

Developing a Model for Open Channel Fluid Flow with a Segment Base having Lateral Inflow Channel

ABSTRACT

An open channel fluid flow is characterized by presence of a free surface. The interface between two homogeneous fluids of different densities is regarded as free surface. It is the surface of the liquid that is in contact with air. Generally, this interface is subject to zero parallel shear stress. The survival of lives and livelihoods has greatly been hampered by occurrence of floods. When there is heavy downpour, accumulation of flooded water has led to bridges being washed away, increased pot holes on the roads and this has led to increased cases of accidents leading to loss of lives. This has posed a huge financial burden to the Government in terms of budgetary allocations to import human capital for maintenance and repair of worn out roads and bridges. This study has developed a model for fluid flow past an open channel with a trapezoidal cross-section with a segment base having lateral inflow channel that has optimal dimensions for maximum discharge. The fluid particles throughout the flow do not crisscross each other and hence the entire flow is assumed to be laminar. The developed model equations are non-dimensionalized, discretized and solved using finite-difference method and numerical values are simulated using Matlab Mathematical software. The findings are discussed, analyzed and presented graphically. It is reported that an increase in length of the lateral channel leads to decrease in flow velocity of the main channel. An angle of inclination of the lateral channel at a range of 30° to 45° exhibit higher values of flow velocity in the main channel compared to other angles. However, maximum velocity at the main channel is attained at an inclination angle of 30° . At this angle, there is minimum shear stress hence less resistance to the flow profile. The results of this study is highly applicable in the design of drainage systems for road construction, sewer building, street drainage, airport construction and dams for electric power plants in Kenya and elsewhere.

Key Words: Lateral Channel, Laminar, Homogeneous, Angle of Inclination, Shear Stress

1. INTRODUCTION

The accumulation of stagnant water occurs when the velocity reaches a steady value where the gravitational force is equal to the resistance to the flow [10]. Water flows through a channel, whose cross-section may be closed or open at the top [14]. The structures with closed tops are referred to as closed channels while those whose tops are in contact with the atmospheric air are called open channels [2].

Open channels have been with man for long time. To divert flood water from flood-stricken areas to the desired areas such as farms, dams and lakes, man has constructed open channels of different cross-sectional shapes where water flows with a free surface [3]. The free surface is the interface between two fluids of different densities [4].

Most road networks, both in urban and rural areas lack efficient drainage systems [4]. When it rains, flooded water as well as overhead run off has led to road carnage, fatalities and other forms of economic devastation [8]. This has impacted negatively on the achievement of Kenya's vision 2030 that aims to create a high-quality, internationally competitive and prosperous Nation around the globe. The government of Kenya aims to accomplish the three main pillars that are anchored on economic, political and social platforms in line with vision 2030 [7]. This study is connected to these pillars in that poor drainage systems directly or indirectly affects the economic, political and social aspects of the nation at large.

Floods at large hamper transport since roads are cut off by runoff and this affects the flow of goods and services [12]. Disease outbreaks and other related health issues pose a danger to the population's health if drainage is poor [5]. As a result, this study seeks to find solutions to these drainage related issues in order to contribute to the vision 2030.

Chagas and Souza [1] sought to provide solution of Saint Venant's Equation to study flood in rivers through Numerical methods. The study emphasized on discretization for the equations that govern the propagation of a flood wave, in natural rivers, with the objective of a better understanding of this propagation process. The findings showed that the hydraulic parameters play important role in the propagation of a flood wave.

Moshirvaziri *et al* [6] examined numerically, the nature of pollutant connectivity between unsealed forest roads and adjacent nearby streams in terms of spatial and temporal

patterns of runoff generation, erosion, and sediment transport with the aim of improving the ability to scale-up the impacts of forest roads on catchment water quality in future works. The study considered the relative effects of rainfall intensity and duration, surface roughness, infiltration rate, macro pore flow and sediment detachment and transport with an objective of identifying the dominant processes and parameters that affect the degree of pollutant connectivity between roads and streams. The study applied St. Venant equations to extract a two-dimensional diffusion wave model with variable conductivity and diffusivity. Through the study, the behavior of flow dynamics was represented mathematically.

