

# Impact of Pre-sale Home Increment on the Volatility Error of Existing Housing Prices

## Abstract

During periods of economic prosperity, real estate developers launch numerous pre-sale housing projects to expand business opportunities, fill market supply gaps, and reduce the demand for completed homes, thereby stabilizing housing prices. However, most people do not perceive housing price as stabilizing. This raises research interest in whether the volume of pre-sale housing projects can mitigate the demand for completed homes and achieve price stability. This study aims to explore the impact of pre-sale housing project volumes on the volatility error of transaction prices of completed homes. Currently, the mainstream product in the market for completed homes is collective housing, typically classified based on building height and the presence of elevator facilities into apartments, mid-rise buildings, and high-rise buildings. Due to changes in the construction environment, the number of new apartment buildings has sharply decreased, making older apartments the primary objects of transactions. The study found that the ARIMA(2,1,0)-GARCH(1,1) model is the best model and exhibits clustering volatility. The GARCH(-1) volatility coefficient  $|\beta_1| = 0.957913 < 1$  indicating that volatility decreases geometrically over successive periods, achieving stability. When the volume of pre-sale housing projects is included in the model, the GARCH(-1) volatility coefficient  $|\beta_1| = 1.032349 > 1$ , indicating that volatility increases geometrically over successive periods, failing to achieve stability. Furthermore, the interference coefficient  $\theta = 0.0012 > 0$  for pre-sale housing project volumes does not reduce volatility. The price volatility model without incorporating pre-sale house project volumes is convergent. However, once pre-sale house project volumes are included, the model fails to effectively stabilize price volatility. This suggests that the market's expectations regarding the volume of pre-sale house projects volumes do not effectively stabilize price volatility.

Keywords: Volatility clustering, Conditional heteroskedasticity, ARCH/GARCH model, Taichung

## 1. Introduction

The pre-sale housing market is a unique transactions system prevalent in Taiwan, Hong Kong, and China, and is one of the common ways for people to purchase homes. Consequently, research on pre-sale housing focuses on these regions. For example, in Taiwan, Lee and Yang (2010) observed that the pre-sale houses possess futures characteristics, and examining the base period gaps and changes helps understand their substitutability. Chiang et al. (2019) observed long-term forecasts of pre-sale and existing home market prices, aiding in price information flow and market price stabilization. Wang et al. (2022) used the cobweb model and forward contract pricing model to illustrate that the demand for spot housing in the real estate market reacts immediately, and due to the lag in the supply of existing homes, demand may shift to the pre-sale housing market, alleviating the demand for existing homes but causing overheating in the pre-sale market. The intense demand for existing homes can impact pre-sale prices, preventing convergence. Whether the price adjustment function of pre-sale supply backfires is debatable. Studies have shown that over the past few decades, the decline in natural interest rates in developed economies has been accompanied by significant increases in housing price volatility, verifying that recent Taiwanese government policies on interest rates and land holding costs have not been suppressive.

Research in China, such as by Wang et al. (2007) on Shanghai housing prices and Chi and Wang (2010) on Yunnan Province housing prices, concluded that the pre-sale housing market stabilizes the existing home market. In Hong Kong, Wong et al. (2006) researched housing price volatility and pointed out that the pre-sale housing market has a stabilizing effect on the prices of existing homes. Most studies agree that the volume of pre-sale housing projects and existing home prices have mutual regulatory functions. Therefore, developers often choose to launch many pre-sale houses during economic booms to effectively seize business opportunities, fill the supply gap in the existing home market, and alleviate the demand for existing homes, expecting this adjustment to stabilize prices. However, most people often do not perceive price stabilization, raising the research question: Can the volume of pre-sale housing projects stabilize the prices of existing homes?

This study aims to observe the volume of pre-sale housing projects and the resulting volatility whether it persists, diverges, or converges to explore the stability of housing price fluctuations and satisfy the research motivation. Pre-sale houses have the characteristic

of supply lag and usually achieve equilibrium prices through periodic price adjustment processes. This process involves the volume of pre-sale housing projects in the current period being determined by the reference price of the previous period's existing homes, while the current period's pre-sale prices are set by the current market prices. The adjustment process forms a cobweb-like convergence trend until price equilibrium is reached.

Furthermore, housing price volatility, due to its asset preservation attribute, often exhibits the following major characteristics: leptokurtic density distribution, conditional variance tending to change over time, strong volatility persistence (large volatility followed by large fluctuations, small fluctuations followed by small volatility), and empirical distribution of asset price series showing a heavy tail phenomenon (more residual values). This phenomenon is known as volatility clustering (Chen, 2020). The conditional variance equation's conditional variance  $\sigma_t^2$  is used to replace risk and uncertainty, and in economic terms,  $\sigma_t^2$  is seen as a quantitative indicator of fluctuation risk, or volatility and volatility clustering. Research in this area often targets average or median transaction prices.  $\sigma_t^2$  is an unobservable variable as a fourth-moment conditional variance but can be estimated using the ARCH/GARCH model. Short-term impacts come from the previous period's residual  $\varepsilon_{t-1}$  as the forecast error of the price average equation. Therefore, the size of unexpected impacts can be estimated by its coefficient representing the degree of impact volatility. The lag term  $\sigma_{t-1}^2$  represents the previous period's conditional variance. The more lag terms there are, the longer the impact time. If the coefficient  $|\beta_1| < 1$ , it will geometrically decrease; the smaller the value, the faster the decrease, and the smaller the impact. Conversely,  $|\beta_1| > 1$  will have the opposite effect.

The mainstream product in the existing home market is collective housing, typically categorized by building height or the presence of an elevator into apartments and high-rise buildings. Due to changes in the construction environment, the construction of new apartment-type housing has significantly decreased, focusing on older properties as the main transaction objects. The market inevitably opts for the best purchases, leading to a downward trend in transaction prices due to successive optimal selections.

