An investigative study of pre-service mathematics teachers' performance on reflection on mathematics teaching and learning after taking the GeoGebra course

Abstract: The advancement of applied mathematics founded upon information technology (IT) represents a pivotal aspect of contemporary mathematical development, with the integration of IT into the mathematics curriculum becoming an indispensable necessity. The study of relevant IT-based courses by pre-service mathematics teachers will contribute to their professional development, as will their reflection on their teaching. This study combines the two, using 40 Master of Education (M.Ed.) students from a university as respondents and employing open-ended interviews to investigate the reflective performance of pre-service mathematics teachers in teaching mathematics after taking the GeoGebra course. Subsequently, the data were analysed from three perspectives: dimensions of reflection, level of reflection and degree of collaboration. The results were concluded as follows: 1. The dimensions of pre-service teachers' reflection was extensive but skewed, and reflection on the students' aspect was not yet evident; 2. The level of reflection was "describes only" (Level 1) and single-angle analytical (Level 2); 3. The degree of collaboration in teaching reflection was predominantly limited to "integration" and "self only"; 4. In conclusion, the GeoGebra programme can be considered an effective tool for assisting pre-service teachers in teaching mathematics. Therefore, it is suggested that: 1. It is recommended that higher education institutions reinforce the training of prospective mathematics teachers by incorporating pertinent GeoGebra courses into their curriculum. 2. Additionally, it is advised that prospective mathematics teachers assume the responsibility of conducting research and reflection to enhance their reflective awareness and capabilities.

Keywords: **pre-service mathematics teachers; GeoGebra; mathematics teaching; teaching reflection**

1. INTRODUCTION

Reflection also plays an important role in teachers' professional growth, and the curriculum standard puts forward requirements for mathematics teachers' reflection,

"The curriculum standard for general high school mathematics (2017 edition)" (Ministry of Education of the People's Republic of China, 2020) points out that "Mathematics teachers should apply theories to guide their practice, and constantly summarise and reflect on their teaching practice "[1], through teaching reflection, teachers are able to review their teaching practice and have more opportunities to identify problems and get solutions through in-depth analyses so as to improve their teaching and accumulate experience. Pre-service mathematics teachers, as aspiring members of the education profession, reflection is equally significant to their professional development as teachers. In the era of information technology, the development of applied mathematics based on information technology is an important feature in the development of mathematics today, and the use of information technology in the mathematics curriculum has become a necessity. GeoGebra software is a powerful software that has emerged in recent years and has a strong supporting effect on the teaching and learning of mathematics. So how do pre-service maths teachers perform in reflecting on the teaching of secondary school mathematics after learning GeoGebra course? What are the new reflections? How will the learning of GeoGebra software and the related courses affect pre-service mathematics teachers' teaching of mathematics? This is a question worth pondering.

2. LITERATURE REVIEW

This paper examines three key areas of research. Firstly, it considers the current state of research on pre-service mathematics teachers' reflection on teaching. Secondly, it reviews the current state of research on the GeoGebra programme. Finally, it presents the analytical framework of this paper, along with the main research questions in the context of the factors affecting teachers' reflection on teaching.

2.1 The current state of research on pre-service teachers' reflection on teaching and learning

In this paper, the term 'pre-service mathematics teachers' refers to trainee teachers who have not yet entered formal teaching and those undergoing new induction training who aspire to teach mathematics. A review of the literature reveals a multitude of research studies that focus on either pre-service or in-service teachers, with few studies encompassing both. A substantial body of research has been conducted on the subject of pre-service teachers' reflection, with the majority of studies focusing on specific aspects of teaching reflection. For example, the dimensions of reflection on teaching, the level of reflection , the ways of reflection on teaching and the sources of reflection on teaching have been the subject of much research.