For open rectangular channels with lateral inflow, the study conducted by Macharia *et al* [2] reported that increasing the length of the lateral inflow channel led to a decrease in the flow velocity in the main channel. Moreover, increasing the velocity of fluid in the lateral inflow channel led to an increase in the fluid flow velocity in the main channel. The study reported that an angle of inclination between thirty and fifty degrees of the lateral inflow channel led to higher fluid flow velocity in the main channel compared to other angles.

Mohammed [13] studied how the discharge coefficient varies with respect to the side of the channel wall in the flow direction for four different angles using an oblique weir. The four angles were 30° , 60° , 75° and 90° of which all were varied along the flow direction. The findings established that maximum discharge was achieved at angle 30° of the side weir compared to that of other angles in consideration

Chirchir *et al* [7] conducted a study on the effect of varying the inclination angle of two lateral inflow channels on the flow velocity of the main rectangular channel. The findings established that the flow velocity at the main channel increases at inclination angles of 30° and 72° , but the flow velocity is at maximum at an inclination angle of 45° to the main channel.

Rotich [8] developed a model of fluid flow past an open channels with parabolic cross-section. The study reported that when the slope of the channel and energy coefficient is increased, the velocity of the fluid flow is increased. Further, when the top width is decreased, the velocity along the channel increases.

Norman *et al* [11] studied the rate of discharge through a culvert and established that

discharge is controlled by inlet or outlet conditions. Inlet control means that the flow through a culvert is limited by culvert entrance characteristics. Outlet means that flow through a culvert is limited by friction between the flowing water and culvert barrel.

Marangu *et al* [9] developed a model on open trapezoidal channel with a segment base. The study reported that increasing the cross sectional area of the flow leads to decrease in the flow velocity. An increase in channel radius and manning coefficient of surface roughness leads to a decrease in flow velocity. As the channel slope increases, the flow velocity along the channel also increases.

The study by Mose *et al* [5] developed a model on fluid flow past an open channel with elliptic cross-section. The findings established that an increase in channel friction resulted to decrease in fluid flow velocity. An increase in hydraulic radius leads to an increase in fluid flow depth.

In order to minimize floods, Engineers have designed channels of different cross-sections to convey maximum discharge to designated areas but still the problem of flooding continue to persist. The analysis of this study focuses on appropriate cross sectional area and surface roughness of the lateral channel to align with the main channel in order to maximize discharge of water from flooded areas, which is a frequent occurrence in rainy seasons. A lot of study appears to have been done in open channels of rectangular, elliptic and circular cross-sections. The study on trapezoidal channel with a base segment and a lateral inflow channel, on the other hand, has received little attention. As a result, this study aims at developing a model of a trapezoidal cross-section with a segment base having a lateral inflow channel that will have maximum discharge of water.

2. RESEARCH OBJECTIVES

- i. To develop a model for fluid flow past an open channel with a trapezoidal cross-section with a segment base having lateral inflow channel.
- ii. To determine the effect of the length of the lateral channel to the flow velocity of the main channel
- iii. To determine the effect of angle of inclination of the lateral channel on the flow velocity of the main channel

3. BASIC ASSUMPTIONS

- i. The fluid has an invariant density.
- ii. The cross-section area of the lateral channel is half that of main channel.
- iii. The fluid in consideration is Newtonian.
- iv. The flow is natural hence is caused by gravitational forces only
- v. The flow is laminar and unsteady.

4. GEOMETRIC REPRESENTATION OF THE MODEL

Figure 1 illustrates the geometric model of the trapezoidal channel with a segment base, having the trapezoidal lateral inflow channel at an angle. The discharge in the main channel and the lateral inflow channel is denoted by Q and q respectively, while θ and L represent the inclination angle and length respectively of the lateral inflow channel.