From August 2012 to June 2023, there were 242,957 residential real transaction records in Taichung City, with 17,348 apartment transactions after sorting. The transaction volume by housing age is shown in Table 1, indicating that properties less than 15 years old have the highest transaction volume for high-rise buildings, while apartments over 30 years old account for 28.26% of transactions, becoming one of the mainstreams of old property

transactions.

Table 1 Age and Transaction Amount of Homes in Taichung City

Housing Age	New construction $\leq 3$	Newpre-owned homes $<3 \sim \leq 15$	Pre-owned homes $15 \sim \leq 30$	Old buildings $30 \leq$
High-riseapartments	4,730	3,982	26,600	3,794
%	7.07%	8.98%	29.85%	8.97%
High-risebuildings	41,943	28,159	41,689	2,555
%	62.70%	63.47%	46.78%	6.04%
Townhouses	20,057	11,696	16,126	24,002
%	29.98%	26.35%	18.10%	56.73%
Apartments	166	526	4,702	11,954
%	0.25%	1.20%	5.27%	28.26%
Total	66,896	44,363	89,117	42,305
%	100%	100%	100%	100%

Compiled by this study

Most of the literature on housing price volatility focuses on property types such as residential, office buildings, retail, factories, and apartments, further divided into one- bedroom and two-bedroom units. Data organization often involves using monthly average transaction prices or median monthly transaction prices. Several ARCH/GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models are tested to identify the most appropriate model for estimation and interpretation.

This study uses transaction data from the Ministry of the Interior's real price registration from August 2012 to June 2023, organized into 131 periods of monthly average data. The volume of pre-sale housing projects is derived from the Ministry of the Interior's building permit statistics, organized as monthly changes, and included in the model as an exogenous variable. First, the data properties are determined; the data in this study are non-stationary and must be differenced once to transform them into stationary data. The ADF (Augmented Dickey-Fuller test) unit root test and residual tests are conducted, and multiple ARCH/GARCH models are established. Using model selection criteria, the most appropriate model is selected.

This study identifies the ARIMA(2,1,0)-GARCH (1,1) as the most suitable model for housing price volatility. The volatility duration meets the significance level but does not converge in the long term. Before incorporating the pre-sale housing project volume into the volatility model, the model converges; after inclusion, it diverges. This suggests that the continuous optimal selection of old apartments for redevelopment results in a growing number of substandard properties that do not meet new development standards, leading to a significant decline in transaction prices and increasing price volatility.

## 2. Literature Review

### (1) Real Estate Transaction Price volatility:

Random price volatility are a normal feature of market operations. In high-frequency trading assets such as stocks, exchange rates, and gold, which are priced daily, large volatility are often followed by even larger volatility, while small volatility are followed by even smaller volatility. This phenomenon, known as volatility clustering, indicates that large volatility are associated with large volatility and small volatility with small volatility. Since the housing market operates with lower frequency transactions, typically reported monthly or quarterly, it is worth investigating whether housing market price fluctuations also exhibit volatility clustering like other assets. The following studies provide relevant insights:

Wang and Hartzel (2020) analyzed real estate price volatility in Hong Kong from February 1993 to February 2019, using monthly transaction data for residential, office, retail, and industrial properties. They found that all four types of real estate exhibited volatility clustering. The factors influencing volatility varied by property type, with residential property volatility primarily affected by exchange rates (RMB and USD), and commercial real estate volatility mainly influenced by unemployment rates. Dufitinema (2020) studied the Finnish housing market with a focus on apartments. The research found that more than half of the cities experienced clustering in apartment prices, and nearly all cities showed asymmetric volatility in apartment prices when impacted by shocks. Additionally, differences in price volatility were observed between different types of housing. Miles (2020) examined the residential markets in all 50 U.S. states and found that over half of the states exhibited volatility clustering. Kaulihowa and Kamiti (2019) investigated the impact of macroeconomic factors on housing price volatility in Namibia from Q1 2007 to Q2 2017 and supported the hypothesis that Namibian housing prices exhibit persistent volatility. Fan et al. (2022) demonstrated that uncertainty in economic policy is a significant factor contributing to housing price volatility. Wang et al. (2022) developed two theoretical models to analyze the long-term equilibrium and short-term dynamics of the pre-sale housing market, finding that the pre-sale housing market in Taiwan experienced signs of overheating in the short term.

### (2) Literature on per-sale Housing Supply:

Hua (2001) posits that both pre-sale prices and existing home prices tend toward long-term equilibrium, but that pre-sale prices adjust more quickly than existing prices. This suggests that the presence of a pre-sale system enhances market efficiency. The background of this research is Taiwan, where the real estate market can be divided into the existing home market and the pre-sale home market. The existing home market can be seen as an inventory

market, while the pre-sale market is a flow market. Somerville (1999) studied the impact of the inventory of unsold homes held by developers on the market. The research found that factors such as the volume of new projects, the cities where these projects are located, market conditions, the types of unsold homes, and pre-sale methods significantly affect the inventory of unsold homes held by developers. Research by Wang et al. (2007) on housing prices in Shanghai and by Qi and Wang (2010) on housing prices in Yunnan Province both concluded that the pre-sale market plays a stabilizing role in the existing home market. Wong et al. (2006) studied housing price volatility in Hong Kong and indicated that the pre-sale market has a stabilizing effect on existing home price volatility. Yang and Chuang (2024) observed that during economic prosperity, the supply of pre-sale homes tends to drive up residential and condominium prices, leading to divergence. Mei and Lin (2017) found that the construction cost index positively affects housing prices.

The inventory of unsold pre-sale housing and construction costs influence the volume of new projects and, indirectly, pre-sale pricing. Since pre-sale prices adjust quickly, they impact the volatility in existing housing market prices. Thus, whether the volume of pre-sale projects can contribute to housing price stability becomes a key indicator, though literature on this issue presents varied findings.

