Impact of Pre-sale Home Increment on the Volatility Error of Existing Housing Prices -Taking apartments of Taichung City as an Example

Abstract

During periods of economic prosperity, real estate developers launch numerous pre-sale housing projects to expand business opportunities, fill market supply gaps, and reduce the demand for completed homes, thereby stabilizing housing prices. However, most people do not perceive housing prices as stabilizing. This raises research interest in whether the volume of pre-sale housing projects can mitigate the demand for completed homes and achieve price stability. This study aims to explore the impact of pre-sale housing project volumes on the volatility error of transaction prices of completed homes. Currently, the mainstream product in the market for completed homes is collective housing, typically classified based on building height and the presence of elevator facilities into apartments, mid-rise buildings, and high-rise buildings. Due to changes in the construction environment, the number of new apartment buildings has sharply decreased, making older apartments the primary objects of transactions .The study found that the ARIMA(2,1,0)-GARCH(1,1) model is the best model and exhibits clustering volatility. The GARCH(-1) volatility coefficient $|\beta_1|$=0.957913<1 indicating that volatility decreases geometrically over successive periods, achieving stability. When the volume of pre-sale housing projects is included in the model, the GARCH(-1) volatility coefficient $|\beta_1|$=1.032349<1, indicating that volatility increases geometrically over successive periods, failing to achieve stability. Furthermore, the interference coefficient θ=0.0012>0 for pre-sale housing projects does not significantly reduce volatility. Before the inclusion of pre-sale housing projects in the volatility model, the trend was convergence; however, after inclusion, it became divergence. This implies that purchasers preferentially buy old properties for reconstruction, repeatedly opting for advantageous transactions. Consequently, the number of low-quality properties not meeting reconstruction standards increases, leading to significant gradual declines in transaction prices and divergent price volatility.

Keywords: Volatility clustering, Conditional heteroskedasticity, ARCH/GARCH model, Taichung

1. Introduction

The pre-sale housing market is a unique transaction system prevalent in Taiwan, Hong Kong, and China, and is one of the common ways for people to purchase homes. Consequently, research on pre-sale housing focuses on these regions. For example, in Taiwan, Lee Ji-Hong and Yang Tsung-Hsien (2010) observed that the pre-sale houses possess futures characteristics, and examining the base period gaps and changes helps understand their substitutability. Jiang Ying-Hui, Zhu Zhi-Yang, and Zhang Jin-E (2019) observed long-term forecasts of pre-sale and existing home market prices, aiding in price information flow and market price stabilization. Wang et al. (2022) used the cobweb model and forward contract pricing model to illustrate that the demand for spot housing in the real estate market reacts immediately, and due to the lag in the supply of existing homes, demand may shift to the pre-sale housing market, alleviating the demand for existing homes but causing overheating in the pre-sale market. The intense demand for existing homes can impact pre-sale prices, preventing convergence. Whether the price adjustment function of pre-sale supply backfires is debatable. Studies have shown that over the past few decades, the decline in natural interest rates in developed economies has been accompanied by significant increases in housing price volatility, verifying that recent Taiwanese government policies on interest rates and land holding costs have not been suppressive.

Research in China, such as by Wang Song-Tao et al. (2007) on Shanghai housing prices and Qi Zheng-Xia and Wang Xu (2010) on Yunnan Province housing prices, concluded that the pre-sale housing market stabilizes the existing home market. In Hong Kong, Wong et al. (2006) researched housing price volatility and pointed out that the pre-sale housing market has a stabilizing effect on the prices of existing homes. Most studies agree that the volume of pre-sale housing projects and existing home prices have mutual regulatory functions. Therefore, developers often choose to launch a large number of pre-sale houses during economic booms to effectively seize business opportunities, fill the supply gap in the existing home market, and alleviate the demand for existing homes, expecting this adjustment to stabilize prices. However, most people often do not perceive price stabilization, raising the research question: Can the volume of pre-sale housing projects stabilize the prices of existing homes?

This study aims to observe the volume of pre-sale housing projects and the resulting volatility—whether it persists, diverges, or converges to explore the stability of housing price fluctuations and satisfy the research motivation. Pre-sale houses have the characteristic of supply lag and usually achieve equilibrium prices through periodic price adjustment processes. This process involves the volume of pre-sale housing projects in the current period being determined by the reference price of the previous period's existing homes, while the current period's pre-sale prices are set by the current market prices. The adjustment process forms a cobweb-like convergence trend until price equilibrium is reached.

Furthermore, housing price volatility, due to its asset preservation attribute, often exhibits the following major characteristics: leptokurtic density distribution, conditional variance tending to change over time, strong volatility persistence (large fluctuations followed by large fluctuations, small fluctuations followed by small fluctuations), and empirical distribution of asset price series showing a heavy tail phenomenon (more residual values). This phenomenon is known as volatility clustering (Chen Xu-Sheng, 2020). The conditional variance equation’s conditional variance σ_t^2 is used to replace risk and uncertainty, and in economic terms, σ_t^2 is seen as a quantitative indicator of fluctuation risk, or volatility and volatility clustering. Research in this area often targets average or median transaction prices. σ_t^2 is an unobservable variable as a fourth-moment condition variance but can be estimated using the ARCH/GARCH model. Short-term impacts come from the previous period's residualε_{t-1} as the forecast error of the price average equation. Therefore, the size of unexpected impacts can be estimated by its coefficient representing the degree of impact volatility. The lag term σ_{t-1}^2 represents the previous period's conditional variance. The more lag terms there are, the longer the impact time. If the coefficient $|\beta_1|<1$, it will geometrically decrease; the smaller the value, the faster the decrease, and the smaller the impact. Conversely, $|\beta_1|>1$will have the opposite effect.