A review of the literature reveals that, in practice, the development of pre-service teachers' reflection on teaching is not as effective as it could be. On the one hand, the content of pre-service teachers' reflections is limited and predominantly at the level of descriptive reflection. For example, Anu Sööt and colleagues employed the technique of analysing reflective journals and evaluated the degree of reflection exhibited by teacher educators in accordance with the Kosselgen onion model. The study revealed that teacher educators engage in reflection at all levels of the onion diagram model,

with the most prevalent level being beliefs, followed by environment, behaviour, competence, identity and mission [2]. This finding is in alignment with the conclusions drawn by Chinese researchers Fu Guanghuai and Tao Li [3]. Another researcher proposed the concept of "broad reflection," which entailed an analysis of trainee teachers' reflective journals to ascertain the evolution of their reflective abilities throughout the training period. The findings indicated that trainee teachers' capacity to reflect continued to mature, as evidenced by an expansion in the scope of their reflections. Initially, they focused on their own experiences as teachers. However, as their training progressed, they began to consider a broader range of factors, including mathematical knowledge, teaching strategies, and students' backgrounds. Eventually, they even delved into the cultural and political nuances of the school environment. However, with regard to the level of reflection, the majority of pre-service teachers were still predominantly at the descriptive level.

Conversely, those undertaking initial teacher training demonstrated a lack of motivation and sources of reflection. Miao Peizhou and other researchers conducted a questionnaire survey on teacher trainees' educational internship and found that, initially, teacher trainees demonstrated a commendable capacity for reflection and engaged in reflective activities with regularity. However, they also identified several shortcomings, including a lack of awareness of reflection, an imbalance in the direction of pedagogical reflection, and a reluctance among pre-service teachers to "self-disclose their shortcomings" and adopt a positive attitude towards reflection. The following issues have been identified as requiring further attention: The source of reflection is fundamental to motivating teachers to engage with specific content [4]. Xie Haiyan (2016) proposed three methods for stimulating reflection based on video repositories: self-discovery, peer pointing, and video comparison[5]. Xie Yuxing (2018) identified the following sources of teachers' reflection: teachers' own summaries and the combination of students' feedback[6]. The dearth of sources of reflection among pre-service teachers has resulted in a lack of awareness of reflection among the majority of these teachers, with the development of reflective skills remaining inadequate.

2.2 The current state of research on the utilisation of GeoGebra software

Since the introduction of GeoGebra software into China, more and more front-line teachers have discovered its superiority in improving classroom teaching efficiency, increasing students' interest in learning, reducing students' cognitive load, and enhancing students' learning ability. Therefore, research on GeoGebra-assisted teaching has gradually increased in recent years. After applying GeoGebra to assist high school mathematics teaching, Kou Hengqing found that the application of GeoGebra can stimulate students' interest in mathematics learning, because with the help of GeoGebra, it can show the marvellousness and beauty of mathematics and the value of the application of mathematics, which enriches the students' way of learning and builds up a platform for students' mathematical exploration after class; the application of GeoGebra optimizes the presentation of teaching content, present teaching content in a diversified way, and dynamically present the formation process of graphs; the application of GeoGebra can also improve the effectiveness of teachers'

work [7]. Sheng Sijia proposed the use of GeoGebra auxiliary teaching, can let the students in the "do" in the "learning", personal experience of the process of knowledge generation to the development of the knowledge system, the construction of knowledge, compared with the traditional teaching, appears to be more intuitively find the laws of mathematics, to overcome the difficulties brought about by the high degree of abstraction of mathematics. Highly abstract and bring difficulties [8]. Zhang Zhiyong proposed that the implementation of visual teaching with the help of GeoGebra can promote students' understanding of the nature of mathematics and the perception of mathematical ideas and methods; teachers pay more attention to the visual limitations, promote the appropriate balance, and in the process of integrating information technology and mathematics teaching, it can help students to form the awareness of the tools, learning awareness, curriculum awareness and development awareness.

In light of the robust functionality of GeoGebra software and its advantageous role in facilitating mathematics instruction, an increasing number of higher education institutions have incorporated related information technology courses into their curriculum. This is with a view to equipping prospective mathematics educators with the requisite information technology skills and abilities, and to fostering information literacy among prospective teachers and the professional development of teachers in the information technology age. In light of the aforementioned, the present study aims to examine the extent to which prospective mathematics educators who have undergone training in the use of GeoGebra-related software have developed novel perspectives and insights pertaining to the teaching and learning of mathematics.

