# Shortest path problem under Pythagorean Fuzzy Network using Dijkstra's Algorithm


## ABSTRACT

In this article investigates the Shortest Path Problem (SPP) in uncertain settings using Pythagorean fuzzy numbers (PFNs). A heuristic methodology using modified Dijkstra's algorithm is developed to exploit error, uncertainty, and partial truth for a low-cost solution. The method calculates minimum distance between starting and destination nodes and evaluates its effectiveness. Pythagorean fuzzy graphs can handle ambiguous situations.



*Keywords: Pythagorean Fuzzy Graph(PFG); Triangular Pythagorean fuzzy number (TriPFN); Ranking functions; Dijkstra algorithm; Triangular Pythagorean fuzzy shortest path problem (TriPFSPP).*




## 1. INTRODUCTION

Problems with the Shortest path (SP) are at the core of routing information. They frequently arise in practise because a wide variety of real-life challenges aim to transmit some commodities among two particular nodes in a network as economically as possible. SPP with the objective of identifying a path with lowest possible cost (time or distance) from source node to the destination node can also be utilised to develop such useful applications. Moreover, it appears that there hasn't been any research on the SPP using information in form of Pythagorean fuzzy numbers (PFN), a more expansive type of fuzzy number (FN), in the research. is paper's main goal being to Google map servicing by posing and suggesting a way for resolving the SPP using triangular Pythagorean Fuzzy arc values and using the ranking functions for TriPFNs. In order to deal with erroneous data in SP problems, Zadeh created the fuzzy set theory. Therefore, multiple attempts have been made by researchers to address various SP challenges in fuzzy contexts. Okada presented a strategy to address the fuzzy SP problem on prospect theory to ascertain the level of possibility for each arc. TawandaT[20], was applying Tawanda's non-iterative optimum tree approach for shortest route issues to determine k-possible critical paths.The TANYAKUMU Labelling Method was expanded by Tawanda, Munapo, Kumar, S., and Nyamugure, P.[21] to compute the shortest paths in directed networks.

Dubois and Prade[2], they are discussed in a fuzzy sets and Systems. Using the fuzzy integer formulation for gradient mean integration, Deng et al. [3] enhanced the Dijkstra algorithm to solve fuzzy SP problems. Other authors focused on applying heuristic methods to determine the SP in a network with various fuzzy arc costs.Kannan.V. and Appasamy.S[7] were identified for applying the Bellman Ford algorithm with score functions to solve the linear Diophantine Fuzzy shortest path problem in network analysis.Mukherjee[9], a well-known Shortest Path Problem, has been investigated in an uncertain environment. Define Dijkstra's algorithm for solving the shortest path issue on networks in an intuitive fuzzy setting.M. Parimala, Broumi, K. Prakash et al. [16] used the Bellman-Ford algorithm to construct shortest path solutions in a network.Shaista Habib, Aqsa Majeed, Muhammad Akram, and Mohammed M. Ali Al-Shamiri[18] discuss the calculation of APSP using dependability, scores, and trapezoidal picture fuzzy additive functions. They compare it to traditional Floyd-Warshall methodologies and investigate Shortest Path Problems in various disciplines. This study by Kumawat, Dudeja, and Kumar[19] explores the use of artificial intelligence in shortest path algorithms, comparing their time complexity and performance, and suggesting future paths.S. Yu and Y. Song[24] used structured recurrent neural networks and the ripple spreading method to solve the time-dependent shortest path issue.

Amna Habib, M. Akram, &CengizKahraman[1] We present the suggested approach's hierarchical clustering implications as the dendrogram in a new Pythagorean fuzzy similarity network. H. Garg(ed). [4], are defined a Pythagorean Fuzzy set. K. Vidhya and A. Saraswathi[5] address imprecise problems in an improved A* search method for the shortest path in a Pyhtagorean fuzzy environment with interval values. Ayesha Shareef, Ahmad N. Al-Kenani, and Muhammad Akram[8] utilized Pythagorean fuzzy occurrence graphs to model a single toll road system, focusing on legal fuzzy occurrence forests for decision-making and comparing it to a previous methodology combining logical deduction and numerical data. The TOPSIS approach is utilized by Akram, Dudek, and Ilyas[10] for multicriteria grouping decisions using Pythagorean fuzzy data, with the best options evaluated using a modified closeness metric. Akram, Habib, and Allahviranloo's[11] technique enhances coherence by comparing Pythagorean fuzzy optimal flows, assessing trends, and evaluating maximal flow methodologies' runtime. The aggregate of the member and non-member degrees, however, can exceed one in some situations. As a result, Yager[22] extended the Pythagorean Fuzzy Set (PFS), a generalisation of IFS where the square sum of the member and non-member degrees is equal to or lower than just one. Yager[23] defined a Pythagorean membership grades in decision making in the problems.