The mainstream product in the existing home market is collective housing, typically categorized by building height or the presence of an elevator into apartments and high-rise buildings. Due to changes in the construction environment, the construction of new apartment-type housing has significantly decreased, focusing on older properties as the main transaction objects. The market inevitably opts for the best purchases, leading to a downward trend in transaction prices due to successive optimal selections.

From August 2012 to June 2023, there were 242,681 residential real transaction records in Taichung City, with 17,348 apartment transactions after sorting. The transaction volume by housing age is shown in Table 1, indicating that properties less than 15 years old have the highest transaction volume for high-rise buildings, while apartments over 30 years old account for 28.26% of transactions, becoming one of the mainstreams of old property transactions.

Table 1 Age and Transaction Amount of Homes in Taichung City

Age / House Type	New construction ≤3	New pre-owned homes <3~≤15	Pre-owned homes 15~≤30	Old buildings 30≤
High-rise apartments	4,730	3,982	26,600	3,794
%	7.07%	8.98%	29.85%	8.97%
High-rise buildings	41,943	28,159	41,689	2,555
%	62.70%	63.47%	46.78%	6.04%
Townhouses	20,057	11,696	16,126	24,002
%	29.98%	26.35%	18.10%	56.73%
Apartments	166	526	4,702	11,954
%	0.25%	1.20%	5.27%	28.26%
Total	66,896	44,363	89,117	42,305
%	100%	100%	100%	100%

Compiled by this study

Most of the literature on housing price volatility focuses on property types such as residential, office buildings, retail, factories, and apartments, further divided into one-bedroom and two-bedroom units. Data organization often involves using monthly average transaction prices or median monthly transaction prices. Several ARCH/GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models are tested to identify the most appropriate model for estimation and interpretation.

This study uses transaction data from the Ministry of the Interior's real price registration from August 2012 to June 2023, organized into 131 periods of monthly average data. The volume of pre-sale housing projects is derived from the Ministry of the Interior's building permit statistics, organized as monthly changes, and included in the model as an exogenous variable. First, the data properties are determined; the data in this study are non-stationary and must be differenced once to transform them into stationary data. The ADF (Augmented Dickey-Fuller test) unit root test and residual tests are conducted, and multiple ARCH/GARCH models are established. Using model selection criteria, the most appropriate model is selected.

This study identifies the ARIMA(2,1,0)-GARCH(1,1) as the most suitable model for housing price volatility. The volatility duration meets the 5% significance level but does not converge in the long term. Before incorporating the pre-sale housing project volume into the volatility model, the model converges; after inclusion, it diverges. This suggests that the continuous optimal selection of old apartments for redevelopment results in a growing number of substandard properties that do not meet new development standards, leading to a significant decline in transaction prices and increasing price volatility.

2. Literature Review

Random price volatility is normal in the market mechanism. For example, stocks, exchange rates and gold in asset markets are transacted at a high frequency of daily pricing. Large fluctuations are often followed by even greater fluctuations, while small fluctuations are followed by smaller fluctuations. This is known as volatility clustering. Housing market involves low frequency transaction and is often presented in monthly transaction data. Relevant research on volatility clustering is organized as follows:

Wang and Hartzel (2020) studied price volatility of real estate in Hong Kong, and analyzed monthly data of residences, office buildings, retail buildings, and factory buildings from February 1993 to February 2019. They found that volatility clustering exists in all four types of real estates.

The determining factors of price volatility vary for several types of real estates. The price volatility of residences in Hong Kong is mainly affected by exchange rate (CNY and USD), while that of commercial real estates is mainly affected by unemployment rate. Dufitinema (2020) studied the housing market of Finland, and found that clustering effects exist in over half of cities and zones among all types of apartments researched. Evidence on the asymmetric influence of impact on the price volatility of residential homes is noticed in residential market of nearly all cities and zones. Moreover, differences of volatility property studied exist in cities and zones as well as among types of apartments. Miles (2020) tested residential market of 50 American states, and found that the GARCH effect of volatility clustering exists in over half of the states. Adam et al. (2021) suggested that impacts lead to a phenomenon of volatility clustering. Under the condition that there exists lower limitation in nominal interest rate, the influence of currency policy of these macroeconomic trends depends on the sources of housing price volatility. Kaulihowa and Kamiti (2019) verified the influence of macroeconomic factor impact on housing prices from the first quarter of 2007 to the second quarter of 2017 in Namibia. Their results support the hypothesis that the housing price of Namibia appears to fluctuate continuously. Fan et al. (2022) indicated that the uncertainty of economic policy is an important determining factor that affects the volatility of housing prices. Wang et al. (2022) established two models to analyze the long-term balance and short-term changes of pre-sale home market. The above discussions are organized into the following.

Chart 1. Economic significance of price volatility

Author	Research Purpose:	Study price volatility of real estate in Hong Kong
Wang, Hartzel (2020)	Data Scope:	Price volatility of real estate in Hong Kong from February 1993 to February 2019.
	Data Object:	Monthly housing price data of residences, office buildings, retail buildings, and factory buildings in Hong Kong.
	Research Method:	ARIMA model/GARCH model.
	Research Results:	1. Volatility clustering exists in all four kinds of real estates, and the determining factors of price volatility vary. 2. The price volatility of residential homes in Hong Kong is mainly affected by exchange rate (CNY and USD), while price volatility of commercial real estates is mainly affected by unemployment rate. 3. Results of index GARCH model indicates that asymmetrical effect does not exist in the real estate market of Hong Kong.
Author	Research Purpose:	Verify whether the housing price of Finland has financial characteristic as with stock and other assets.
Dufitinema (2020)	Data Scope:	15 major zones from the first quarter of 1988 to the fourth quarter of 2018.
	Data Object:	Type of residence studied is block of flats (consists of one-bedroom type, two-bedroom type and housing type with more than three bedrooms).
	Research Method:	ARIMA model/GARCH model.
	Research Results:	1. The clustering effects exist in housing types of over half of cities and zones among all types of apartments researched 2. The significant risk-return relationship observed in cities with all the three types of apartments is a mixed result. 3. The volatility properties studied vary for different apartment types. 4. The evidence of asymmetric influence on price volatility of residence is noticed in residential markets of nearly all cities and zones.