2.3 Main research questions and analytical framework of this study

2.3.1 Main research questions of this study

This study is concerned with the performance of pre-service mathematics teachers in terms of their reflections on mathematics teaching and learning following the completion of the GeoGebra course. In particular, it seeks to ascertain the perspectives from which pre-service mathematics teachers reflect on their teaching practice and the level of reflection achieved following the completion of the GeoGebra course. The objective is to ascertain the specific impact of the GeoGebra course on the teaching of mathematics by pre-service teachers. In light of the objectives of this study and the focus of existing research, this paper will examine the impact of the GeoGebra programme on the teaching and learning performance of pre-service mathematics teachers with regard to three key areas: the dimensions of reflection, the level of reflection and the degree of collaboration. Three perspectives for analysing the performance of mathematics teachers' reflections on teaching and learning were identified in the literature review[9-10].

In this context, the dimensions of reflection and the level of reflection have the same meaning as most of the existing studies, i.e., the focus and depth of reflection on teaching and learning. The dimensions of reflection on teaching and learning can focus on teachers; students; mathematics; teacher-student; teacher-mathematics; student-mathematics; teacher-student-mathematics; and others. The level of reflection is divided into three levels: describes only; Single-angle analysis; and multiple

perspective analysis. The degree of collaboration is to examine how well pre-service mathematics teachers support each other within the group in the context of learning the GeoGebra course, i.e. whether or not they reflect on teaching and learning in conjunction with the actions or evaluations of others, and is divided into the three dimensions of self only, others only, and integration. Therefore, the main research question of this paper is:

1. How pre-service mathematics teachers reflect on their teaching and learning after the GeoGebra course, with a particular focus on three analytical perspectives: reflective dimensions, level of reflection and degree of collaboration?

2. What impact does studying the GeoGebra programme have on the teaching of mathematics by pre-service mathematics teachers?

2.3.2 Analytical framework of the study

In light of the findings of existing research in this field, this study will employ the dimensions of reflection, level of reflection and degree of collaboration as the points of departure for examining the teaching reflection of pre-service mathematics teachers. The specific analytical framework is presented in **Table 1**.

Table 1: Analytical framework of the study

Analytical Perspectives	Form	Description of content
Dimensions of reflection	Teachers only	Content related to teachers only. Examples include basic teaching skills, teachers' own status, teacher-teacher collaboration, classroom management and control, teaching strategies or teaching effectiveness.
	Students only	Content related to students only. For example, students' learning status and classroom performance, and analyses of general student learning.
	Maths only	Content related to mathematics only. E.g. correct and standardised presentation of mathematical content, discussion of mathematical knowledge and methods (not related to teaching).
	Teachers - students	Teachers' attention and analysis of students, design of student activities, and interaction and communication with students.
	Teachers - Maths	Teachers' instructional design and implementation of mathematical content, teachers' mathematical correctness, etc. (weak relationship with students).
	Students - Maths	Incorporate the analysis of the learning situation of the specific mathematical content, the students' state of learning on the mathematical content and their classroom

		performance (weak relationship with the teacher).
	Teachers - Students - Maths	Teachers design and implement the teaching and learning related to the mathematical content by analysing the students, and teachers and students communicate and interact with each other around the mathematical content.
	Other	Other aspects that are not relevant to teachers, students, or maths.
Level of reflection	Describes only (Level I)	Simply describing ideas, events or improvements without further reasoning or analysis or attempting to make connections.
	Single-angle analysis (Level II)	Attempts to reason and analyse or make connections to ideas, events or improvements based on descriptions, but from a single perspective.
	Multi-perspective analysis (Level III)	Combines multiple elements from other related ideas, events, improvements, theories, causes, effects, etc., to provide a comprehensive analysis of ideas, events, or improvements from multiple perspectives.
Degree of collaboration	Self only	Does not incorporate the behaviours or evaluations of others and bases his/her reflections solely on his/her own ideas.
	Others only	Only commenting on the behaviour of others but not relating it to themselves, or only repeating what others have said about themselves without any new thinking of their own.
	Integration	Reflects on the behaviour or evaluations of others and relates to himself or herself or has new thoughts of his or her own.