Mohammad Enayattabar, Ali Ebrahimnejad, HomayunMotameni[12] are define into a Dijktra's algorithm for shortest path problem under interval-valued Pythagorean fuzzy environment. AsimBasha, Mohammed Jabarulla, and Said Broumi[13] used Pythagorean Fuzy Triangular Number to solve the shortest path problem. M. AsimBasha and M. Mohammed Jabarulla's[14] study on triangle I-V, using Pythagorean fuzzy numbers for edge weights, provides a graphical representation for explanation. M. AsimBasha, M. Mohammed Jabarulla and Said Broumi[15] developed a strategy using the Neutrosophic Pythagorean fuzzy Triangular number to find the shortest path between nodes. Broumi et al. [17] propose a new method for calculating neutrosophic shortest path using interval-valued neutrosophic numbers, validated through numerical examples and compared to existing methods. K. Vidhya, A. Saraswathi and Said Broumi[6] address an efficient approach for solving time-dependent shortest path problem under FermeateanNeutrosophic environment.

As a result, the objective of this study was to suggest a strategy for resolving SP issues in a TriPF setting. To accomplish this, we first provide the numerical solution for SP issues that expresses the price of traversing arcs in terms of TriPFNs. Moreover, in order to construct a solution algorithm, we present the sufficient conditions in TriPF networks. To do this, TriPFNs are utilised as the arc costs to compare the prices of various paths using a ranking function. The cost of TriPFSP is then determined by extending the conventional Dijkstra's technique. The digital mapping service in a network with a TriPF environment serves as an illustration of the proposed algorithm.

## 2. PRELIMINARIES

Consider some fundamental definitions of Pythagorean fuzzy graphs (PFGs), PFN, and Ranking function of numerical properties in the following section.

**Definition 1**

A fuzzy Pythagorean graph on a set that is not empty E is two $\tilde{G} = (\tilde{\xi}, \tilde{\zeta})$, where $\tilde{\xi}$ is a fuzzy Pythagorean set on E &$\tilde{\zeta}$ is a fuzzy Pythagorean relation on E, and were

$$\tilde{\eta}_{\tilde{\zeta}}(xy) \leq \tilde{\eta}_{\tilde{\xi}}(x) \wedge \tilde{\eta}_{\tilde{\xi}}(y)$$

$$\tilde{\Gamma}_{\tilde{\zeta}}(xy) \leq \tilde{\Gamma}_{\tilde{\xi}}(x) \wedge \tilde{\Gamma}_{\tilde{\xi}}(y)$$

Where, $\tilde{\eta}_{\tilde{\zeta}}: E \times E \to [0,1]$&$\tilde{\Gamma}_{\tilde{\zeta}}: E \times E \to [0,1]$ appropriately reflect the member and non-member functions of $\tilde{\zeta}$is $0 \leq \tilde{\eta}^2{}_{\tilde{\zeta}}(xy) + \tilde{\Gamma}^2{}_{\tilde{\zeta}}(xy) \leq 1 \; \forall xy \in E$.

**Definition 2**

The relation of $\tilde{x}'$ to $\boldsymbol{\eta}_{\tilde{P}}$ is degree of uncertainty is given by $\boldsymbol{\Pi}_{\eta_{\tilde{P}}}(\boldsymbol{x}) = \sqrt{\mathbf{1} - \left(\tilde{\boldsymbol{\eta}}_{\tilde{\zeta}}(\boldsymbol{x})\right)^2 - \left(\tilde{\boldsymbol{\Gamma}}_{\tilde{\zeta}}(\boldsymbol{x})\right)^2}$. If $\boldsymbol{\eta}_{\tilde{P}} = (\tilde{\boldsymbol{\eta}}_{\tilde{\zeta}}, \tilde{\boldsymbol{\Gamma}}_{\tilde{\zeta}})$is a PFN then the degree of indeterminacy of the number$\boldsymbol{\Pi}_{\eta_{\tilde{P}}}(\boldsymbol{x}) = \sqrt{\mathbf{1} - (\tilde{\boldsymbol{\eta}}_{\tilde{\zeta}})^2 - (\tilde{\boldsymbol{\Gamma}}_{\tilde{\zeta}})^2}$, where $(\tilde{\boldsymbol{\eta}}_{\tilde{\zeta}}, \tilde{\boldsymbol{\Gamma}}_{\tilde{\zeta}}) \in [\mathbf{0}, \mathbf{1}]$ and $\mathbf{0} \leq (\tilde{\boldsymbol{\eta}}_{\tilde{\zeta}})^2 + (\tilde{\boldsymbol{\Gamma}}_{\tilde{\zeta}})^2 \leq \mathbf{1}$.