Author	Research Purpose:	Determine the optimal model forecast return of housing price and changes of volatility in the real estate market of Finland
Dufitinema (2021)	Data Scope:	Quarterly housing price index of 15 major zones of Finland from 1988 Q_1 to 2018 Q_4
	Data Object:	Real estate market of Finland
	Research Method:	ARMA, ARFIMA and EGARCH model
	Research Results:	GARCH model (CGARCH and FIGARCH) is superior to EGARCH model in the aspect of predicting price volatility. Price volatility has long-term dependence, and features of housing price volatility can be captured and predicted.
Author	Research Purpose:	Explore macro factors affecting the housing price volatility of Namibia
Kaulihowa and Kamiti (2019)	Data Scope:	Housing price of Namibia from 2007 Q_1- 2017 Q_2
	Research Method:	ARIMA model/)/ GARCH.
	Research Results:	Results support the hypothesis that the housing price of Namibia appears to fluctuate continuously.
Author	Research Purpose:	Test the residential market of 50 states with GARCH and develop an independent GARCH model.
Miles (2020)	Data Scope:	Housing price in 50 American states.
	Research Method:	ARIMA model / GARCH model.
	Research Results:	The clustering effects exist in over half of cities and zones among all types of apartments researched.
Author	Research Method:	Explore the relationship between housing price volatility and uncertainty of economic policies in G7 countries.
Fan et al. (2022)	Data Scope:	Housing price of G7 countries.
	Research Method:	GARCH-MIDAS model, newly introduced econometric techniques, is applied to sample size from January 1998 to May 2021.
	Research Results:	Uncertainty of economic policies is an important determining factor of housing price. The uncertainty of economic policies

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		is a valid predictive factor for the samples. GARCH-MIDAS has better predictive ability.
Author	Research Method:	Explain why overheating phenomenon often appears in pre-sale home market in short term with two theoretical models taking real estate market of Taiwan as an example.
Wang, Lin, and Tsai (2022)	Data Scope:	Data of Taipei City, New Taipei City, Taichung City and Kaohsiung City are used in this research. Data of pre-sale home market are extracted from possible transaction price data provided by Cathay Real Estate Development, Center for Real Estate Research, College of Social Sciences, and National Chengchi University (2000 Q1 - 2021 Q1).
	Research Method:	The cobweb model and forward contract pricing model are adopted.
	Research Results:	Short-term volatility of pre-sale home market was explained with the forward contract pricing model. Taiwanese government should cope with overheating issue of pre-sale home market by raising interest rate and land holding cost.

The economic significance of price volatility is often interpreted using time series models, also known as Random Walk (RW) models. The explanations for these models are as follows:1.**RW Model without Intercept**: The economic implication of this model is that if the supply and demand conditions in any market remain unchanged, market participants can use the price from the previous period as a direct reference for future price expectations. In such cases, the time series data of prices (and even trading volumes) may follow an RW model without an intercept.2.**RW Model with Intercept:** This model implies that when market supply increases due to continuous technological advancements or demand steadily increases due to factors like population growth or rising incomes, the time series data of prices (or trading volumes) may follow an RW model with an intercept.3.**RW Model with Random Disturbance**: When factors such as input, technological progress, and innovation are random but have a persistent and cumulative impact, the resulting time series data of income or prices may follow an RW model with random disturbances (Yang Yi-Nong, 2017).

Based on the literature, the economic significance of volatility confirms that asset prices exhibit volatility clustering. The process of volatility adjustment implicitly reflects the economic significance of endogenous variables. The literature indicates that the main exogenous factors affecting volatility include housing types, exchange rates, interest rates, economic policies, and pre-sale housing indices, which are indirect factors related to housing prices. This study uses the volume of pre-sale housing projects, which is directly related to housing prices, as an exogenous factor. This approach enhances the explanatory power of the model and provides a more accurate interpretation of housing price dynamics.

3. Data Source

(1) Transaction data size

There are 489,448 pieces of transaction data registered based on the transaction prices of residential homes in Taichung City from August 2012 to June 2023.

(2) Screening and classification

Through the data screening steps in Table 2, excluding missing and outlier data, a total of 242,681 valid data points were obtained.

Table 2 Data Screening Process

Step	Number of Removed Data	Valid Data (pieces)
(1) Population data	-	489,448
(2) Integration of house and land	142,999	346,449
(3) Housing product	100,453	245,996
(4) Removed missing value	3,015	242,705
(5)Removed deviation value (take 3 standard deviations)	24	242,681
Total	246,491	242,681

These data are classified according to housing types, namely, high-rise apartments, high-rise buildings, townhouses, and apartments. Difference between high-rise apartments and high-rise buildings is only building height, while all other conditions are the same. Hence, the data of high-rise apartments and high-rise buildings are combined. As shown in Table 3, there are 242,681 pieces of housing transactions, of which 153,452 pieces are high-rise apartments and high-rise buildings, (39,106 pieces of high-rise buildings and 114,346 pieces of high-rise apartments), 71,881 pieces are townhouses and 17,348 pieces are apartments (no elevator, no more than 5 stories).