3. THEORETICAL BASIS

3.1 The meaning and characteristics of reflection on teaching and learning

The earliest scholar to introduce the concept of "reflection" in the field of teaching was Dewey, who believed that reflection could cause problem-solving action and purposeful inquiry, arising from the confusion and doubt in the context of direct experience, and that it was a specialised form of thinking, with the characteristics of continuity, initiative, and thoroughness, etc. In the 1980s, Schon proposed the concept of reflective teaching and studied reflective practice in depth, but his research ignored the social conditions and their influence on teaching practice, focusing only on

individual teachers' teaching behaviours and their impact on teaching practice. In the 1980s, Schon proposed the concept of reflective teaching and studied reflective practice in depth, but his research ignored the social conditions and their impact on teaching practice, focusing only on the individual teacher's teaching behaviour and process. Since then, reflective practice thinking in the field of teacher education has become a hotspot of concern [11-12]. Currently, there is no unified statement about the definition of teaching reflection, Professor Xiong Chuanwu pointed out in his book On Reflective Teaching that reflection is based on exploring and solving teaching problems with the purpose of seeking rationality [13]. Shen Jiliang and Liu Jiaxia defined teaching reflection from the purpose of teaching reflection, the object of reflection and the process of reflection, which believed that the purpose of teaching reflection is to achieve effective teaching and learning, the object of reflection refers to the theories and assumptions behind the teaching and learning activities that have taken place or are taking place as well as these activities, and the process of reflection is to think about the above objects, clearly characterise the problems and seek ways to solve the problems [14].

Based on the definitions of reflection on teaching by different scholars, the analysis has come up with the characteristics of teachers' reflection on teaching: firstly, the initial motivation for the emergence of reflection on teaching is confusion and doubt. When there is a blockage in the process of carrying out teaching activities, teachers think about and analyse possible problems, deconstruct plans, decisions and results that have already been made, and reconstruct them in a new form. Secondly, reflection on teaching is a process of introspection and critique from a systemic perspective. On the basis of questioning, the teacher is able to seriously review and sort out his/her awareness, ideas, feelings, attitudes and behaviours in his/her teaching and learning activities with an open mind, so as to achieve the purpose of unification. Third, teaching reflection is a kind of active thinking process of teachers. Teachers are strongly driven by a sense of responsibility and subjective self-awareness to communicate with students and peers or to learn advanced theories, to examine their own teaching concepts and practices in a timely manner, and to continuously improve themselves in teaching.

3.2 Theoretical basis for reflection on teaching and learning

The theoretical foundations of reflection on teaching and learning are primarily based on metacognition theory. The term 'metacognition' was first introduced by the American developmental psychologist Flavelli in the 1970s. In Flavelli's view, metacognition refers to an individual's awareness of their own cognitive processes and outcomes. It encompasses the capacity to actively monitor and continuously regulate these processes in order to achieve specific goals or complete tasks. Other researchers posit that the essence of metacognition is people's cognitive adjustment of ongoing cognitive activities and the regulation of cognitive activities. The necessity for metacognitive skills is thus evident. Scholars have identified four metacognitive skills: metacognitive planning, metacognitive monitoring, metacognitive regulation, and metacognitive evaluation. Individuals must plan the cognitive actions that will be taken, as well as the cognitive activities and the results of the process of assessment.

They must then use the process evaluation of the cognitive activities to improve and enhance. The advent of the information age has necessitated the formulation of more rigorous standards for the professional development of educators. In this digital age, teachers are required to enhance their metacognitive abilities, given that their reservoir of knowledge remains comparatively limited in comparison to the vast expanse of information available on the Internet. It is, therefore, incumbent upon educators to equip their students with the necessary skills to navigate and harness the wealth of knowledge available to them. [15] The enhancement of teachers' metacognitive abilities can prompt educators to assume a more proactive role in optimising pedagogical practices. In particular, teachers' metacognitive knowledge base can facilitate the objective evaluation of their professional competencies and a comprehensive understanding of their students' circumstances. This enables educators to tailor their instruction to align with students' aptitudes and foster the optimal development of their potential. Educators with metacognitive proficiency tend to engage in ongoing reflection on their cognitive processes. Teachers with metacognitive skills will be aware of their own ongoing cognitive activities in practice. They will provide real-time feedback on the results of their teaching in accordance with the teaching objectives, identify any teaching problems and implement appropriate remedial measures. In this process, teachers will also develop a positive mood and a high sense of self-efficacy, which will lead to continuous reflective activities. Consequently, metacognitive abilities constitute a crucial foundation for educators to engage in reflective practice. By cultivating a consciousness of self-reflection, developing robust metacognitive expertise, and honing stable metacognitive monitoring and regulation skills, educators can enhance their capacity for reflection, which is instrumental to their professional growth and development. [16-18].