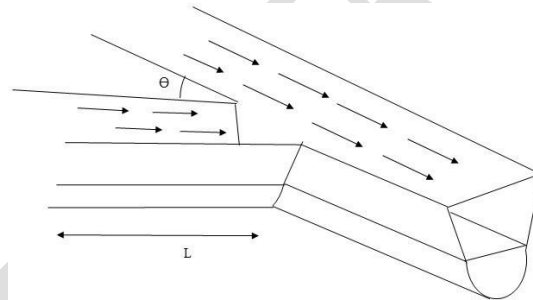


Figure 1: Geometric representation of the model

5. THE MATHEMATICAL FORMULATION OF THE MODEL

5.1 Continuity Equation

The continuity equation is based on the principal of conservation of mass, which states that mass cannot be created nor destroyed. For any given shape of an open channel with a lateral inflow channel, the continuity equation for unsteady incompressible fluid flow is given by equation (1.1)

$$\frac{A}{T} \frac{\partial V}{\partial x} + \frac{\partial y}{\partial t} + V \frac{\partial y}{\partial x} = \frac{q}{LT} \sin \theta \quad (1.1)$$

Consider Figure 2, the cross section of a trapezoidal channel with a segment base.

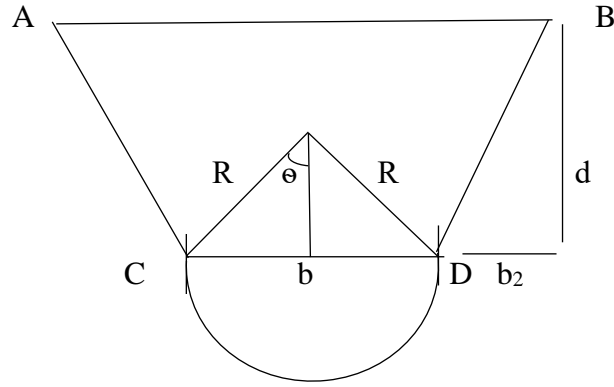


Figure 2: Cross-section of a trapezoidal channel with a segment base

The cross-sectional area of trapezium ABCD (figure 2) is given by equation 1.2

$$A_1 = (b_2d^2 + bd) \quad (1.2)$$

while that of the bottom segment is given by equation 1.3

$$A_2 = R^2(-\sin \theta \cos \theta + \theta) \quad (1.3)$$

Total area of the cross-section is thus given by equation 1.4

$$A = (b_2d^2 + bd) + R^2(-\sin \theta \cos \theta + \theta) \quad (1.4)$$

Let the length b increase by a factor b_1 , and Radius R increase by a factor R_1 . The new cross section area, A is given by equation 1.5

$$A = [b_2d^2 + (b + b_1)d] + [(R + R_1)^2(-\sin \theta \cos \theta + \theta)] \quad (1.5)$$

Let the angle of inclination be defined by $\frac{\pi}{m}$ where m is the set of positive integers and the length of the channel be $\gamma+L$ where γ is the set of positive integers. The lateral discharge along the direction of flow will be defined by $\frac{1}{2} \frac{q}{(\gamma+L)T} \sin \frac{\pi}{m}$. Therefore, the modified continuity equation becomes equation 1.6

$$\frac{[b_2d^2 + (b + b_1)d] + [(R + R_1)^2(-\sin \theta \cos \theta + \theta)]}{T} \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = \frac{1}{2} \frac{q}{(\gamma+L)T} \sin \frac{\pi}{m} \quad (1.6)$$

5.2 Momentum Equation

The equation is derived from the Newton's second law of motion that relates the sum of forces acting on the fluid element to the rate of change of momentum. According to the conservation law, the momentum equation is defined by equation 1.7

$$V \frac{\partial V}{\partial x} + g \frac{\partial v}{\partial x} + \frac{\partial V}{\partial t} + g(S_f - S_0) = \frac{q}{AL} \sin \theta (u \cos \theta - V) \quad (1.7)$$

Where A is the cross section area and u is the flow velocity of the channel.