Table 3 Classification of Residence

Property Category	Number of Transactions
(1) High-rise apartments and high-rise buildings	153,452
(2) High-rise apartments (with elevator, no more than 10 stories)	39,106
(3) High-rise buildings (with elevator, more than 11 stories)	114,346
(4) Townhouses (single street number for the entire building)	71,881
(5) Apartments (no elevator, no more than 5 stories)	17,348
Total	242,681

(3) Source of pre-sale homes:

The number of construction licenses approved and issued for residential buildings over the years is sourced from the construction statistics published by the National Land Management Agency from August 2012 to June 2023. There are 244,116 households from August 2012 to June 2023. The lowest number of licenses approved and issued is 413 in February 2016 and the highest number is 6,792 in January 2021. The average monthly number is 1,864. Statistics of all these months are organized into Table 4.

To address the issue of different calculation units and the disparity in volume of this variable, we use the absolute value of the monthly rate of change: |(Current month quantity - Previous month quantity) / Previous month quantity|. This represents the absolute monthly increment of pre-sold houses.

Table 4 Monthly Statistics Table of Construction Licenses for Residential Buildings (Unit: Household)

Period	2012M8	2012M9	2012M10	2012M11	2012M12	2013M1	2013M2	2013M3	2013M4	2013M5	2013M6	2013M7	2013M8	2013M9	2013M10	2013M11	2013M12
Number	1244	2199	1018	1248	1837	2076	1012	1391	1700	1965	1184	1486	1550	2498	2241	1082	1493
Period	2014M1	2014M2	2014M3	2014M4	2014M5	2014M6	2014M7	2014M8	2014M9	2014M10	2014M11	2014M12	2015M1	2015M2	2015M3	2015M4	2015M5
Number	876	1099	960	1471	954	1137	1561	1190	604	1113	822	940	879	1099	967	1471	954
Period	2015M6	2015M7	2015M8	2015M9	2015M10	2015M11	2015M12	2016M1	2016M2	2016M3	2016M4	2016M5	2016M6	2016M7	2016M8	2016M9	2016M10
Number	1137	1561	1190	604	1113	822	940	1131	413	1228	616	864	611	1280	1862	799	860
Period	2016M11	2016M12	2017M1	2017M2	2017M3	2017M4	2017M5	2017M6	2017M7	2017M8	2017M9	2017M10	2017M11	2017M12	2018M1	2018M2	2018M3
Number	770	2224	1258	963	684	1076	1997	1123	1419	1711	1501	920	1333	1400	1628	797	2097
Period	2018M4	2018M5	2018M6	2018M7	2018M8	2018M9	2018M10	2018M11	2018M12	2019M1	2019M2	2019M3	2019M4	2019M5	2019M6	2019M7	2019M8
Number	1573	1590	1897	938	2098	1241	2459	2109	2851	4376	792	3642	2068	1144	913	2805	2187
Period	2019M9	2019M10	2019M11	2019M12	2020M1	2020M2	2020M3	2020M4	2020M5	2020M6	2020M7	2020M8	2020M9	2020M10	2020M11	2020M12	2021M1
Number	2433	4080	1472	3130	2410	3498	2531	1456	1831	4002	3291	2633	3519	2505	2710	3655	6792
Period	2021M2	2021M3	2021M4	2021M5	2021M6	2021M7	2021M8	2021M9	2021M10	2021M11	2021M12	2022M1	2022M2	2022M3	2022M4	2022M5	2022M6
Number	1527	2550	1996	1849	2765	3364	1067	1096	2050	3362	4066	1433	1603	2729	3403	2522	4312
Period	2022M7	2022M8	2022M9	2022M10	2022M11	2022M12	2023M1	2023M2	2023M3	2023M4	2023M5	2023M6					
Number	2173	2792	3226	2772	4308	2374	1549	3241	1083	1438	1185	4397					

4. Results and Discussion of Empirical Analysis

The ARCH model and GARCH models are employed in this research. The absolute value increment of pre-sale homes is further expanded as an exogenous variable to discuss the influence of ARCH model and GARCH model on the absolute value increment of pre-sale homes. The optimal model for impact time in this research is as follows:

$$y_t|\Omega \sim N(x, a, \sigma^2) \quad \text{Eq. (4-1)}$$

$$\varepsilon_t = y_t - X_t a \quad \text{Eq. (4-2)}$$

$$\sigma^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 \quad \text{Eq. (4-3)}$$

where X_t is the vector of regression equation independent variable; a is the coefficient vector of regression equation; and q is the order of lagging terms.

X_{ta} denotes the linear combination of variables acquired by Ω_t $(a_0 + a_1 X_{1t} + a_2 X_{2t} + \cdots + a_k X_{kt})$. Eq.(4-2)is the mean equation and Eq. (4-3) is the variance equation. In mean equation, X_t is the single time series ARMA, including y_t which is the deferred lagging term, and n is the deferred lagging term of MA. Heteroscedasticity is ARCH(q), which can express the mean equation and variance equation ARCH as ARMA(m,n)-ARCH(q).

$$y_t = a_0 + \sum_n^m a_i\, y_{t-i} + \varepsilon_t + \sum_n^m b_i\, \varepsilon_{t-1} \ldots\ldots\ldots\ldots \text{Eq. (4-4)}$$

$$\sigma^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots\ldots + \alpha_q \varepsilon_{t-q}^2 \ldots\ldots\ldots \text{Eq. (4-5)}$$