4. RESEARCH METHODS

4.1 Participants

In order to accurately assess the impact of GeoGebra courses on pre-service mathematics teachers' reflection on secondary school mathematics teaching, a sample of 40 Master of Education postgraduates, 35 female and 5 male, majoring in Subject Teaching (Mathematics) in the class of 2023 at the School of Mathematics and Statistics, Shandong Normal University, was selected as the target respondents of this study. All of the respondents have completed the requisite GeoGebra courses and will utilise the software to assist with their future teaching of mathematics.

4.2 Instrument

This study was investigated using the open-ended interview method, with a question designed into the interview outline: "After taking the GeoGebra course, what are your new thoughts and reflections on the integration of GeoGebra and mathematics teaching and learning?" Please provide a detailed and comprehensive account of your thoughts on this matter. This question was selected to ascertain the impact of the GeoGebra course on the teaching of mathematics by pre-service mathematics teachers. The impact was primarily observed in the form of reflection on teaching. The

open-ended interview method was selected for its expediency, flexibility, and capacity to elicit in-depth responses beyond the scope of written language.

4.3 Data collection

In order to ensure the reliability of the study, the open-ended interview method was used to interview each of the 40 M.Ed. postgraduate students individually, and the interviews were audio-recorded in their entirety after asking for each other's permission.

4.4 Data Processing

The content of the analytical framework presented above was initially divided and coded using the symbols A, B and C to denote the three dimensions of reflective dimensions, level of reflection and degree of collaboration, respectively. This was followed by the introduction of the symbols A1-A8 to denote the eight dimensions of reflective content, namely: A1 Teachers ; A2 Students ; A3 Maths only; A4 Teachers-Students; A5 Teachers-Maths; A6 Students-Maths; A7 Teachers-Students-Maths, A8 Other 8 dimensions respectively. B1-B3 denote the three levels of reflection level, i.e. B1 Describes only (level 1); B2 Single angle analysis (level 2); B3 Multi-perspective analysis (level 3). C1 Self only; C2 Others only; C3 Integration. Subsequently, further coding divisions were made according to the specific content contained in each section. For example, A11, which denotes Teachers' Basic Teaching Skills, denotes a further division under the indicator A1, which denotes Teachers Only. From this, a total of three aspects of A–C and 31 items of A11–C15 were divided, and the specific contents are shown in Table 2. Subsequently, the contents of the interview recordings were converted into text, and the original words spoken during the interviews were removed and organised strictly according to the interviews, and compared with the coded contents one by one. Once the content had been compared individually, it was deemed appropriate to consider the content of a similar meaning as a reflection of the survey respondents. This was determined by establishing the level of reflection and the degree of cooperation based on the survey respondents' reflections. Ultimately, the number of individuals referenced in each item and the aggregate number of individuals referenced in each indicator were tabulated, the corresponding percentages were calculated, and a statistical table was constructed.

Table 2: Content coding

Category		Label	Description of content
A Reflective dimensions	**A1 Teachers only**	A11	Teachers' Basic Teaching Skills
		A12	Teachers' own status
		A13	Teachers' teaching strategies or effectiveness
		A14	Teachers' classroom management and control
	A2 Students only	A21	Students' learning status and classroom performance
		A22	Analyses of student learning
		A23	Reflections on student responses in the

			classroom
		A24	Reflection on students' abilities in all areas
	A3 Maths only	A31	Reflections on the application of understanding of specific mathematical knowledge
		A32	Discussion of mathematical knowledge and methods
	A4 Teachers - Students	A41	Teachers' attention and analysis of students
		A42	Teachers design student activities
		A43	Teachers' interaction with students
	A5 Teachers - Maths	A51	Teachers' design and implementation of mathematics content
		A52	Teachers' mathematical knowledge base and application of correct
	A6 Students - Maths	A61	Learning situation analysis in relation to specific mathematical content
		A62	Students' state of learning in maths content
		A63	Reflection on students' reactions in class
	A7 Teachers - Students - Maths	A71	Teachers design and implement mathematics content-related teaching through analysing students
		A72	Teachers and students interacting around maths content
	A8 Other	A81	Aspects not directly related to teachers, students, or mathematics
B Level of reflection	**B1 Describes only (Level I)**	B11	Only describes ideas, events without further reasoning or analysis and without attempting to make connections
		B12	Only describing improvements without further reasoned analysis or attempts to make connections
	B2 Single angle analysis (Level II)	B21	Reasoning and analysing or making connections to ideas and events based on descriptions, but from a single perspective
		B22	Reasoned analyses or linkages to improvements based on descriptions, but from a single perspective
	B3 Multi-perspecti	B31	Reflection combines multiple elements of other relevant ideas, events,