With previous assumptions and that the cross-section area of the lateral channel is half the cross-section area of the main channel, the modified momentum equation becomes equation (1.8)

$$V \frac{\partial V}{\partial x} + g \frac{\partial v}{\partial x} + \frac{\partial V}{\partial t} + g(S_f - S_0) = \frac{q}{[b_2 d^2 + (b + b_1)d] + [(R + R_1)^2(-\sin \theta \cos \theta + \theta)] (\gamma + L)} \sin \frac{\pi}{m} (u \cos \frac{\pi}{m} - V) \quad (1.8)$$

6. METHODOLOGY

The model equations are first expressed in dimensionless form, discretized and then solved using finite difference method. Finite difference method has the advantage over other methods because of its highly consistency, convergence and stability for one dimensional fluid flows. Numerical values are simulated using Matlab mathematical software, where the velocity profiles of the main channel are plotted against time as various parameters are varied. The results are demonstrated graphically for various values of the parameters involved in the study.

6.1 Model Equations in finite difference form

In order to express the model equations in finite difference form, the Taylor series approximations are taken as equations 1.9a, 1.9b, 1.9c, 1.9d.

$$\frac{\partial v}{\partial t} = \frac{v(i, j + 1) - v(i, j)}{\Delta t} \quad (1.9a)$$

$$\frac{\partial y}{\partial t} = \frac{y(i, j + 1) - y(i, j)}{\Delta t} \quad (1.9b)$$

$$\frac{\partial v}{\partial x} = \frac{v(i+1,j) - v(i-1,j)}{2\Delta x} \quad (1.9c)$$

$$\frac{\partial y}{\partial x} = \frac{y(i+1,j+1) - y(i-1,j)}{2\Delta x} \quad (1.9d)$$

Substituting these equations in our model Continuity equation we get equation 1.10

$$\begin{aligned} v(i,j) \frac{y(i+1,j) - y(i-1,j)}{2\Delta x} + \left(\frac{[b_2 d^2 + (b + b_1)d] + [(R + R1)^2(-\sin \theta \cos \theta + \theta)](\gamma + L)}{T} \right) \frac{v(i+1,j) - v(i-1,j)}{2\Delta x} \\ + \frac{y(i,j+1) - y(i,j)}{\Delta t} = \frac{1}{2} \frac{q}{(\gamma + L)T} \sin \frac{\pi}{m} \end{aligned} \quad (1.10)$$

Rearranging equation 1.10 yields equation 1.11

$$\begin{aligned} y(i, j + 1) = \Delta t \left\{ -v(i, j) \frac{y(i+1, j) - y(i-1, j)}{2\Delta x} \right\} - \\ \Delta t \left\{ \left(\frac{[b_2 d^2 + (b + b_1)d] + [(R + R1)^2(-\sin \theta \cos \theta + \theta)](\gamma + L)}{T} \right) \frac{v(i+1, j) - v(i-1, j)}{2\Delta x} + \frac{1}{2} \frac{q}{(\gamma + L)T} \sin \frac{\pi}{m} \right\} + y(i, j) \end{aligned} \quad (1.11)$$

Substituting equation 1.9 in our model Momentum equations we get equation 1.12

$$\begin{aligned} \frac{v(i, j+1) - v(i, j)}{\Delta t} + v(i, j) \frac{v(i+1, j) - v(i-1, j)}{2\Delta x} + g \frac{y(i+1, j) - y(i-1, j)}{2\Delta x} + g(S_f - S_0) = \\ \frac{1}{2} \frac{q}{[b_2 d^2 + (b + b_1)d] + [(R + R1)^2(-\sin \theta \cos \theta + \theta)](\gamma + L)} \sin \frac{\pi}{m} \left\{ (u \cos \frac{\pi}{m} - V(i, j)) \right\} \end{aligned} \quad (1.12)$$

Rearranging the above equation yields

$$v(i, j + 1) = \Delta t \left\{ -v(i, j) \frac{v(i+1, j) - v(i-1, j)}{2\Delta x} - g \frac{y(i+1, j) - y(i-1, j)}{2\Delta x} - g(S_f - S_0) \right\} +$$

$$\Delta t \left\{ \frac{1}{2} \frac{q}{[b_2 d^2 + (b + b_1)d] + [(R + R1)^2(-\sin \theta \cos \theta + \theta)] (\gamma + L)} \sin \frac{\pi}{m} \left[u \cos \frac{\pi}{m} - V(i, j) \right] \right\} + v(i, j) \quad (1.13)$$