GARCH model is the generalized ARCH model and typical GARCH (p,q) can be expressed as follows:

$$y_t|\Omega \sim N(X_t, a, \sigma^2) \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. (4-6)}$$

$$\varepsilon_t = y_t - X_t a \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. (4-7)}$$

$$\sigma^2 = \alpha_0 + \sum_{i=1}^q a_1\, \varepsilon_{t-1}^2 + \sum_p^q \beta_1\, \sigma_{t-1}^2 \ldots\ldots\ldots\ldots \text{Eq. (4-8)}$$

where p,q is the order of GARCH, if q=0 , the model is ARCH(q). ARMA(2,0) is AR(2) and the model formula of AR(2)-GARCH(1,1) is as follows

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. (4-9)}$$

$$\sigma^2 = \alpha_0 + \sum_{i=1}^q a_1\, \varepsilon_{t-1}^2 + \sum_{i=1}^q \beta_1\, \sigma_{t-1}^2 \ldots\ldots\ldots\ldots \text{Eq. (4-10)}$$

where σ^2 is the conditional variance. It is an unobservable variable, but it can be observed via the GARCH model and estimated. This variable is often regarded as substitute variable of risk or uncertainty, so σ^2 is a quantitative index used to measure shifting risk or is called volatility. ε_{t-1}^2 is regarded as the square number of the expected error of last period of mean equation. α_1 is the short-term accidental volatility; a higher value indicates greater influence of short-term unexpected factors, and vice versa. Long-term continuous volatility is the accumulation of short-term volatility, i.e. coefficient β_i plus coefficient a_i, $a_0 + a_1\beta_i$. Therefore, if $|\beta_i|<1$, the influence decreases geometrically, which is a weak stationary hypothesis. It can be obtained that long-term variance $\sigma^2 = \frac{a_0}{1-(a_1+\beta_1+\beta_2)}$ (Yang, 2017).

The mean equation of this paper is $y_t = a_0 + a_1 y_t + at + a_2 y_{t-2}$

GARCH-IN- Variance Regressors model

$$\sigma^2 = \alpha_0 + \sum_q^p a_1\ \varepsilon_{t-1}^2 \alpha_1 + \sum_q^p \beta_1\ \sigma_{t-1}^2 + \theta[\Delta x_t] \text{......Eq. (4-11)}$$

(1) Building of empirical model

In this study, the data consists of the monthly average price of apartments and the number of pre-sale housing projects. The steps for testing are as follows:

Observe Figure 1, which depicts the trend of apartment prices. It appears that there might be an intercept term. Perform a unit root test to confirm if it is significant.

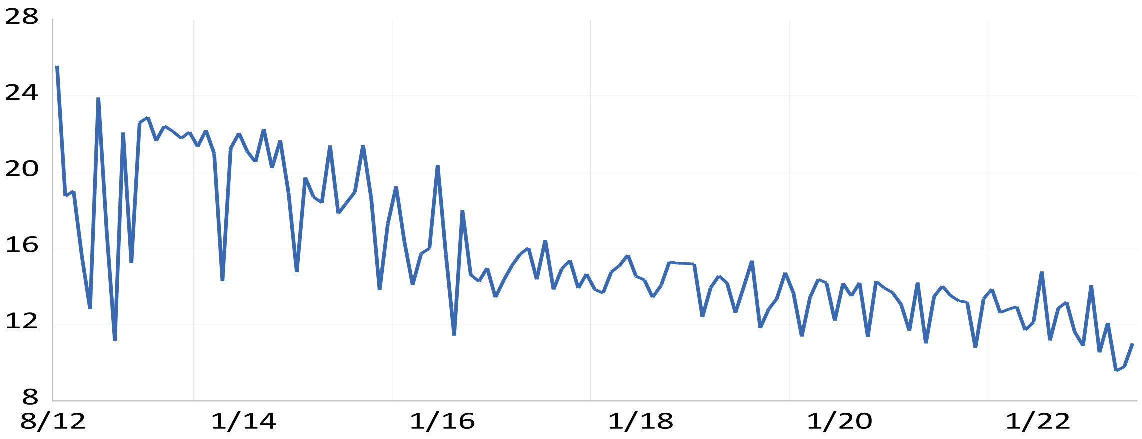

Figure 1 Apartment transaction price trend

1. Stationarity or non-stationarity

The Augmented Dickey-Fuller (ADF) test for unit root has the null hypothesis H_0 "accept unit root" and the alternative hypothesis H_1: "reject unite root." Acceptance or rejection of these hypotheses determines whether the monthly average price of apartments is stationary. Observing Figure 1, the monthly average price of apartments in this study appears to follow an equation with an intercept term. The unit root test does not reach the 5% significance level, thus we accept the null hypothesis H_0: "Presence of a unit root." This indicates that the data variable is non-stationary and can be transformed into a stationary variable through differencing. The relevant test results are as follows:

(1) ADF value of monthly apartment transaction t-Statistic data-0.398808 is greater than 5% critical value (-2.884477). H_0 null hypothesis that there is unit root is accepted, as shown in Table 5.

Table 5. Apartment Stationarity Test

Augmented Dickey-Fuller test statistic		Test critical values:		
t-Statistic	-0.398808	1% level	5% level	10% level
Prob	0.9048	-3.482879	-2.884477	-2.57908

Note: $\mathcal{P}^{*} \leq 0.05$; $\mathcal{P}^{**} \leq 0.01$; $\mathcal{P}^{***} \leq 0.000$

(2) Non-stationary data is transferred into stationary data

The above-mentioned non-stationary variable data, after differential verification, the test results are as shown in Table 6. The ADF value -11.84077 is less than the 5% critical value -2.884477. H_1 "rejection of a single root" is accepted, reaching a significant level of more than 1%, 5%, and 10%. It is a stationary variable, and its trend is as shown in Figure 2 below.