### (3) Economic Significance of Housing Price volatility

The economic significance of price volatility is often interpreted using time series models, also known as Random Walk (RW) models. The explanations for these models are as follows:

1. RW Model without Intercept: The economic implication of this model is that if the supply and demand conditions in any market remain unchanged, market participants can use the price from the previous period as a direct reference for future price expectations. In such cases, the time series data of prices (and even trading volumes) may follow an RW model without an intercept.
2. RW Model with Intercept: This model implies that when market supply increases due to continuous technological advancements or demand steadily increases due to factors like population growth or rising incomes, the time series data of prices (or trading volumes) may follow an RW model with an intercept.
3. RW Model with Random Disturbance: When factors such as input, technological progress, and innovation are random but have a persistent and cumulative impact, the resulting time series data of income or prices may follow an RW model with random disturbances (Yang, 2017).

Based on the aforementioned literature, it is understood that housing price fluctuations exhibit clustering phenomena. Some research indicates that the volume of pre-sale projects can stabilize housing price fluctuations, while other studies suggest that it may exacerbate these

fluctuations. The presence or absence of an intercept term in price fluctuation models carries different economic implications.

### 3. Datum Source

#### (1) transaction datum

There are 489,448 pieces of transaction data registered based on the transaction prices of residential homes in Taichung City from August 2012 to June 2023.

#### (2) Screening and classification

Through the data screening steps in Table 2, excluding missing and outlier data, a total of 242,681 valid data points were obtained.

Table 2 Data Screening Process

Step	Number of Removed Data	Valid Data (pieces)
(1) Population data	-	489,448
(2) Integration of house and land	142,999	346,449
(3) Housing product	100,453	245,996
(4) Removed missing value	3,015	242,981
(5) Removed deviation value (take 3 standard deviations)	24	242,957
Total	246,491	242,957

These data are classified according to housing types, namely, high-rise apartments, high-rise buildings, townhouses, and apartments. Difference between high-rise apartments and high-rise buildings is only building height, while all other conditions are the same. Hence, the data of high-rise apartments and high-rise buildings are combined. As shown in Table 3, there are 242,681 pieces of housing transactions, of which 153,452 pieces are high-rise apartments and high-rise buildings, (39,106 pieces of high-rise buildings and 114,346 pieces of high-rise apartments), 71,881 pieces are townhouses and 17,348 pieces are apartments (no elevator, no more than 5 stories).

Table 3 Classification of Residence

Property Category	Number of Transactions
(1) High-rise apartments and high-rise buildings	153,452
(2) High-rise apartments (with elevator, no more than 10 stories)	39,106
(3) High-rise buildings (with elevator, more than 11 stories)	114,346
(4) Townhouses (single street number for the entire building)	72,157
(5) Apartments (no elevator, no more than 5 stories)	17,348
Total	242,957

#### (1) Source of pre-sale homes:

The number of construction licenses approved and issued for residential buildings over the years is sourced from the construction statistics published by the National Land Management Agency from August 2012 to June 2023. There are 244,116 households from August 2012 to June 2023. The lowest number of licenses approved and issued is 413 in February 2016 and the highest number

is 6,792 in January 2021. The average monthly number is 1,864. Statistics of all these months are organized into Table 4.

To address the issue of different calculation units and the disparity in volume of this variable, we use the absolute value of the monthly rate of change:  $|(Current\ month\ quantity - Previous\ month\ quantity) / Previous\ month\ quantity|$ . This represents the absolute monthly increment of pre-sale houses.

Table4MonthlyStatisticsTableofConstructionLicensesforResidentialBuildings(Unit:Household)

Period	2012M8	2012M9	2012M10	2012M11	2012M12	2013M1	2013M2	2013M3	2013M4	2013M5	2013M6	2013M7	2013M8	2013M9	2013M10	2013M11	2013M12
Number	1244	2199	1018	1248	1837	2076	1012	1391	1700	1965	1184	1486	1550	2498	2241	1082	1493
Period	2014M1	2014M2	2014M3	2014M4	2014M5	2014M6	2014M7	2014M8	2014M9	2014M10	2014M11	2014M12	2015M1	2015M2	2015M3	2015M4	2015M5
Number	876	1099	960	1471	954	1137	1561	1190	604	1113	822	940	879	1099	967	1471	954
Period	2015M6	2015M7	2015M8	2015M9	2015M10	2015M11	2015M12	2016M1	2016M2	2016M3	2016M4	2016M5	2016M6	2016M7	2016M8	2016M9	2016M10
Number	1137	1561	1190	604	1113	822	940	1131	413	1228	616	864	611	1280	1862	799	860
Period	2016M11	2016M12	2017M1	2017M2	2017M3	2017M4	2017M5	2017M6	2017M7	2017M8	2017M9	2017M10	2017M11	2017M12	2018M1	2018M2	2018M3
Number	770	2224	1258	963	684	1076	1997	1123	1419	1711	1501	920	1333	1400	1628	797	2097
Period	2018M4	2018M5	2018M6	2018M7	2018M8	2018M9	2018M10	2018M11	2018M12	2019M1	2019M2	2019M3	2019M4	2019M5	2019M6	2019M7	2019M8
Number	1573	1590	1897	938	2098	1241	2459	2109	2851	4376	792	3642	2068	1144	913	2805	2187
Period	2019M9	2019M10	2019M11	2019M12	2020M1	2020M2	2020M3	2020M4	2020M5	2020M6	2020M7	2020M8	2020M9	2020M10	2020M11	2020M12	2021M1
Number	2433	4080	1472	3130	2410	3498	2531	1456	1831	4002	3291	2633	3519	2505	2710	3655	6792
Period	2021M2	2021M3	2021M4	2021M5	2021M6	2021M7	2021M8	2021M9	2021M10	2021M11	2021M12	2022M1	2022M2	2022M3	2022M4	2022M5	2022M6
Number	1527	2550	1996	1849	2765	3364	1067	1096	2050	3362	4066	1433	1603	2729	3403	2522	4312
Period	2022M7	2022M8	2022M9	2022M10	2022M11	2022M12	2023M1	2023M2	2023M3	2023M4	2023M5	2023M6					
Number	2173	2792	3226	2772	4308	2374	1549	3241	1083	1438	1185	4397					