	ve analysis (Level III)		improvements, theories, causes, effects, etc.
		B32	Multi-faceted and comprehensive analyses of ideas, events or improvements
C Degree of collaboration	**C1 Self only**	C11	Reflection based on one's own ideas without incorporating the behaviour or evaluations of others
	C2 Others only	C21	Commenting only on the behaviour of others without relating it to oneself
		C22	Repeating what others have said about themselves and their impact without new thinking of their own.
	C3 Integration	C31	Reflecting on the behaviour or evaluations of others and making connections and new reflections of their own.

5. RESULTS

5.1 On the reflective dimensions

According to the analytical framework, a total of 21 items were classified as related to the reflective dimensions, and pre-service mathematics teachers were able to reflect on 20 of them, accounting for 95.23% of the total, which indicates that the content of the listed reflections was largely covered. In terms of different aspects, the indicator "Teachers only" had the highest total number of reflections, with 32 persons, or 80% of the total; followed by the indicator "Teachers-students-maths", which was mentioned by 22, or 55% of the total; and again by the indicator "Teachers-maths", with 22 persons, or 55% of the total. The indicator "Teacher - Maths" was mentioned by 20 people, or 50% of the total; the indicator "Other" was the lowest, with 3 people, or 7.5% of the total; and the rest of the indicators had a total number of reflections ranging from 20 to 35 per cent of the total number of reflections. With regard to the points classified under each indicator, under the indicator "Teacher-student-maths", the point "Teachers design and implement mathematics content-related teaching through analysing students" had the highest number of reflections, with 21 persons, accounting for 52.5% of the total number of reflections. This was followed by "Teachers' design and implementation of mathematics content" under the indicator "Teachers - Maths", with 13 persons (32.5%) of the total number of reflections. The indicator "students only" was divided into four points, of which only three were recognised, with no mention of "reflection on students' reactions in class", while all other aspects were covered.

It can be seen that the content of the current pre-service mathematics teachers' teaching reflections after studying the GeoGebra course is not comprehensive and biased, and the reflections are mostly focused on individual points. In terms of

specific content, most of the pre-service teachers can reflect on combining GeoGebra with students and mathematics for instructional design, and reflect more on the basic skills of the teachers' own state, while reflecting on the students and their performance on the mathematical content is not yet in place. The details are shown in **Table 3**.

Table 3: Statistical data on the results of reflective content

Category		Label	Number	Percentage (%)	Total number of reflections for each indicator	Percentage (%)
A Reflective dimensions	A1 Teachers only	A11	10	25	32	80
		A12	9	22.5		
		A13	11	27.5		
		A14	2	5		
	A2 students only	A21	1	2.5	10	25
		A22	2	5		
		A23	0	0		
		A24	7	17.5		
	A3 Maths only	A31	7	17.5	14	35
		A32	7	17.5		
	A4 Teachers - Students	A41	5	12.5	9	22.5
		A42	3	7.5		
		A43	1	2.5		
	A5 Teachers - Maths	A51	13	32.5	20	50
		A52	7	17.5		
	A6 Students - Maths	A61	8	20	12	30
		A62	3	7.5		
		A63	1	2.5		
	A7 Teachers - Students - Maths	A71	21	52.5	22	55
		A72	1	2.5		
	A8 Other	A81	3	7.5	3	7.5

5.2 On the level of reflection

According to the analytical framework, the components related to the level of reflection were divided into three levels and six areas. In terms of the different levels, the highest number of people, 17 or 42.5% of the total, reflected on the outcome of "describes only (level 1)"; 16 persons or 40% of the total reflected on the outcome of

"single-angle analysis (level 2)"; and "Multi-perspective analysis (Level III)" for 7 people (17.5% of the total). In terms of the content of the six aspects of the three levels, most of the reflections stayed at Level 1 "Only describes ideas, events without further reasoning or analysis and without attempting to make connections", and at Level 2 "Reasoning and analysing or making connections to ideas and events based on descriptions, but from a single perspective".