Table 6 Apartment Difference Stationarity Test

Augmented Dickey-Fuller test statistic		Test critical values:		
t-Statistic	-11.84077	1% level	5% level	10% level
Prob.*	0.0000***	-3.482879	-2.884477	-2.579080

Note: $\mathcal{P}^{*} \leq 0.05$; $\mathcal{P}^{**} \leq 0.01$; $\mathcal{P}^{***} \leq 0.000$

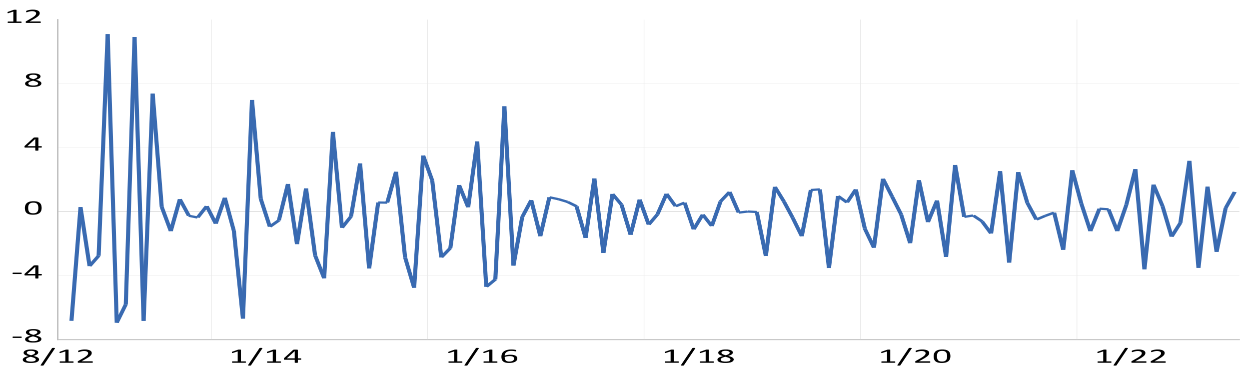

Figure 2: Apartment transaction price difference stationary state

2. Residual test and model-based estimation

Additionally, observe the ADF test estimation to check for autocorrelation in the residuals. Typically, the lag length is adjusted to estimate the appropriate number of lags. If autocorrelation is present, increase the number of lags until no autocorrelation is detected. The residual test should confirm the failure to reject the null hypothesis H0 at the 5% significance level, indicating no autocorrelation and the presence of white noise.

Furthermore, discuss whether the equation includes an intercept and trend term. If these terms do not reach the 5% significance level, they should be excluded. The model fit should be evaluated based on the Schwarz Criterion (S.C.), Akaike Information Criterion (AIC), and the explanatory power (R^2). Choose the model that best fits according to these criteria.

(1) Residual of estimator for apartment ADF test is as shown in Table 7. The number of lagging periods needed for variables to reach white noise is determined. The statistics of the third lagging period (t-statistic) is 4.078789 in Table 7. Its significance level surpasses 5% critical value. $\mathcal{P}$−value 0.000^{***} has reached significance level and residual is white noise. The variables of the fourth lagging period fail to reach a significance level and are not listed in the table. Statistics of intercept term (−1.00138) fails to reach the significance level of 5% and are removed from the equation.

Table 7 Residual of Estimator for Apartment ADF Test

variable	coefficient	t-statistic	$\mathcal{P}$−VALUE
D(AP(-1))	-3.413119	-11.84077	0.000^{***}
D(AP(-1),2)	1.553688	6.513138	0.000^{***}
D(AP(-2),2)	0.730720	4.592502	0.000^{***}
D(AP(-3),2)	0.316846	4.078789	0.000^{***}
C	-0.175503	-1.001380	0.319
R^2	0.844202	AIC	4.216179
Log likelihood	-260.6193	SBC	4.32873

Note: $\mathcal{P}^* \leq 0.05$; $\mathcal{P}^{**} \leq 0.01$; $\mathcal{P}^{***} \leq 0.000$

(2) Estimate the number of lagging periods

This study then confirms if all lagging periods reach significance level with OLS method based on ADF test results in Table 8. The third lagging period and intercept term are removed for not reaching significance level. Estimator for the second lagging period (Lag Length=2) is

$$d(AP_t) = \gamma AP_{t-1} + \beta_1(AP_{t-1}) + \beta_2 d(AP_{t-2}) + \varepsilon_t \text{......Eq. (4-11)}$$

Put coefficients of all variables into Table 8 in Eq. (4-11) to obtained. (4-12)

$$(AP_t) = -0.014632AP_{t-1} - 0.685238\Delta(AP_{t-1}) - 0.458108\Delta(AP_{t-2}) + \varepsilon_t \text{......Eq. (4-12)}$$

Table 8 Estimation with Ordinary Least Squares (OLS)

Variable	Coefficient	t-statistic	$\mathcal{P}$−VALUE
AP(-1)	-0.014632	-1.193933	0.235
D(AP(-1))	-0.685238	-8.751284	0.000***
D(AP(-2))	-0.458108	-5.993017	0.000***
R^2	0.401243	AIC	4.438951
Log likelihood	-281.0929	SBC	4.505795

Note: $\mathcal{P}^{*} \leq 0.05$; $\mathcal{P}^{**} \leq 0.01$; $\mathcal{P}^{***} \leq 0.000$

Box-Pierce Q statistic is shown in Table 9. Significance level has not passed the 5% critical value. Thus, null hypothesis H_0 that there is no existence of self-relevant residual sequence is accepted. In Table 10, Jarque Bera statistics is 49.66572. According to the normality test, P-value is up to 0.000*** significance level. Null hypothesis H_0 that it can be alternative model because it has “normal distribution” is rejected. Further discussion on the GARCH model is as follows.