**Example 3**

A Pythagorean fuzzy graph (PFG) is defined similarly to a conventional fuzzy graph, with the distinction that the sum of the squares of the membership and non-membership degrees must be less than or equal to 1 for each edge.

Here's a simple example:

Graph Definition

- Nodes: $\mathbf{4}\ (\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\ )$
- Edges: $\mathbf{1}-\mathbf{2}, \mathbf{1}-\mathbf{3}, \mathbf{2}-\mathbf{4}, \mathbf{3}-\mathbf{4}$
- Membership Degrees:
  $\boldsymbol{\mu}(\mathbf{1},\mathbf{2}) = (\mathbf{0}.\mathbf{6}), \boldsymbol{\mu}(\mathbf{1},\mathbf{3}) = (\mathbf{0}.\mathbf{7}), \boldsymbol{\mu}(\mathbf{2},\mathbf{4}) = (\mathbf{0}.\mathbf{5}), \boldsymbol{\mu}(\mathbf{3},\mathbf{4}) = (\mathbf{0}.\mathbf{8})$
- Non-membership Degrees:
  $\boldsymbol{\nu}(\mathbf{1},\mathbf{2}) = (\mathbf{0}.\mathbf{5}), \boldsymbol{\nu}(\mathbf{1},\mathbf{3}) = (\mathbf{0}.\mathbf{4}), \boldsymbol{\nu}(\mathbf{2},\mathbf{4}) = (\mathbf{0}.\mathbf{6}), \boldsymbol{\nu}(\mathbf{3},\mathbf{4}) = (\mathbf{0}.\mathbf{4})$

Condition Check

For each edge, $\mathbf{0} \leq \boldsymbol{\mu}^2 + \boldsymbol{\nu}^2 \leq \mathbf{1}$:

- $\mathbf{0} \leq (\mathbf{0}.\mathbf{6})^2 + (\mathbf{0}.\mathbf{5})^2 = \mathbf{0}.\mathbf{36} + \mathbf{0}.\mathbf{25} = \mathbf{0}.\mathbf{61} \leq \mathbf{1}$
- $\mathbf{0} \leq (\mathbf{0}.\mathbf{7})^2 + (\mathbf{0}.\mathbf{4})^2 = \mathbf{0}.\mathbf{49} + \mathbf{0}.\mathbf{16} = \mathbf{0}.\mathbf{65} \leq \mathbf{1}$
- $\mathbf{0} \leq (\mathbf{0}.\mathbf{5})^2 + (\mathbf{0}.\mathbf{6})^2 = \mathbf{0}.\mathbf{25} + \mathbf{0}.\mathbf{36} = \mathbf{0}.\mathbf{61} \leq \mathbf{1}$

♠ $0 \le (0.8)^2 + (0.4)^2 = 0.64 + 0.16 = 0.80 \le 1$

The conditions are satisfied.

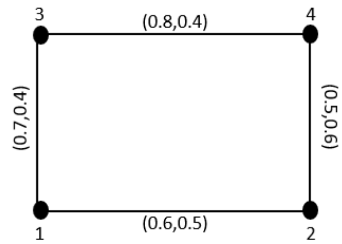



**Figure 1**: *Pythagorean Fuzzy Graph.*

**Definition 4**

Let two TPFNs, $\mathfrak{I}_{\tilde{P}'} = \langle[0,0,0],[1,1,1]\rangle \& \mathcal{O}_{\tilde{P}'} = \langle[1,1,1],[0,0,0]\rangle$ are, in two numbers, the smallest and largest TriPFNs, respectively.