## 4. Result and Discussion of Empirical Analysis

Dufitinema (2020), Miles (2020), and Wang and Hartzel (2020) have identified the GARCH effect of price volatility clustering in housing markets. Wang, Lin, and Tsai (2022) use the spider web model and forward contract pricing model to explain the frequent occurrence of overheating in the pre-sale housing market in the short term. Yang and Chuang (2024) indicate that the supply of pre-sale homes contributes to the rise and divergence of residential and condominium prices. Research by Wang .et al. (2007), Chii and Wang. (2010), and Wong. et al. (2006) suggests that the pre-sale market has a stabilizing effect on the volatility of existing home prices. Wang.et.al (2022) also note that the pre-sale housing market often experiences overheating in the short term. Based on the above literature, this study establishes the following hypotheses:1. To verify whether there is a clustering phenomenon in the price volatility of apartments in Taichung City.2. To examine whether the impact of pre-sale project volumes on the price of apartments in Taichung City can achieve convergence.

The ARCH model and GARCH models are employed in this research. The absolute value increment of pre-sale housing is further expanded as an exogenous variable to discuss the influence of ARCH model and GARCH model on the absolute value increment of pre-sale housing. The optimal model for impact time in this research is as follows:

$$y_t | \Omega_t \sim N(x, a, \sigma^2) \dots \dots \dots \text{Eq. (4-1)}$$

$$\varepsilon_t = y_t - X_t a \dots \dots \dots \text{Eq. (4-2)}$$

$$\sigma^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 \dots \dots \text{Eq. (4-3)}$$

Here  $X_t$  is the vector of regression independent variable;  $a$  is the coefficient vector of regression equation; and  $q$  is the order of lagging terms.

$X_{ta}$  denotes the linear combination of variables acquired by  $\Omega_t$ . ( $a_0 + a_1 X_{1t} + a_2 X_{2t} + \dots + a_k X_{kt}$ ). Eq. (4-2) is the mean equation and Eq. (4-3) is the variance equation. In mean equation,  $X_t$  is the single time series ARMA, including  $y_t$  which is the deferred lagging term, and  $n$  is the deferred lagging term of MA. Heteroscedasticity is ARCH( $q$ ), which can express the mean equation and variance equation ARCH as ARMA( $m, n$ )-ARCH( $q$ ).

$$y_t = a_0 + \sum_n^m a_i y_{t-i} + \varepsilon_t + \sum_n^m b_i \varepsilon_{t-1} \dots \dots \dots \text{Eq. (4-4)}$$

$$\sigma^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 \dots \dots \dots \text{Eq. (4-5)}$$

GARCH model is the generalized ARCH model and typical GARCH( $p, q$ ) can be expressed as follows:

$$y_t|\Omega_t \sim N(X_t, a, \sigma^2) \dots \dots \dots \text{Eq (4-6)}$$

$$\varepsilon_t = y_t - X_t a \dots \dots \dots \text{Eq(4-7)}$$

$$\sigma^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 \dots \dots \dots \text{Eq.(4-8)}$$

Where, q is the order of GARCH, if q=0, the model is ARCH(q). ARMA(2,0) is AR(2) and the model formula of AR(2)-GARCH(1,1) is as follows

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t \dots \varepsilon_t \sim N(0, \sigma^2) \dots \dots \dots \text{Eq.(4-9)}$$

$$\sigma^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \dots \dots \dots \text{Eq (4-10)}$$

Where  $\sigma^2$  is the conditional variance. It is an unobservable variable, but it can be observed via the GARCH model and estimated. This variable is often regarded as substitute variable of risk or uncertainty, so  $\sigma^2$  is a quantitative index used to measure shifting risk or is called volatility.  $\varepsilon_{t-1}^2$  is regarded as the square number of the expected error of last period of mean equation.  $\alpha_1$  is the short-term accidental volatility; a higher value indicates greater influence of short-term unexpected factors, and vice versa. Long-term continuous volatility is the accumulation of short-term volatility, i.e., coefficient  $\beta_i$  plus coefficient  $\alpha_i$ ,  $a_0 + a_1 \beta_i$ . Therefore, if  $|\beta_i| < 1$ , the influence decreases geometrically, which is a weak stationary hypothesis. It can be obtained that long-term variance.

$$\sigma^2 = \frac{\alpha_0}{1 - (\alpha_1 + \beta_1 + \beta_2)} \text{ (Yang, 2017) } \circ$$

The mean equation of this paper is  $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \dots + \varepsilon_t$

GARCH-IN- Variance Regressors model

$$\sigma^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 + \theta [\Delta x_t] \dots \dots \dots \text{Eq (4-11)}$$

**(1) Building of empirical model**

In this study, the data consists of the monthly average price of apartments and the number of pre-sale housing projects. The steps for testing are as follows: Observe Figure 1, which depicts the trend of apartment prices. There might be an intercept term. Perform a unit root test to confirm if it is significant.