Table 9 Box-Pierce Q Statistic

NO	AC	PAC	Q-Stat	Prob*
1	-0.085	-0.085	0.9428	0.332
5	0.2	0.176	8.5199	0.13
10	-0.036	-0.092	13.185	0.214
15	-0.04	0.054	18.245	0.25
20	0.061	0.058	22.348	0.322
30	0.013	0.022	30.414	0.445
35	0.015	-0.065	31.821	0.622

Note: $\mathcal{P}^{*} \leq 0.05$; $\mathcal{P}^{**} \leq 0.01$; $\mathcal{P}^{***} \leq 0.000$

Table 10 Jarque Bera Residual Test

Normality Test Jarque Bera			
Jarque Bera	49.66572	Prob.	0.000***

Note: $\mathcal{P}^{*} \leq 0.05$; $\mathcal{P}^{**} \leq 0.01$; $\mathcal{P}^{***} \leq 0.000$

(3) Model estimation

From Table 11, we observe the estimation results for the following models: ARIMA(2,1,0)-GARCH(1,0), ARIMA(2,1,0)-GARCH(0,1), and ARIMA(2,1,0)-GARCH(1,1). Table 11 shows the residual equations for the three models. In the ARIMA(2,1,0)-GARCH(1,1) model, the ARCH(-1) statistic is -0.891478, which does not reach the 5% significance level. In contrast, the GARCH(-1) statistics for the other models are 109.5751***, 3.340904**, and 40.78289***, all of which are significant at the 5% level.

Table 11 Estimators of All Models

Equation	ARIMA(2,1,0)-GARCH(1,0)			ARIMA(2,1,0)-GARCH(0,1)			ARIMA(2,1,0)-GARCH(1,1)		
Variable	Coefficient	z-Statistic	Prob.	Coefficient	z-statistic	Prob.	Coefficient	z-statistic	Prob.
AP(-1)	-0.014681	-1.423194	0.155	0.003324	0.320777	0.748	-0.013284	-1.252870	0.2103
D(AP(-1))	-0.669262	-6.781668	0.000***	-0.388361	-5.461878	0.000***	-0.682337	-7.362895	0.000***
D(AP(-2))	-0.444162	-5.209973	0.000***	-0.322571	-3.803193	0.000***	-0.448484	-5.345641	0.000***
C	0.095727	2.772980	0.006**	2.075634	6.944265	0.000***	0.101946	3.855146	0.000***
RESID(-1)^2							-0.021470	-0.891478	0.373
GARCH(-1)	0.940337	109.5751	0.000***	0.669992	3.340904	0.001**	0.957913	40.78289	0.000***
R-squared	0.407794			0.402248			0.312995		
Log likelihood	-254.5719			-253.0576			-268.8035		
Akaike info criterion	4.071435			4.063400			4.293804		
Schwarz criterion	4.205124			4.219370			4.427493		

Note: $\mathcal{P}^{*} \leq 0.05$; $\mathcal{P}^{**} \leq 0.01$; $\mathcal{P}^{***} \leq 0.000$

(2) Residual Test of Models:

1. Residuals of all models are presented in Table 12. There is no autocorrelation residual sequence. Correlogram squared Residual is tested. All tested models accept null hypothesis H_0 that there is no existence of "self-relevant residual sequence", which fails to reach the significance level of 5% critical value.
2. As shown in Table 12, there is no obvious ARCH phenomenon in all residuals tested with ARCH –LM. As a result, all of these three models accept null hypothesis H_0 that there is no "ARCH phenomenon" or ARCH effect.
3. Normality test is shown in Table 12. Assessed value of ARIMA(2,1,0)-ARCH(0,1) is 0.000011^{***}. Null hypothesis H_0 that there is "normal distribution" is rejected. Both tested values of the other two models are 0.211471 and 0.146178, respectively. Null hypothesis H_0 that there is "normal distribution" is accepted. Only assessed value of ARIMA(2,1,0)-ARCH(0,1) reached significance level and reject the existence of normal distribution.

Table 12 Residual Test of All Models

Test / Model	Correlogram squared Residual Test	Normality Test Jarque Bera Probability	ARCH-LM F-statistic Probability/F -statistic/Obs*R-squared
ARIMA(2,1,0)-ARCH(1,0)	0.052-.0.402	0.211471	0.114/0.2083
ARIMA(2,1,0)-ARCH(0,1)	0.296-0.982	0.000011***	0.3688/0.3648
ARIMA(2.1,0)-ARCH(1,1)	0.057-0.435	0.146178	0.3120/0.3082

Note: $\mathcal{P}* \leq 0.05$; $\mathcal{P}** \leq 0.01$; $\mathcal{P}*** \leq 0.000$

Correlogram squared Residual Test is applicable for test of small samples, so this test is adopted to determine whether the three models pass residual test when exogenous variables not affected by independent variables are added. Exogenous variables are increment of pre-sale homes and regarded as supply quantity or variance regressors of residences. The last test is then repeated.