**Definition 5**

Let $\tilde{\mathcal{A}}_{\tilde{P}'} = \langle[\widetilde{x_1}',\widetilde{y_1}',\widetilde{z_1}'],\left[\widetilde{a_1}',\widetilde{b_1}',\widetilde{c_1}'\right]\rangle$ and $\tilde{\mathcal{B}}_{\tilde{P}'} = \langle[\widetilde{x_2}',\widetilde{y_2}',\widetilde{z_2}'],\left[\widetilde{a_2}',\widetilde{b_2}',\widetilde{c_2}'\right]\rangle$ are the two TriPFNs. Then the result of adding two numbers is

$$\tilde{\mathcal{A}}_{\tilde{P}'} \oplus \tilde{\mathcal{B}}_{\tilde{P}'} = \left\langle\left[\begin{array}{c}\sqrt{\left(\widetilde{x_1}'^2 + \widetilde{x_2}'^2\right) - \left(\widetilde{x_1}'^2\right)\left(\widetilde{x_2}'^2\right)}, \sqrt{\left(\widetilde{y_1}'^2 + \widetilde{y_2}'^2\right) - \left(\widetilde{y_1}'^2\right)\left(\widetilde{y_2}'^2\right)}, \\ \sqrt{\left(\widetilde{z_1}'^2 + \widetilde{z_2}'^2\right) - \left(\widetilde{z_1}'^2\right)\left(\widetilde{z_2}'^2\right)}, (a_1 a_2, b_1 b_2, c_1 c_2)\end{array}\right]\right\rangle.$$

**Definition 6**

If it's for all TriPFNs are $\tilde{\mathcal{A}}_{\tilde{P}'} = \langle[\widetilde{x_1}',\widetilde{y_1}',\widetilde{z_1}'],\left[\widetilde{a_1}',\widetilde{b_1}',\widetilde{c_1}'\right]\rangle$ we have $\tilde{\mathcal{A}}_{\tilde{P}'} \oplus \mathcal{O}_{\tilde{P}'} = \tilde{\mathcal{A}}_{\tilde{P}'}$.

**Definition 7**

If any TriPFNs score and precise function are $\tilde{\mathcal{A}}_{\tilde{P}'} = \langle[\widetilde{x_1}',\widetilde{y_1}',\widetilde{z_1}'],\left[\widetilde{a_1}',\widetilde{b_1}',\widetilde{c_1}'\right]\rangle$and $\tilde{\mathcal{B}}_{\tilde{P}'} = \langle[\widetilde{x_2}',\widetilde{y_2}',\widetilde{z_2}'],\left[\widetilde{a_2}',\widetilde{b_2}',\widetilde{c_2}'\right]\rangle$ along with is referred to as $\tilde{\mathcal{S}}(\tilde{\mathcal{A}}_{\tilde{P}'}) = \frac{1}{4}\left[\left(\left(\widetilde{x_1}'^2\right) + 2\left(\widetilde{y_1}'^2\right) + \left(\widetilde{z_1}'^2\right)\right) - \left(\left(\widetilde{x_2}'^2\right) + 2\left(\widetilde{y_2}'^2\right) + \left(\widetilde{z_2}'^2\right)\right)\right]$ $\forall\, \tilde{\mathcal{S}}(\tilde{\mathcal{A}}_{\tilde{P}'}) \in [-1,1]$.

And $\tilde{\mathcal{H}}(\tilde{\mathcal{A}}_{\tilde{P}'}) = \frac{1}{4}\left[\left(\left(\widetilde{x_1}'^2\right) + 2\left(\widetilde{y_1}'^2\right) + \left(\widetilde{z_1}'^2\right)\right) + \left(\left(\widetilde{x_2}'^2\right) + 2\left(\widetilde{y_2}'^2\right) + \left(\widetilde{z_2}'^2\right)\right)\right]$ $\forall\, \tilde{\mathcal{H}}(\tilde{\mathcal{A}}_{\tilde{P}'}) \in [0,1]$.

The $\tilde{\mathcal{A}}_{\tilde{P}'} = \langle[\widetilde{x_1}',\widetilde{y_1}',\widetilde{z_1}'],\left[\widetilde{a_1}',\widetilde{b_1}',\widetilde{c_1}'\right]\rangle$ and$\tilde{\mathcal{B}}_{\tilde{P}'} = \langle[\widetilde{x_2}',\widetilde{y_2}',\widetilde{z_2}'],\left[\widetilde{a_2}',\widetilde{b_2}',\widetilde{c_2}'\right]\rangle$ are then TriPFNs.

## 3. EXTENDED DIJKSTRA'S ALGORITHM UNDER TRIPF ENVIRONMENT

The Extended Dijkstra's method in TriPF Environment is discussed in this section. Then the study was concerned a set is $\tilde{\mathcal{Z}}_{\tilde{P}} = \tilde{\mathcal{C}}_{\tilde{P}'} = \langle[0,0,0],[1,1,1]\rangle$ Source Node (SN) is one and the Predecessors node (PN) one is zero.

**Initialization Step:**

Consider the all edges in the set $(F, \tilde{E}) = \{(i,j): i \in \tilde{S}, j\ in\ \tilde{E}\}$ and let $\tilde{E} = F - \tilde{S}$.