Figure1 Apartment Transaction Price Trend

Fig 1: Stationarity or non-stationarity

The Augmented Dickey-Fuller (ADF) test for unit root has the null hypothesis  $H_0$ : "accept unit root" and the alternative hypothesis  $H_1$ : "reject unit root." Acceptance or rejection of these hypotheses determines whether the monthly average price of apartments is stationary. Observing Figure 1, the monthly average price of apartments in this study appears to follow an equation with an intercept term. The unit root test does not reach the significance level, thus we accept the null hypothesis  $H_0$ : "Presence of a unit root." This indicates that the data variable is non-stationary and can be transformed into a stationary variable through differencing. The relevant test results are as follows:

- (1) ADF value of monthly apartment transaction-statistic -0.398808 is greater than critical value (-2.884477).  $H_0$  null hypothesis that there is unit root is accepted, as shown in Table 5.

Table 5. Apartment Stationarity Test

Augmented Dickey-Fuller test statistic		Test critical values:		
t-Statistic	-0.398808	1% level	5% level	10% level
Prob	0.9048	-3.482879	-2.884477	-2.57908

Note:  $\mathcal{P}^* \leq 0.05$  ;  $\mathcal{P}^{**} \leq 0.01$  ;  $\mathcal{P}^{***} \leq 0.000$

- (2) Non-stationary data is transferred into stationary data

The above-mentioned non-stationary variable data, after differential verification, the test results are as shown in Table 6. The ADF value -11.84077 is less than the critical value -2.884477.  $H_1$  "rejection of a single root" is accepted, reaching a significant level of more than 1%, 5%, and 10%. It is a stationary variable, and its trend is as shown in Figure 2 below.

Table 6 Apartment Difference Stationarity Test

Augmented Dickey-Fuller test statistic		Test critical values:		
t-Statistic	-11.84077	1% level	5% level	10% level
Prob.*	0.0000***	-3.482879	-2.884477	-2.579080

Note:  $\mathcal{P}^* \leq 0.05$  ;  $\mathcal{P}^{**} \leq 0.01$  ;  $\mathcal{P}^{***} \leq 0.000$

## 2. Residual test and model-based estimation

- (1) Residual of estimator for apartment ADF test is as shown in Table 7. The number of lagging periods needed for variables to reach white noise is determined. The statistics of the third lagging period (t-statistic) is 4.078789 in Table 7. Its significance level  $\mathcal{P}$ -value 0.000\*\*\* has reached significance level and residual is white noise.

The variables of the fourth lagging period fail to reach a significance level and are not listed in the table. Statistics of intercept term (-1.00138) fails to reach the significance level and are removed

from the equation.

Table 7 Residual of Estimator for Apartment ADF Test

variable	coefficient	t-statistic	p-VALUE
D(AP(-1))	-3.413119	-11.84077	0.000***
D(AP(-1),2)	1.553688	6.513138	0.000***
D(AP(-2),2)	0.730720	4.592502	0.000***
D(AP(-3),2)	0.316846	4.078789	0.000***
C	-0.175503	-1.001380	0.319
R <sup>2</sup>	0.844202	AIC	4.216179
Log likelihood	-260.6193	SBC	4.32873

Note:  $\mathcal{P}^* \leq 0.05$  ;  $\mathcal{P}^{**} \leq 0.01$  ;  $\mathcal{P}^{***} \leq 0.000$

(2) Estimate the number of lagging periods

This study then confirms if all lagging periods reach significance level with OLS method based on ADF test results in Table 8. The third lagging period and intercept term are removed for not reaching significance level. Estimator for the second lagging period (Lag Length=2) is

$$d(AP_t) = \gamma AP_{t-1} + \beta_1 d(AP_{t-1}) + \beta_2 d(AP_{t-2}) + \varepsilon_t \dots \dots \dots \text{Eq (4-11)}$$

Put coefficients of all variables into Table 8 in Eq. (4-11) to obtained. (4-12)

$$d(AP_t) = -0.014632 AP_{t-1} - 0.685238 \Delta(AP_{t-1}) - 0.458108 \Delta(AP_{t-2}) + \varepsilon_t \dots \text{Eq(4-12)}$$

Table 8 Estimation with Ordinary Least Squares (OLS)

Variable	Coefficient	t-statistic	p-VALUE
AP(-1)	-0.014632	-1.193933	0.235
D(AP(-1))	-0.685238	-8.751284	0.000***
D(AP(-2))	-0.458108	-5.993017	0.000***
R <sup>2</sup>	0.401243	AIC	4.438951
Log likelihood	-281.0929	SBC	4.505795

Note:  $\mathcal{P}^* \leq 0.05$  ;  $\mathcal{P}^{**} \leq 0.01$  ;  $\mathcal{P}^{***} \leq 0.000$

(3) Model estimation

From Table 9, we observe the estimation results for the following models: ARIMA (2,1,0)-GARCH (1,0), ARIMA(2,1,0)-GARCH(0,1), and ARIMA(2,1,0)-GARCH(1,1). Table 9 shows the residual equations for the three models. In the ARIMA (2,1,0)-GARCH (1,1) model, the ARCH(-1) statistic is -0.891478, which does not reach the significance level. In contrast, the GARCH (-1) statistics for the other models are 109.5751\*\*\*, 3.340904\*\*, and 40.78289\*\*\*, all of which are significant level.

