studies, such as those by Mao (1997), Shen et al. (2021), and Wei (2019). However, the connections between these studies and the manuscript's contributions need to be more explicitly stated.</li><li>6. The role of Mao (1967) is mentioned but not adequately contextualized. The abstract lacks a clear explanation of the broader implications of the findings.</li><li>7. No keyword under the section of abstract is mentioned.</li></ol>	
<p><b>Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form.</b></p>	<p><b>Weaknesses and Areas for Improvement:</b></p> <ol style="list-style-type: none"><li>1. The title emphasizes "stochastic optimal control" and "Ito Brownian noise." While foundational references are included, more works on stochastic <b>optimal control frameworks</b> and explicit treatment of <b>Ito Brownian motion</b> could strengthen the list.</li><li>2. While there are works from 2021, more publications from 2022 onward would make the references feel more up-to-date. This is especially critical in a fast-evolving field like stochastic analysis.</li><li>3. To Search two or three publications from 2022 onward to ensure the manuscript reflects the latest advances.</li><li>4. Recent studies specifically addressing stochastic optimal control and stabilization techniques with Brownian noise, such as:<ol style="list-style-type: none"><li>I. Journals: SIAM Journal on Control and Optimization, Journal of Dynamics and Differential Equations.</li><li>II. Authors working on similar topics (search for recent works by experts in stochastic control and Ito processes).</li></ol></li></ol>	

Review Form 3

<p><b>Is the language/English quality of the article suitable for scholarly communications?</b></p>	<p><b>Issues with the Language:</b></p> <p><b>1. Grammar and Syntax:</b> Several sentences are awkwardly phrased, impacting readability and clarity.</p> <p>Example: "This can never happen in the deterministic optimal control delay differential equations where noise is absence." Correction: "This phenomenon does not occur in deterministic optimal control delay differential equations where noise is absent."</p> <p><b>2. Word Choice:</b> Some terms are improperly used or inconsistent: "White noise" and "Brownian noise" need clarification to avoid confusion. Replace "e t c" with "etc." or a more formal phrase like "among others."</p> <p><b>3. Redundancy and Repetition:</b> Ideas are repeated unnecessarily, making the text verbose. Example: The concept of stability is elaborated multiple times without adding new information.</p> <p><b>4. Lack of Formality:</b> Informal phrases and conversational tone detract from the scholarly tone. Example: "occur everywhere in our daily real life situations." Correction: "are prevalent in various real-world scenarios."</p> <p><b>5. Missing Transitions:</b> The text lacks smooth transitions between sentences and paragraphs, leading to abrupt shifts in topics.</p> <p><b>6. Punctuation and Formatting:</b> Incorrect punctuation usage, such as missing commas, periods, and unnecessary spaces. Example: "Delay differential equations can be applied in the modeling of the dynamics of control systems, the stability of mechanical structures and the behavior of materials, in biological systems, delay differential equations can be used..." Correction: "Delay differential equations are used to model the dynamics of control systems, the stability of mechanical structures, and the behavior of materials. In biological systems, they are employed to model..."</p>	
<p><u>Optional/General</u> comments</p>	<p><b>Strengths:</b></p> <p>1. The article addresses an important area of research: the stabilization of deterministic nonlinear delay differential systems under stochastic influences. This topic is relevant in both theoretical and applied mathematics, particularly in engineering, physics, biology, and economics.</p> <p>2. Use of Established Techniques such as the application of Lyapunov methods, stochastic perturbations, and specific stochastic control techniques demonstrates familiarity with advanced mathematical tools.</p> <p><b>Suggestions for Improvement:</b></p> <p><b>Language Editing:</b> The manuscript should undergo thorough proofreading to improve grammar, syntax, and clarity. Using professional editing services or tools would enhance readability.</p> <p><b>Improved Structure:</b> Include distinct sections for: Abstract Introduction</p>	

**Review Form 3**

	Literature Review Methodology Results Discussion Conclusion Clearly articulate the research question and the paper's unique contributions. <b>Focus on the Research Contributions:</b> Provide a clear explanation of how they advance the field. Compare the proposed techniques and results with existing approaches to demonstrate the improvement. <b>Abstract Refinement:</b> Rewrite the abstract to provide a concise summary of the problem, methodology, key results, and significance.	
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**PART 2:**

	<u>Reviewer's comment</u>	<u>Author's comment (if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)</u>
<u>Are there ethical issues in this manuscript?</u>	<u>(If yes, Kindly please write down the ethical issues here in details)</u>	

**Reviewer Details:**

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