The initial and boundary conditions were taken as;

Initial conditions:

$$y(0, x) = 0.5 \quad v(0, x) = 0.1$$

boundary conditions:

$$y(t, x_{\text{initial}}) = 1 \quad v(t, x_{\text{initial}}) = 1$$

$$y(t, x_{\text{final}}) = 12 \quad v(t, x_{\text{initial}}) = 20$$

The following constants were also taken

$$T = 1, \quad S_0 = 0.002, \quad L = 1, \quad \theta = \frac{\pi}{3.33}, \quad q = 0.3, \quad R = 0.4,$$

$$B = 0.5, \quad g = 9.82, \quad d = 2$$

7. RESULTS AND DISCUSSIONS

The Matlab software is used to simulate the equation by varying i and j at various nodal points. Then the graphs were plotted using the values of the velocity against time at a certain location. Various flow parameters of length and angle of inclination of the lateral channel are varied to determine how they affect the fluid velocity in the main channel. The graphs are plotted, analyzed, discussed and then conclusions drawn.

7.1 A graph of velocity against time with surface length γ varying

Figure 3 illustrates the effects of the length γ of the lateral channel to the flow velocity on the main channel.

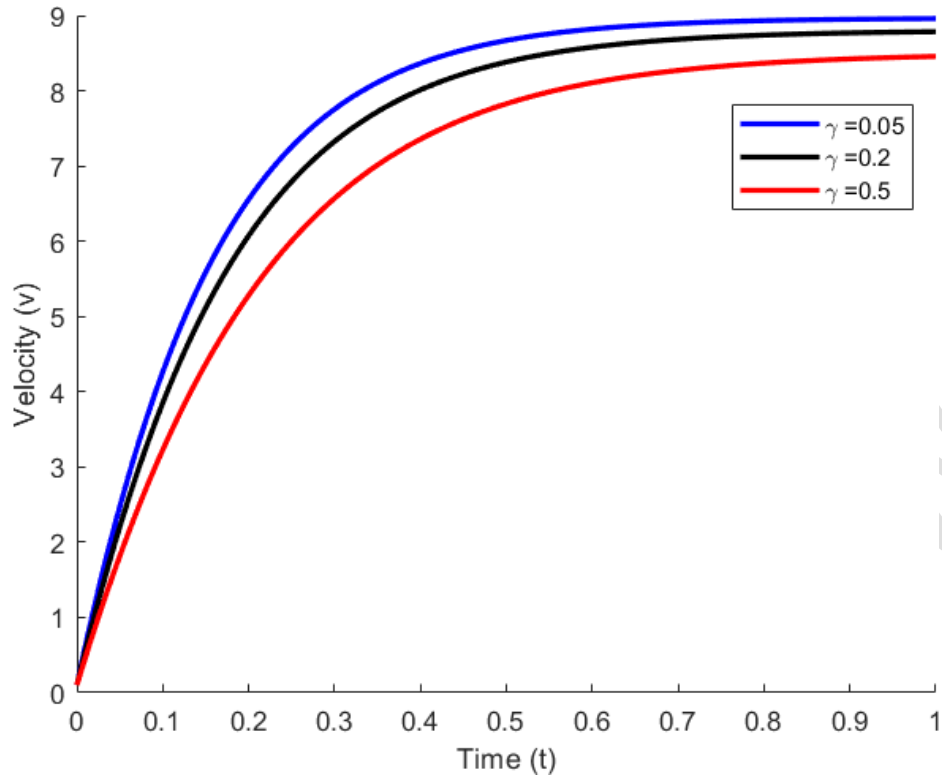


Figure 3: A graph of velocity against time with length γ varying

From the figure 3, as the length of the lateral channel increases from $\gamma = 0.05$ to $\gamma = 0.5$, the flow velocity of the main channel decreases from 10 to 8. This is because increasing the length of the lateral channel increases the surface of contact between the walls and the bottom segment of the channel since the liquid is spread over a wide surface area.