***Main Step:***

Let $\tilde{S}\left(\tilde{Z}_{\tilde{P}_x} \oplus \tilde{I}_{\tilde{P}_{xy}}\right) = \min\left\{\tilde{S}\left(\tilde{Z}_{\tilde{P}_i} \oplus \tilde{I}_{\tilde{P}_{ij}}\right): (i,j) \in (F, \tilde{E})\right\}$

Set $\tilde{Z}_{\tilde{P}_y} = \tilde{Z}_{\tilde{P}_x}$, Processors node $y$ in $x$ and $F = F \cup \{y\}$.

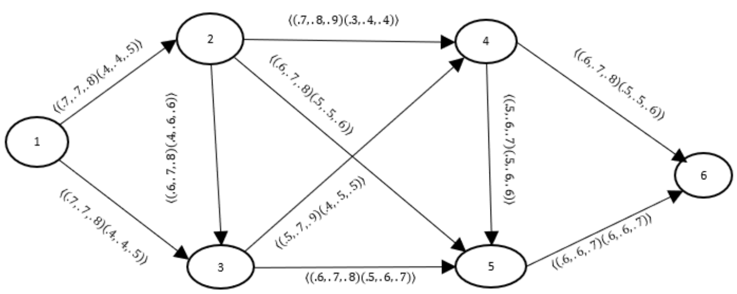
After that, repeat themain step accurately (m-1) times, and then stop the process. The matching TriPFSP and its TriPF cost are available.

**Remark 1**

Take note that node $i$ is a previous node is provided by predecessor $i$.

**Network Terminology 2**

Consider using Google Maps' digital mapping services in this section if you have ever looked for the distance between two cities or between your current position and a nearby destinations. Due to the fact that there are numerous routes and paths linking them but only one needs to demonstrate the shortest distance, The shortest route among any two places along the path is found using Dijkstra's approach. Consider this: Imagine India as a graph, where each city or location is represented by a vertex, and the connections between them are represented by lines. A TriPSP is what the route seeks to identify in order to ensure that messages can effectively travel between the cities or locations. Using the suggested extended Dijkstra's approach, TriPFSP cost and corresponding triangular Pythagorean fuzzy shortest path can be found




as follows:

**Figure 2.** *TriPFSP Network for the digital mapping service*

| Edges | TriPF distance |
|---|---|
| 1-2 | ⟨(.7,.7,.8)(.5,.5,.6)⟩ |
| 1-3 | ⟨(.7,.7,.8)(.4,.4,.5)⟩ |
| 2-3 | ⟨(.6,.7,.8)(.4,.6,.6)⟩ |
| 2-4 | ⟨(.7,.8,.9)(.3,.4,.4)⟩ |
| 2-5 | ⟨(.6,.7,.8)(.5,.5,.6)⟩ |
| 3-4 | ⟨(.5,.7,.9)(.4,.5,.5)⟩ |
| 3-5 | ⟨(.6,.7,.8)(.5,.6,.7)⟩ |
| 4-5 | ⟨(.5,.6,.7)(.5,.6,.6)⟩ |
| 4-6 | ⟨(.6,.7,.8)(.5,.5,.6)⟩ |
| 5-6 | ⟨(.6,.6,.7)(.6,.6,.7)⟩ |

**Table 1.** *The edges of TriPFNs.*

**3.2 Numerical Explanation**

**Initialization Step:**

Let us consider $\tilde{Z}_{\tilde{P}} = \tilde{C}_{\tilde{P}} = \langle(0,0,0),(1,1,1)\rangle$ then the SN is one and predecessor node one is equal to zero.

***Main step:***

Let us consider the following iterations are given below:

**Iteration 1.** Let $\tilde{E} = F - \tilde{S}$ = {2,3,4,5,6}and $\left(F, \tilde{E}\right) = \{(i, , j): i \in F, j \in \tilde{E}\}$ = {(1,2), (1,3)}. Then, we have,

$\left(\tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{12}}\right) = \langle(0,0,0), (1,1,1)\rangle \oplus \langle(.7, .7, .8)(.5, .5, .6)\rangle = \langle(.7, .7, .8)(.5, .5, .6)\rangle$

$\left(\tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{13}}\right) = \langle(0,0,0), (1,1,1)\rangle \oplus \langle(.7, .7, .8)(.4, .4, .5)\rangle = \langle(.7, .7, .8)(.4, .4, .5)\rangle$

Accordingly,

$\left(\tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{12}}\right) = \langle(.7, .7, .8)(.5, .5, .6)\rangle = 0.063.$

$\left(\tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{13}}\right) = \langle(.7, .7, .8)(.4, .4, .5)\rangle = 0.158$

Since, $\left(\tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{12}}\right) = \min\{(0.063, 0.158)\} = (0.063)$, Then we have

$\tilde{Z}_{\tilde{P}_2} = \tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{12}} = \langle(.7, .7, .8)(.5, .5, .6)\rangle$, Predecessors node 2 is equal to 1 and then $\tilde{S}$ = {1,2}.