It can be seen that the level of pedagogical reflection on mathematics teaching by current pre-service mathematics teachers after studying the GeoGebra course still needs to be improved, and most of them only carry out descriptive reflections or reflections with a single angle of analysis on the viewpoints and measures for improvement without combining with other elements to carry out comprehensive analyses from multiple perspectives. Details are shown in **Table 4**.

Table 4: Statistical data on the results of the level of reflection

Category		Label	Number	Percentage (%)	Total number of reflections for each indicator	Percentage (%)
B Level of reflection	**B1 describes only (Level I)**	B11	9	22.5	17	42.5
		B12	8	20		
	B2 single angle analysis (Level II)	B21	5	12.5	16	40
		B22	11	27.5		
	B3 Multi-perspective analysis (Level III)	B31	5	12.5	7	17.5
		B32	2	5		

5.3 On the degree of collaboration

According to the analytical framework, the components related to the degree of cooperation in reflection were divided into three areas with four items. In terms of the different dimensions, the indicator "Self only" had the highest total number of reflections, with 24, or 60% of the total, while the indicator "Integration" had the second highest total number of reflections, with 12, or 30% of the total; The indicator "others only" had the lowest total number of reflections, with 2, or 5 % of the total. Specifically, the majority of pre-service teachers were in the position of "Reflection based on one's own ideas without incorporating the behaviour or evaluations of others"; some pre-service teachers were able to "reflect on the behaviour or comments of others and relate them to themselves, with their own new thinking"; no one was "only commenting"; and no one was "Reflecting on the behaviour or evaluations of others and making connections and new reflections of their own.". None "Commenting only on the behaviour of others without relating it to oneself".

It can be seen that the current level of collaboration among pre-service mathematics teachers in their pedagogical reflections on mathematics teaching and learning after the GeoGebra course is predominantly on their own and in integration, with very little just describing the behaviour of others or recapitulating the comments of others.The GeoGebra course provides pre-service mathematics teachers with a number of opportunities to collaborate in their learning, for example, by sharing in small groups, and by each pre-service teacher engaging in mutual evaluation and discussion. Based on the access to others' evaluations, pre-service mathematics teachers would choose to integrate others' information or return to themselves and focus only on themselves for teaching reflection. Details are shown in **Table 5**.

Table 5: Statistical data on the results of the degree of collaboration

Category		Label	Number	Percentage (%)	Total number of reflections for each indicator	Percentage (%)
C Degree of collaboration	**C1 Self only**	C11	24	60	24	60
	C2 Others only	C21	0	0	2	5
		C22	2	5		
	C3 Integration	C31	12	30	12	30

6. DISCUSSION

6.1 On the Reflective dimensions

From the above data analysis, it can be seen that most of the teaching reflections of the current pre-service mathematics teachers after learning GeoGebra courses are able to combine GeoGebra with students and mathematics, and they can reflect on the use of GeoGebra technology to present knowledge in a timely manner in accordance with the learning situation and the content of mathematics. A number of pre-service teachers reflected on the incorporation of GeoGebra technology into their instructional design to help students better grasp knowledge, with the help of examples such as "Teaching Exponential Functions", "Moving Points Problems", and "Angle Theorem of a Circle". Secondly, pre-service teachers also reflected more on teachers' own basic skills and status, for example, pre-service teachers' mathematical and technical knowledge reserves; teachers' own qualities; teachers' attitudes; and teaching styles. However, the pre-service teachers' reflections on the students' aspects were lacking, and they could consider that the integration of GeoGebra could help to improve the students' literacy skills but not in depth enough, precisely because the pre-service teachers were not formally engaged in teaching, so they did not have in-depth reflections on the students' learning situation, classroom responses, and students' abilities in various aspects.