This increases the shear stress which in turn sets the friction that increases the resistance to the flow, leading to decrease in the flow velocity of the main channel. From the results, maximum velocity of 9 is attained when the length of the lateral channel is 0.05 m. In this regard, Engineers are advised to consider the shorter lengths of lateral channels for optimum discharge to be achieved on the main channel.

7.2 A graph of Velocity against time with angle of inclination $\frac{\pi}{m}$ varying

Figure 4 illustrates the effects of the angle of inclination, $\frac{\pi}{m}$ of the lateral channel to the flow velocity on the main channel.

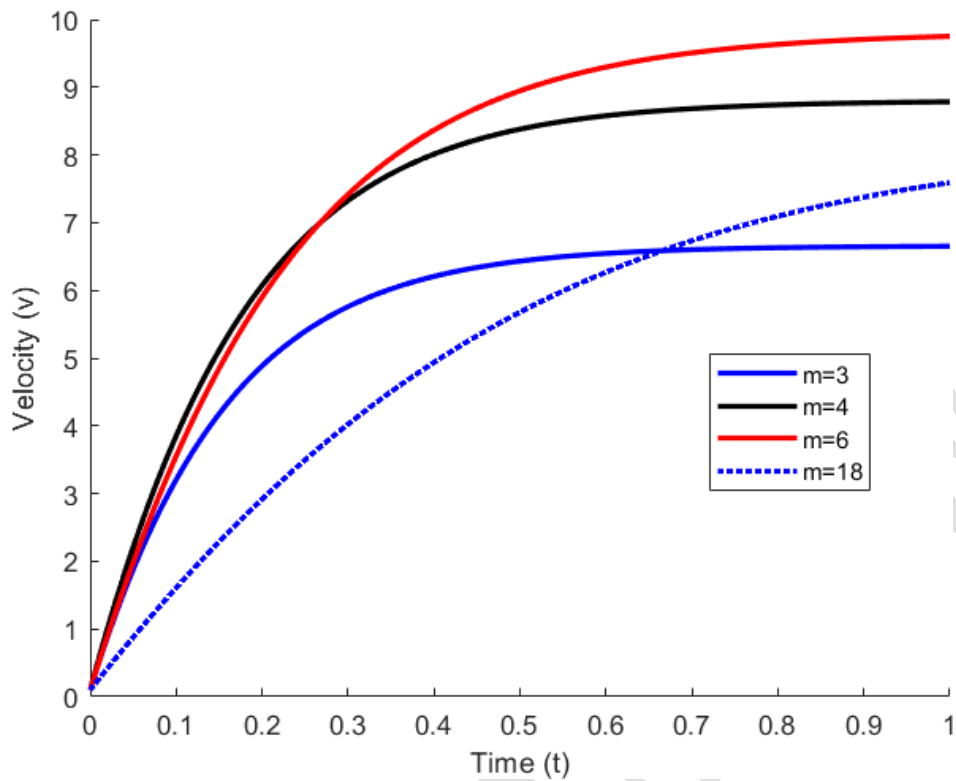


Figure 4: velocity against time with angle of inclination, $\frac{\pi}{m}$ varying

From Figure 4, increase in the angle of inclination of the lateral channel above 30° leads to a decrease in the flow velocity of the main channel. A decrease in the angle of inclination below 30° leads to decrease in the flow velocity. An angle of 30° results to maximum flow velocity in the main channel. This is because at this angle, there is minimal shear stress of the fluid particles at the entrance, which results to minimal friction hence less resistance to the flow compared to other angles. An inclination angle of 10° records low velocity value at the initial stages which progressively but slowly increases with time up to a maximum of 7.5. At the initial stages, the shear stress is rampant at the entrance which increases resistance leading to decreased flow velocity. This slow but gradual velocity poses a challenge as it may encourage clogging and accumulation of debris at initial stages which in turn could lower the flow velocity at the main channel. In this regard, Engineers are advised to implement 30° as the angle of inclination of the lateral channel to the main channel for maximum efficiency of the channel to be achieved.