**Iteration 2.** Let $\tilde{E} = F - \tilde{S}$ = {3,4,5,6}and $\left(F, \tilde{E}\right) = \{(i, , j): i \in F, j \in \tilde{E}\}$ = {(1,3), (2,3), (2,4), (2,5)}. Then, we have,

$$\left(\tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{13}}\right) = \langle(0,0,0), (1,1,1)\rangle \oplus \langle(.7, .7, .8)(.4, .4, .5)\rangle = \langle(.7, .7, .8)(.4, .4, .5)\rangle\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{23}}\right)$$
$$= \langle(.7, .7, .8)(.5, .5, .6)\rangle \oplus \langle(.6, .7, .8)(.4, .6, .6)\rangle = \langle(.764, .862, 1.018)(.2, .3, .36)\rangle$$

$$\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{24}}\right) = \langle(.7, .7, .8)(.5, .5, .6)\rangle \oplus \langle(.7, .8, .9)(.3, .4, .4)\rangle = \langle(.862, .976, 1.118)(.15, .2, .24)\rangle$$

$\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{25}}\right) = \langle(.7, .7, .8)(.5, .5, .6)\rangle \oplus \langle(.6, .7, .8)(.5, .5, .6)\rangle = \langle(.764, .862, 1.018)(.25, .25, .36)\rangle$

Accordingly,

$\left(\tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{13}}\right) = \langle(.7, .7, .8)(.4, .4, .5)\rangle = 0.158$

$\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{23}}\right) = \langle(.764, .862, 1.018)(.2, .3, .36)\rangle = 0.685$

$\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{24}}\right) = \langle(.862, .976, 1.118)(.15, .2, .24)\rangle = 0.902$

$\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{25}}\right) = \langle(.764, .862, 1.018)(.25, .25, .36)\rangle = 0.693$

Since, $\left(\tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{13}}\right) = \min\{(0.158, 0.685, 0.902, 0.693)\} = (0.158)$, Then we have

$\tilde{Z}_{\tilde{P}_3} = \tilde{Z}_{\tilde{P}_1} \oplus \tilde{I}_{\tilde{P}_{13}} = \langle(.7, .7, .8)(.4, .4, .5)\rangle$, Predecessors node 3 is equal to 1 and then $\tilde{S}$ = {1,2,3}.

**Iteration 3.** Let $\tilde{E} = F - \tilde{S}$ = {4,5,6}and $\left(F, \tilde{E}\right) = \{(i, , j): i \in F, j \in \tilde{E}\}$ = {(2,4), (2,5), (3,4), (3,5)}. Then, we have,

$\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{24}}\right) = \langle(.7, .7, .8)(.5, .5, .6)\rangle \oplus \langle(.7, .8, .9)(.3, .4, .4)\rangle = \langle(.862, .976, 1.118)(.15, .2, .24)\rangle$

$\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{25}}\right) = \langle(.7, .7, .8)(.5, .5, .6)\rangle \oplus \langle(.6, .7, .8)(.5, .5, .6)\rangle = \langle(.764, .862, 1.018)(.25, .25, .36)\rangle$

$\left(\tilde{Z}_{\tilde{P}_3} \oplus \tilde{I}_{\tilde{P}_{34}}\right) = \langle(.7, .7, .8)(.4, .4, .5)\rangle \oplus \langle(.5, .7, .9)(.4, .5, .5)\rangle = \langle(.68, .862, 1.118)(.16, .2, .25)\rangle$

$\left(\tilde{Z}_{\tilde{P}_3} \oplus \tilde{I}_{\tilde{P}_{35}}\right) = \langle(.7, .7, .8)(.4, .4, .5)\rangle \oplus \langle(.6, .7, .8)(.5, .6, .7)\rangle = \langle(.764, .862, 1.018)(.2, .24, .35)\rangle$

Accordingly,

$\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{24}}\right)$ = ⟨(.862, .976,1.118)(.15, .2, .24)⟩ = 0.902

$\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{25}}\right)$ = ⟨(.764, .862,1.018)(.25, .25, .36)⟩ = 0.693