Table 9 Estimators of All Models

Equation	ARIMA(2,1,0)-GARCH(1,0)			ARIMA(2,1,0)-GARCH(0,1)			ARIMA(2,1,0)-GARCH(1,1)		
Variable	Coefficient	z-Statistic	Prob.	Coefficient	z-statistic	Prob.	Coefficient	z-statistic	Prob.
AP(-1)	-0.014681	-1.423194	0.155	0.003324	0.320777	0.748	-0.013284	-1.252870	0.2103
D(AP(-1))	-0.669262	-6.781668	0.000***	-0.388361	-5.461878	0.000***	-0.682337	-7.362895	0.000***
D(AP(-2))	-0.444162	-5.209973	0.000***	-0.322571	-3.803193	0.000***	-0.448484	-5.345641	0.000***

C	0.095727	2.772980	0.006**	2.075634	6.944265	0.000***	0.101946	3.855146	0.000***
RESID(-1)^2							-0.021470	-0.891478	0.373
GARCH(-1)	0.940337	109.5751	0.000***	0.669992	3.340904	0.001**	0.957913	40.78289	0.000***
R-squared		0.407794			0.402248			0.312995	
Log likelihood		-254.5719			-253.0576			-268.8035	
Akaikeinfocriterion		4.071435			4.063400			4.293804	
Schwarzcriterion		4.205124			4.219370			4.427493	

Note:  $\mathcal{P}^* \leq 0.05$  ;  $\mathcal{P}^{**} \leq 0.01$  ;  $\mathcal{P}^{***} \leq 0.000$

**(2)ResidualTestof Models**

1.Residuals of all models are presented in Table 10. There is no autocorrelation residual sequence. Correlogram squared Residual is tested. All tested models accept null hypothesis  $H_0$ that there is no existence of “self-relevant residual sequence”, which fails to reach the significance levele.2.As shown inTable 10, there is no obvious ARCH phenomenon in all residuals tested with ARCH–LM.As a result, these three models accept null hypothesis  $H_0$ that there is no “ARCH phenomenon” or ARCH effect.3. Normality test is shown in Table 10. Assessed value of ARIMA (2,1,0)-ARCH (0,1) is 0.000011\*\*\*. Null hypothesis  $H_0$ that there is “normal distribution” is rejected. Both tested values of the other two models are 0.211471 and 0.146178, respectively. Null hypothesis  $H_0$ that there is “normal distribution” is accepted. Only assessed value of ARIMA(2,1,0)- ARCH(0,1) reached significance level and reject the existence of normal distribution.

Table 10 Residual Test of All Models

Model	Test	Correlogram squared Residual Test	Normality Test Jarque Bera Probability	ARCH-LMF-statistic/Probability/F-statistic/Obs*R-squared
ARIMA(2,1,0)-ARCH(1,0)		0.052-0.402	0.211471	0.114/0.2083
ARIMA(2,1,0)-ARCH(0,1)		0.296-0.982	0.000011***	0.3688/0.3648
ARIMA(2,1,0)-ARCH(1,1)		0.057-0.435	0.146178	0.3120/0.3082

Note:  $\mathcal{P}^* \leq 0.05$  ;  $\mathcal{P}^{**} \leq 0.01$  ;  $\mathcal{P}^{***} \leq 0.000$

Correlogram squared Residual Test is applicable for test of small samples, so this test is adopted to determine whether the three models pass residual test when exogenous variables not affected by independent variables are added. Exogenous variables are increment of pre-sale homes and regarded as supply quantity or variance regressors of residences. The last test is then repeated.

**(3) GARCH in Variance Regressors and Residual Test**

The increment of pre-sale housing is an exogenous variable and is included in residual equation for observation. In ARIMA (2,1,0)-GARCH (1,0) column of Table 11, z-statistic of pre-sale housing increment (PREDQ) is 2.49479, coefficient is 0.000170 and significance level is above critical value (0.025\*\*), which is of significance. As for the other two models, for ARIMA (2,1,0)- GARCH(1,1), z-statistic is 1.625313, coefficient is 0.000120 and P-VALE=0.1041, which fails to reach significance level; for ARIMA(2,1,0)-GARCH(0,1), z-statistic is - 1.081251, P- VALE=0.280, which is below the significance level of critical value.

Therefore, only ARIMA (2,1,0)-GARCH (1,0) reaches the significance level.

Table11GARCHinVariance Regressors

equation variable	ARIMA(2,1,0)-GARCH(1,0)			ARIMA(2,1,0)-GARCH(0,1)			ARIMA(2,1,0)-GARCH(1,1)		
	Coefficient	z-Statistic	Prob.	Coefficient	z-statistic	Prob.	Coefficient	z-statistic	Prob.
AP(-1)	-0.012076	-1.254408	0.2097	0.001753	0.155753	0.876	-0.008885	-0.966633	0.3337
D(AP(-1))	-0.651681	-6.571648	0.000***	-0.397623	-5.663564	0.000***	-0.687251	-9.222193	0.000***
D(AP(-2))	-0.430212	-4.955348	0.000***	-0.331632	-3.903802	0.001**	-0.427384	-5.953039	0.000***
C	-0.119295	-1.403471	0.161	2.416863	5.869995	0.000***	-0.008745	-0.098420	0.9216
RESID(-1)^2				0.608438	3.222528	0.001**	-0.101085	-15.08624	0.000***
GARCH(-1)	0.958039	116.0923	0.000***				1.032349	4675.166	0.000***
PREDQ	0.000170	2.249479	0.025*	-0.000260	-1.081251	0.280	0.000120	1.625313	0.1041
R-squared	0.399952			0.325245			0.399225		
Log likelihood	-254.3775			-268.5708			-247.0239		
Akaikeinfocriterion	4.068399			4.290169			3.969124		
Schwarzcriterion	4.202088			4.423858			4.125094		

Note:  $\mathcal{P}^* \leq 0.05$  ;  $\mathcal{P}^{**} \leq 0.01$  ;  $\mathcal{P}^{***} \leq 0.000$

Table12 ResidualTestwithGARCHinVarianceRegressorsModel

Test	Model	ARIMA(2,1,0)- ARCH(1,0)	ARIMA(2,1,0)- ARCH(0,1)	ARIMA(2,1,0)- ARCH(1,1)
CorrelogramsquaredResidualTest		0.091-0.499	0.327-0.951	0.579-0.956
NormalityTestJarqueBeraProbability		0.319031	0.000***	0.228376
ARCH-LMF-statisticProbability		0.2037/0.2007	0.3381/0.3342	0.5872/0.5837
F-statistic/Obs*R-square				