6.2 On the level of reflection

From the analysis of the above data, it can be seen that nearly half of the pre-service teachers' reflection level was at the level of "describes only", which only describes the ideas, events or improvement measures without further reasoning and analyses. For example, they only indicated that the use of GeoGebra in mathematics teaching was helpful, without further analysis in relation to relevant mathematical knowledge and students' situation. Nearly 40% of the pre-service teachers conducted a "Single angle analysis", some pre-service teachers only considered what types of topics or knowledge GeoGebra was suitable for, without analysing them in the context of the students, teachers, and so on. This shows that there is still room for improvement in the level of reflection on mathematics teaching by pre-service mathematics teachers after learning GeoGebra, and that they should conduct comprehensive reflection and analysis based on the consideration of teachers and students, mathematical content, real resources and teachers' own practice experience, so as to better integrate GeoGebra with mathematics teaching.

6.3 On the degree of collaboration

From the analysis of the above data, it can be seen that the degree of collaboration in teaching reflection among pre-service teachers after taking the GeoGebra course was mainly focused on "self only" and "integration", accounting for 60% and 30% respectively, while "others only" accounted for less than 10%. It can be seen that pre-service teachers, through the GeoGebra course, will integrate, for example, group sharing and reading and learning from related literature to reflect, so as to put forward their own different opinions and new thinking. Alternatively, on the basis of the course learning, they may return to themselves and reflect only from their own perspective, thereby improving their teaching and laying the foundation for their own professional development.

7.CONCLUSION

The purpose of this study was to find out the reflective performance of pre-service mathematics teachers on secondary school mathematics teaching after taking the GeoGebra course. For this purpose this study selected 40 Master of Education students from Shandong Normal University (China) who had taken the GeoGebra course and investigated the reflective performance of pre-service mathematics teachers using the open-ended interview method. By collecting relevant information, then organising and analysing the data according to the questions, the research conclusions were finally drawn. The following research conclusions are given in the order of the research questions: 1. In terms of the dimensions of reflection, pre-service mathematics teachers' reflections on their teaching and learning after studying the GeoGebra course covered a wide range of topics but were also biased. Most of the pre-service teachers' reflections were able to integrate GeoGebra with students and mathematics, although the reflection on the students' aspect was not yet in place; in terms of the level of reflection, the pre-service teachers' level of reflection was concentrated at Level 1 and Level 2, i.e., reflection was at the level of description only or single-angle analysis; in terms of the degree of collaboration, the degree of

collaboration in teaching reflection was concentrated at the levels of "self only" and "integration". The study of the GeoGebra programme provides many opportunities for pre-service mathematics teachers to cooperate in learning, with some teachers reflecting from themselves and some teachers integrating the evaluation of others. 2. Overall speaking, the study of the GeoGebra programme can effectively assist pre-service teachers in teaching maths. Specifically, with the help of Geogebra's dynamic demonstration, it can show the process of knowledge formation, stimulate students' interest in learning, and help them to understand and master; Secondly, Geogebra can guide students to observe the changes due to "number" changes to "shape" changes, experience from the special to the general process of inquiry, to train students in the combination of number and shape of the idea; Finally, Geogebra to assist in teaching, you can achieve the effects of traditional teaching methods can not be achieved to help students solve the key problems, to facilitate the teacher's teaching and improve teaching efficiency. However, while using Geogebra to assist teaching, teachers should not completely rely on information technology and should use it at the right time.

From the above conclusions, it is recommended that: 1. Teachers and experts related to the training of pre-service high school mathematics teachers should pay more attention to information technology, such as Geogebra teaching, and strengthen the training of pre-service mathematics teachers through the addition of relevant courses; 2. Pre-service mathematics teachers should take the initiative to conduct learning, research, and reflection, and to improve their own awareness and ability to reflect, so that they can: (1) Strengthen their own mathematical knowledge and technical knowledge, and improve their own quality and professionalism; (2) Conduct multiple analyses by considering both teachers, students, and mathematics. knowledge reserves, improve their own quality and professionalism; (3) pay attention to both group work and be able to think independently to identify problems [19-20].

The research population of this survey was 40 Master of Education students, which is a relatively small sample size and concentrated on the same grade level in the same institution, and the sample selection was narrower as other types of pre-service mathematics teacher groups were not involved. Therefore, in the future, it is necessary to expand the scope of the research sample and further conduct a more in-depth study on the performance of pre-service mathematics teachers' reflection on secondary school mathematics teaching and learning after studying the GeoGebra course by adopting a variety of research methods, with a view to finding out more detailed and comprehensive results.

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