$\left(\tilde{Z}_{\tilde{P}_3} \oplus \tilde{I}_{\tilde{P}_{34}}\right)$ = ⟨(.68, .862,1.118)(.16, .2, .25)⟩ = 0.725

$\left(\tilde{Z}_{\tilde{P}_3} \oplus \tilde{I}_{\tilde{P}_{35}}\right)$ = ⟨(.764, .862,1.018)(.2, .24, .35)⟩ = 0.703

Since, $\left(\tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{25}}\right)$ = min{(0.902,0.693,0.725,0.703)} = (0.693), Then we have, Predecessors node 5 is equal to 2 and then $\tilde{S}$ = {1,2,3,5}.

**Iteration 4.** Let $\tilde{E} = F - \tilde{S}$ = {4,6}and $\left(F, \tilde{E}\right) = \{(i, , j): i \in F, j \in \tilde{E}\}$ = {(3,4), (5,6)}.

Then, we have,

$\left(\tilde{Z}_{\tilde{P}_3} \oplus \tilde{I}_{\tilde{P}_{34}}\right)$ = ⟨(.7, .7, .8)(.4, .4, .5)⟩ ⊕ ⟨(.5, .7, .9)(.4, .5, .5)⟩ = ⟨(.68, .862,1.118)(.16, .2, .25)⟩

$\left(\tilde{Z}_{\tilde{P}_5} \oplus \tilde{I}_{\tilde{P}_{56}}\right)$ = ⟨(.764, .862,1.018)(.25, .25, .36)⟩ ⊕ ⟨(.6, .6, .7)(.6, .6, .7)⟩ = ⟨(.914, .955,1)(.15, .15, .252)⟩

Accordingly,

$\tilde{Z}_{\tilde{P}_3}\tilde{Z}_{\tilde{P}_5} = \tilde{Z}_{\tilde{P}_2} \oplus \tilde{I}_{\tilde{P}_{25}}$ = ⟨(.764, .862,1.018)(.25, .25, .36)⟩ ⊕ ⟨(.68, .862,1.118)(.16, .2, .25)⟩ = 0.725

$\left(\tilde{Z}_{\tilde{P}_5} \oplus \tilde{I}_{\tilde{P}_{56}}\right)$ = ⟨(.914, .955,1)(.15, .15, .252)⟩ = 0.885

Since, $\left(\tilde{Z}_{\tilde{P}_3} \oplus \tilde{I}_{\tilde{P}_{34}}\right)$ = min{(0.725,0.819)} = (0.725), Then we have

$\tilde{Z}_{\tilde{P}_4} = \tilde{Z}_{\tilde{P}_3} \oplus \tilde{I}_{\tilde{P}_{34}}$ = ⟨(.68, .862,1.118)(.16, .2, .25)⟩, Predecessors node 4 is equal to 3 and then $\tilde{S}$ = {1,2,3,5,4}.

**Iteration 5.** Let $\tilde{E} = F - \tilde{S}$ = {6}and $\left(F, \tilde{E}\right) = \{(i, j): i \in F, j \in \tilde{E}\}$ = {(4,6), (5,6)}. Then, we have,

$\left(\tilde{Z}_{\tilde{P}_4} \oplus \tilde{I}_{\tilde{P}_{46}}\right)$ = ⟨(.68, .862,1.118)(.16, .2, .25)⟩ ⊕ ⟨(.6, .7, .8)(.5, .5, .6)⟩ = ⟨(.874, .988, .958)(.08, .1, .15)⟩

$\left(\tilde{Z}_{\tilde{P}_5} \oplus \tilde{I}_{\tilde{P}_{56}}\right)$ = ⟨(.764, .862,1.018)(.25, .25, .36)⟩ ⊕ ⟨(.6, .6, .7)(.6, .6, .7)⟩ = ⟨(.914, .955,1)(.15, .15, .252)⟩

Accordingly,

$\left(\tilde{Z}_{\tilde{P}_4} \oplus \tilde{I}_{\tilde{P}_{46}}\right)$ = ⟨(.874, .988, .958)(.08, .1, .15)⟩ = 0.906

$\left(\tilde{Z}_{\tilde{P}_5} \oplus \tilde{I}_{\tilde{P}_{56}}\right)$ = ⟨(.914, .955,1)(.15, .15, .252)⟩ = 0.885

Since, $\left(\tilde{Z}_{\tilde{P}_5} \oplus \tilde{I}_{\tilde{P}_{56}}\right)$ = min{(0.906,0.885)} = (0.885), Then we have