Note:  $\mathcal{P}^* \leq 0.05$  ;  $\mathcal{P}^{**} \leq 0.01$  ;  $\mathcal{P}^{***} \leq 0.000$

The resultsoftheresidual testsforthethreemodelsareasshown inTable12:

1. Statistical test of Correlogram squared Residual Test: The observed value of the 36th lagging period is assessed to determine whether there exists self- relevant residual sequence. All these three models have residual terms and accept  $H_0$  that there is” no existence of self-relevant residual sequence”. 2. Normality Test Jarque Bera Probability: Two models accept null hypothesis  $H_0$  that there is “normal distribution” for observation values. Only ARIMA (2.1,0)-ARCH (0,1) rejects the null hypothesis  $H_0$  that there is “normal distribution”. 3. ARCH-LM Test: The test determines if there is heteroscedasticity of autocorrelation residual sequence. Null hypothesis  $H_0$  is without “ARCH effect”. With significance level reaching critical value, these three models accept null hypothesis  $H_0$  “without ARCH effect”.

#### (4) Model Selection Criteria

BasedonTable13,thebettermodelwasselected.TheARIMA(2,1,0)-GARCH (1,0)modelhas the highest R-squared value at 0.399952, indicating the greatest explanatory power. The second highestistheARIMA (2,1,0)-GARCH (1,1)modelwithanR-squaredof0.399225,withnegligible difference between the two. For the AIC (Akaike Information Criterion) and SC (Schwarz Criterion),thesmallervaluesarepreferred.TheARIMA (2,1,0)-GARCH (1,0)modelhasanAIC of4.068399andSCof4.202088,whicharelargerthantheAICof3.969124andSCof4.125094 for the ARIMA(2,1,0)-GARCH(1,1) model. Therefore, the ARIMA(2,1,0)-GARCH(1,1) model is

chosen as the most appropriate model.

Table 13 Model Selection Criteria

Model Goodness of fit Test	ARIMA(2,1,0)-ARCH(1,0)	ARIMA(2,1,0)-ARCH(0,1)	ARIMA(2,1,0)-ARCH(1,1)
R-squared	0.399952	0.325245	0.399225
Log likelihood	-254.3775	-268.5708	-247.0239
Akaike info criterion	4.068399	4.290169	3.969124
Schwarz criterion	4.202088	4.423858	4.125094

**(5) Discussion**

This paper estimates ARIMA(2,1,0)-GARCH(1,1) as the optimal model to explain the volatility of apartment price, and its relevant coefficients are shown in Table 14.

Table 14 Optimal Model of ARCH/GARCH

equation	variable	AP(-1)	D(AP(-1))	D(AP(-2))	C	RESID(-1)^2	GARCH(-1)	PREDQ
ARIMA(2,1,0)-GARCH(1,1)	Coefficient	-0.008885	-0.687251	-0.427384	-0.008745	-0.101085	1.032349	0.000120
	z-statistic	-0.966633	-9.222193	-5.953039	-0.09842	-15.08624	4675.166	1.625313
	Prob.	0.3337	0.000***	0.000***	0.92160	0.000***	0.000***	0.104100

Note:  $\mathcal{P}^* \leq 0.05$  ;  $\mathcal{P}^{**} \leq 0.01$  ;  $\mathcal{P}^{***} \leq 0.000$

For example, the residual equation of Eq. (4-13) can also be included into impact equations of exogenous variable, such as Eq. (4-14). Its influence on volatility is observed and its degree of influence is discussed. For example, the rate of relative change for supply quantity of pre-sale housing is included in this paper, take absolute value as  $|\Delta\chi_t|$ . If coefficient of this exogenous variable  $\theta > 0$ , it means that the influence of current,  $\sigma_t^2$  on volatility increases. On the contrary, if  $\theta < 0$ , then influence degree of current,  $\sigma_t^2$  on current volatility decreases.

Residual equation of GARCH (1,0) is  $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \theta |\Delta\chi_t|$ .....Eq (4-13)

Put coefficients in Table 14 into Eq. (4-13)

$$\sigma_t^2 = -0.008745 - 0.101085 \varepsilon_{t-1}^2 + 1.032349 \sigma_{t-1}^2 + 0.00012 |\Delta\chi_t|$$
.....Eq (4-14)

The clustering of apartment price volatility in Taichung City is comparable to the volatility clustering observed in real estate markets in Hong Kong, 15 regions in Finland, and half of the real estate markets in 50 U.S. states. Apartments, with their low-density use, are considered non-mainstream properties in metropolitan areas. The empirical results of this study indicate that residential price volatility in Taichung City exhibits clustering phenomena, consistent with existing literature. Kaulihowa and Kamari (2019) provide empirical evidence that macroeconomic factors support the persistence of volatility. Wang et al. (2022) established housing price models using indices from four cities: Taipei, New Taipei, Taichung, and Kaohsiung, revealing that the pre-sale housing market frequently experiences overheating in the short term. This study further finds that incr

easing the supply of pre-sale housing during periods of strong demand does not effectively stabilize price volatility.

The addition of pre-sale project volumes does not help to converge the previously declining apartment prices. Under the opportunity for redevelopment of old apartments, well-located apartments become preferred targets for developers, leading to a growing number of apartments deemed unsuitable for redevelopment. Consequently, the downward price trend continues and does not achieve convergence.

## 5. Conclusion

In this study, residential buildings are categorized into high-rise apartments and condominiums. Using the transaction prices of condominiums in Taichung City as the empirical observation object, the most suitable model after various tests is the ARIMA(2,1,0)-GARCH(1,1) price volatility model. When excluding the volume of pre-sale housing projects, the GARCH(-1) volatility parameter  $|\beta_1| = 0.957913 < 1$  indicates that price volatility can converge. However, when including the volume of pre-sale housing projects, the GARCH(-1) volatility parameter  $|\beta_1| = 1.032349 > 1$  suggests that volatility is divergent. The impact coefficient  $\theta = 0.000120 > 0$ , with a p-value not reaching the 10% significance level, indicates that the volume of pre-sale housing projects does not effectively reduce the level of volatility.