8. CONCLUSIONS

This study primarily sets to determine the effect of length and angle of inclination of the lateral channel to the flow velocity on the main channel. The developed model

equations are solved using the finite difference iterative scheme. To generate numerical values, simulations are carried out using the MATLAB Mathematical software. The results are then presented in graphs. The following are the summary of the findings;

- i. Increasing the length of the lateral channel decreases the flow velocity of the main channel
- ii. An angle of inclination of 30° records the highest velocity value at the main channel. Other angles above or below this records low velocity values at the main channel.

9. RECOMMENDATIONS

It is recommended that future study be carried out on:

- i. The flow of the same orientation in three Dimensions.
- ii. The effect of length and cross-section area of the lateral channel to the flowrate at the main channel when the flow is assumed to be turbulent.

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APPENDIX: MATLAB CODE FOR SIMULATING VELOCITY PROFILES FOR VARIOUS FLOW PARAMETERS

```
function OpenChan_v2() clear all;clc;
%% =====constants===== g=9.81; L=1; T=1;d=2; R=0.4;
b=0.5; So=0.002;
%% =====parameters===== b1=0.2; % 0.1; 0.2; 0.3;
theta=pi/3.3; % pi/2; pi/3.3; pi/4 b2=b-2*R*sin(theta);
R1=0.2; % 0.03; 0.2; 0.3
gamma=0.2; % 0.05; 0.2; 0.5
n=0.05; % 0.012; 0.05; 0.1
```

```

m=18;          % 3; 4; 6; 18;
u=15;          % 10; 15; 20;
%% =====parameters=====
color='b'; % blue(b); black(k); red(r)

x0=0;xN=2;N=51; dx=(xN-x0)/(N-1); x=x0:dx:xN; t0=0;tK=2;K=151; dt=(tK-t0)/(K-1); t=t0:dt:tK;

%% constitutive relations
A=((b+b1)*d+b2*d*d)+((R+R1)^2) * (theta-sin(theta)*cos(theta)); h=sqrt(b2*b2+d*d);
P=2*h+2*R*theta; Rs=A/P;
q=(A/2)*u; % verify the other 1/2 in equation

%% boundary and initial conditions y0=0.5; v0=0.1;
y=zeros(N,K); v=zeros(N,K);
%% evaluation of finite difference scheme

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UNDER PEER REVIEW

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for j=1:K-1 for i=2:N-1
v(:,1)=v0; y(:,1)=y0; %IC

y(i,j+1)=0.5*(y(i-1,j)+y(i+1,j))-dt*(v(i,j)*(y(i+1,j)-
y(i-1,j))/(2*dx)... +(A/T)*((v(i+1,j)-v(i-1,j))/(2*dx))- (q/(T*(gamma+L)))*sin(pi/m));

v(i,j+1)=0.5*(v(i-1,j)+v(i+1,j))-dt*(v(i,j)*(v(i+1,j)-
v(i-1,j))/(2*dx)... +g*(y(i+1,j)-y(i-1,j))/(2*dx)+
g*((n*n/(2*(Rs^4/3)))) *0.5 * (v(i-1,j)^2 + v(i+1,j)^2) - So)...
-(q/(A*(gamma+L)))*sin(pi/m)*(u*cos(pi/m)-v(i,j));

v(1,:)=v(2,:); y(1,:)=y(2,:); %BC at x=x0

v(N,:)=v(N-1,:); y(N,:)=y(N-1,:); %BC at x=xN

end
end
figure(1);
hold on; plot(t,y(N,:),color,'linewidth',2); xlabel('Time (t)'); ylabel('Depth (y)');
hold off figure(2);
hold on; plot(t,v(N,:),color,'linewidth',2); xlabel('Time (t)'); ylabel('Velocity (v)');
hold off figure(3)
hold on; plot3(y(N,:),v(N,:),t,color,'linewidth',2); xlabel
('Depth (y)');ylabel('Velocity (v)'); view(2); hold off
end

```