$\tilde{Z}_{\tilde{P}_6} = \tilde{Z}_{\tilde{P}_5} \oplus \tilde{I}_{\tilde{P}_{56}}$ = ⟨(.914, .955,1)(.15, .15, .252)⟩, Predecessors node 6 is equal to 5 and then $\tilde{S}$ = {1,2,3,5,4,6}.

| Edges | Minimum Cost of the nodes |
|---|---|

| | |
|---|---|
| 1-2 | 0.063 |
| 1-3 | 0.158 |
| 2-3 | 0.685 |
| 2-4 | 0.902 |
| 2-5 | 0.693 |
| 3-4 | 0.725 |
| 3-5 | 0.703 |
| 4-6 | 0.906 |
| 5-6 | 0.885 |

**Table 2.** *Minimum cost of the Pythagorean values.*

The calculations above show that the SP is a path $\mathbf{1} \to \mathbf{2} \to \mathbf{5} \to \mathbf{6}$, and the distance in between the SN and the Destination Nodes is $\langle(.914, .955, 1)(.15, .15, .252)\rangle$

Then the corresponding TriPFSP are as follows, predecessors node 6 is equal to 5, predecessors node 5 is equal to 2 and predecessors node 2 is equal to 1.

Hence, a TriPFSPis $\mathbf{1} \to \mathbf{2} \to \mathbf{5} \to \mathbf{6}.$

### 3.3 Comparison Analysis:

- **Uncertainty Representation:**

  **Intuitionistic Fuzzy Graph (IFG):** Represents uncertainty with a simple linear constraint on the sum of membership and non-membership.

  **Pythagorean Fuzzy Graph (PFG):** A quadratic (Pythagorean) constraint is used, which allows for greater flexibility and a wider range of values when modelling uncertainty.

- **Mathematical Structure:**

  IFG: $0 \le \tilde{\eta}_{\tilde{\zeta}} + \tilde{\Gamma}_{\tilde{\zeta}} \le 1$

  PFG: $0 \le \left(\tilde{\eta}_{\tilde{\zeta}}\right)^2 + \left(\tilde{\Gamma}_{\tilde{\zeta}}\right)^2 \le 1$

- **Application Suitability:**

  **IFG:** Useful if an easy, linear explanation for uncertainty is sufficient.

  **PFG:** Preferred in higher-level scenarios that necessitate a richer, more flexible representation of uncertainty and reluctance.

- **Example of IFG and PFG:**

| S.No | Values of IFG | Explanation of IFG | Values of PFG | Explanation of PFG |
|---|---|---|---|---|
| **1** | [0.4,0.5] | $0 \le 0.4 + 0.5 \le 1$<br>$0 \le 0.9 \le 1$ | [0.5,0.6] | $0 \le (0.5)^2 + (0.6)^2 \le 1$<br>$0 \le 0.25 + 0.36 \le 1$<br>$0 \le 0.61 \le 1$ |
| **2** | [0.3,0.7] | $0 \le 0.3 + 0.7 \le 1$<br>$0 \le 1.0 \le 1$ | [0.7,0.7] | $0 \le (0.7)^2 + (0.7)^2 \le 1$<br>$0 \le 0.49 + 0.49 \le 1$<br>$0 \le 0.98 \le 1$ |
| **3** | [0.2,0.6] | $0 \le 0.2 + 0.6 \le 1$<br>$0 \le 0.8 \le 1$ | [0.6,0.8] | $0 \le (0.6)^2 + (0.8)^2 \le 1$<br>$0 \le 0.36 + 0.64 \le 1$<br>$0 \le 1 \le 1$ |
| **4** | [0.3,0.5] | $0 \le 0.3 + 0.5 \le 1$<br>$0 \le 0.8 \le 1$ | [0.6,0.7] | $0 \le (0.6)^2 + (0.7)^2 \le 1$<br>$0 \le 0.36 + 0.49 \le 1$<br>$0 \le 0.85 \le 1$ |

**Table 3.** *Values of IFG and PFG.*

Both IFG and PFG are effective strategies for dealing with uncertainty in graph-based models, with PFG providing more sophisticated features in some difficult cases.

## 4. CONCLUSION

This study explores a novel technique for solving SPP in uncertain environments using Fuzzy numbers. It examines a TriPF arc cost SPP, initially using TriPFNs as arc costs. The TriPFSP and associated TriPF cost are determined using Dijkstra's algorithm. The approach is illustrated using a tiny route/path network and will be applied to more challenging network issues.

Disclaimer (Artificial intelligence)

Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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Details of the AI usage are given below:

1.

2.

3.



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