The contributions of this paper are as follows: 1. Provision of an Alternative Risk Measurement Approach: It offers a method for predicting price trend convergence based on the persistence of transaction price volatility, allowing for quantitative data to make mutually exclusive risk choices. 2. Verification of Price Trends During Economic Prosperity: The results show that not all housing prices necessarily rise during periods of economic prosperity; in Taichung City, the trend in condominium transaction prices indicates a downward trend. 3. Expansion of Perspectives on Old Housing Issues: It broadens the discussion on old housing problems, suggesting future research topics such as: Investigating whether the volatility of high-rise apartments and condominiums differs. Exploring the relationship between new and old houses, and whether there is a cointegration relationship based on building age.

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## 6. References

- Chen, S.S. (2020). *Applied Time-Series Econometrics for Macroeconomics and Finance*. Dong Hua Books, 2<sup>nd</sup> ed. 4<sup>th</sup> prim., Taipei City; P319-328.
- Chi, C. X., Wang, S. (2010). *Research on Influence of Pre-sale House Price on Price of Spot House Taking Real Estate of Yunnan Province as an Example*. *China Economic Review* (10): 94-95.
- Chiang, Y. H., Chu, C. Y., Chang, C. O. (2019). *A Mass Appraisal Modeling of the Pre-Sale Housing*. *Journal of Housing Studies*, 28(2), 17 -36.
- Ching-Chun Hua, Chin-Oh Chang, Chengho Hsieh. (2001). *The Price-Volume Relationships between the Existing and the Pre-Sales Housing Markets in Taiwan*. *International Real Estate Review* Vol. 4 No. 1: p. 80 – 94.
- Ho-Wen Yang, Chia-Yu Chuang. (2024). *Will the Release of Presale Housing moderate or exacerbate residential price volatility? - Taking Apartment complexes Units in Taichung City as an Example*. *Technium Social Sciences Journal*. VOL.59. P218.
- Josephine Dufitinema. (2020). *Volatility clustering, risk-return relationship and asymmetric adjustment in the Finnish housing market*. *The current issue and full text archive of this journal*

- is available on Emerald Insight at: <https://www.emerald.com/insight/1753-8270.htm>.
- Josephine Dufitinema.(2021)."*Forecasting the Finnish house price returns and volatility: a comparison of timeseries models*". International Journal of Housing Markets and Analysis Vol. 15 No. 1, 2022 pp. 165-187
- Klaus Adam, Oliver Pfauti, TimoReinelt.(2021)."*Falling Natural Rates, Rising Housing Volatility, and the Optimal*".Inlow risen Target June 21.
- Lee,J.H.,Yang,C.H.(2010)."*TheRelationshipofPriceDifferencewithPre-saleandExisting House-take Taipei Cityand Taipei Countyas an Example*". Journal of Architecture and Planning, 11(1), 1-14.
- Mei Qiang, Shang-Yi Lin. (2017)."*The Impact of Macroeconomics Variables on the House Prices of the Six Cities in Taiwan*". Asia-Pacific Economic and Management Review Vol. 21, No. 1, September 2017.
- Shizhen Wang, David Hartzell. (2020). "*What influences real estate volatility in Hong Kong? An ARMA-GARCH approach*". The current issue and full text archive of this journal is available on Emerald Insight at: <https://www.emerald.com/insight/1753-8270.htm>
- Somerville, C.T. (1999)."*Residential Construction Costs and the Supply of New Housing: Endogeneity and Bias in Construction Cost Indexes*". Journal of Real Estate Finance and Economics, 18(1), pp.43-62.
- TeresiaKaulihowa,KatrinaKamari(2019)."*DeterminantsofhousepricevolatilityinNamibia*". InternationalJournalofHousingMarketsandAnalysis.ISSN:1753-8270.
- Ting Fan, Asadullah Khaskheli, Syed Ali Raza and Nida Shah (2022)."*The role of economic policyuncertaintyinforecastinghousepricesvolatilityindevelopedeconomies: evidence fromaGARCH-MIDASapproach*". InternationalJournalofHousingMarketsand AnalysisIssue(s)available:74–FromVolume:1 Issue:1,toVolume:16Issue:7.
- Wang, C. T., Liu, H. Y., Chen, W., Lung, F. C. (2007)."*Stabilizing effect of the pre-sale market on spot house prices*". Journal of Tsinghua University (Science and Technology) 47(6): 781-784.
- Wen-KaiWang,Che-ChunLin,I-ChunTsai(2022)."*Long-andshort-termpricebehaviorsin pre-sale housing*".Economic Analysis and Policy 74 350–364journal homepage: [www.elsevier.com/locate/eap](http://www.elsevier.com/locate/eap).
- Willian Miles (2020)."*Volatility Clustering in U.S. Home Prices, Journal of Real Estate*

- Research*". Pages 73-90 | Published online: 17 Jun 2020.
- Wong, S. K., C. Y. Yiu & K. S. Tse. (2006). "Do the Forward Sales of Real Estate Stabilize Spot Prices?" .The Journal of Real Estate Financial and Economics. 32(3): 289-304.
- Wu, J. L., Huang, H. C., Wang, C. N., Wu, R. W. (2012). "Revisiting to Taiwan's Foreign Exchange Rate Policies". Taiwan Economic Review, 40(2), P273-284.
- Yang, Y. N. (2017). "Time Series Analysis in Economics and Finance". 3rd ed., Yeh Yeh Book Gallery, P164-171; P341-345. Time Series: Economic and Financial Applications