(3) GARCH in Variance Regressors and Residual Test

The increment of pre-sale homes is an exogenous variable, and is included in residual equation for observation. In ARIMA(2,1,0)-GARCH(1,0) column of Table 13, z-statistic of pre-sale home increment (PREDQ) is 2.49479, coefficient is 0.000170 and significance level is above 5% critical value (0.025**), which is of significance. As for the other two models, for ARIMA(2,1,0)-GARCH(1,1), z-statistic is 1.625313, coefficient is 0.000120 and P-VALE=0.1041, which fails to reach significance level; for ARIMA(2,1,0)-GARCH(0,1), z-statistic is − 1.081251, P-VALE=0.280, which is below the significance level of 5% critical value. Therefore, only **ARIMA(2,1,0)-GARCH(1,0)** reaches the significance level**.**

Table 13 GARCH in Variance Regressors

equation	ARIMA(2,1,0)-GARCH(1,0)			ARIMA(2,1,0)-GARCH(0,1)			ARIMA(2,1,0)-GARCH(1,1)		
variable	Coefficient	z-Statistic	Prob.	Coefficient	z-statistic	Prob.	Coefficient	z-statistic	Prob.
AP(-1)	-0.012076	-1.254408	0.2097	0.001753	0.155753	0.876	-0.008885	-0.966633	0.3337
D(AP(-1))	-0.651681	-6.571648	0.000***	-0.397623	-5.663564	0.000***	-0.687251	-9.222193	0.000***
D(AP(-2))	-0.430212	-4.955348	0.000***	-0.331632	-3.903802	0.001**	-0.427384	-5.953039	0.000***
C	-0.119295	-1.403471	0.161	2.416863	5.869995	0.000***	-0.008745	-0.098420	0.9216
RESID(-1)^2				0.608438	3.222528	0.001**	-0.101085	-15.08624	0.000***
GARCH(-1)	0.958039	116.0923	0.000***				1.032349	4675.166	0.000***
PREDQ	0.000170	2.249479	0.025*	-0.000260	-1.081251	0.280	0.000120	1.625313	0.1041
R-squared	0.399952			0.325245			0.399225		
Log likelihood	-254.3775			-268.5708			-247.0239		
Akaike info criterion	4.068399			4.290169			3.969124		
Schwarz criterion	4.202088			4.423858			4.125094		

Note: $\mathcal{P}* \leq 0.05$; $\mathcal{P}** \leq 0.01$; $\mathcal{P}*** \leq 0.000$

Table 14 Residual Test with GARCH in Variance Regressors Model

Test \ Model	ARIMA(2,1,0)-ARCH(1,0)	ARIMA(2.1,0)-ARCH(0,1)	ARIMA(2,1,0)-ARCH(1,1)
Correlogram squared Residual Test	0.091-0.499	0.327-0.951	0.579-0.956
Normality Test Jarque Bera Probability	0.319031	0.000***	0.228376
ARCH-LM F-statistic Probability F -statistic/Obs*R-square	0.2037/0.2007	0.3381/0.3342	0.5872/0.5837

Note: $\mathcal{P}* \leq 0.05$; $\mathcal{P}** \leq 0.01$; $\mathcal{P}*** \leq 0.000$

The results of the residual tests for the three models are as shown in Table 14:

1. Statistical test of Correlogram squared Residual Test.

   The observed value of the 36th lagging period is tested to determine whether there exists self-relevant residual sequence. All these three models have residual terms and accept H_0 that there is" no existence of self-relevant residual sequence".

2. Normality Test Jarque Bera Probability.

   Two models accept null hypothesis H0 that there is "normal distribution" for observation values. Only ARIMA(2.1,0)-ARCH(0,1) rejects the null hypothesis H0 that there is "normal distribution" .

3. ARCH-LM Test

The test determines if there is heteroscedasticity of autocorrelation residual sequence. Null hypothesis H_0 is without "ARCH effect". With significance level reaching 5% critical value, these three models accept null hypothesis H_0 "without ARCH effect".

(4) Model Selection Criteria

Based on Table 15, the better model was selected. The ARIMA(2,1,0)-GARCH(1,0) model has the highest R-squared value at 0.399952, indicating the greatest explanatory power. The second highest is the ARIMA(2,1,0)-GARCH(1,1) model with an R-squared of 0.399225, with negligible difference between the two. For the AIC (Akaike Information Criterion) and SC (Schwarz Criterion), the smaller values are preferred. The ARIMA(2,1,0)-GARCH(1,0) model has an AIC of 4.068399 and SC of 4.202088, which are larger than the AIC of 3.969124 and SC of 4.125094 for the ARIMA(2,1,0)-GARCH(1,1) model. Therefore, the ARIMA(2,1,0)-GARCH(1,1) model is chosen as the most appropriate model.

Table 15 Model Selection Criteria

Model Goodness of fit Test	ARIMA(2,1,0)-ARCH(1,0)	ARIMA(2.1,0)-ARCH(0,1)	ARIMA(2,1,0)-ARCH(1,1)
R-squared	0.399952	0.325245	0.399225
Log likelihood	-254.3775	-268.5708	-247.0239
Akaike info criterion	4.068399	4.290169	3.969124
Schwarz criterion	4.202088	4.423858	4.125094

(5)Summary

This paper estimates ARIMA(2,1,0)-GARCH(1,1) as the optimal model to explain the volatility of apartment price, and its relevant coefficients are shown in Table 16.

Table 16 Optimal Model of ARCH/GARCH

equation	variable	AP(-1)	D(AP(-1))	D(AP(-2))	C	RESID(-1)^2	GARCH(-1)	PREDQ
ARIMA(2,1,0)-GARCH(1,1)	Coefficient	-0.008885	-0.687251	-0.427384	-0.008745	-0.101085	1.032349	0.000120
	z-statistic	-0.966633	-9.222193	-5.953039	-0.09842	-15.08624	4675.166	1.625313
	Prob.	0.3337	0.000***	0.000***	0.92160	0.000***	0.000***	0.104100

Note: $\mathcal{P}^{*} \leq 0.05$; $\mathcal{P}^{**} \leq 0.01$; $\mathcal{P}^{***} \leq 0.000$

For example, the residual equation of Eq. (4-13) can also be included into impact equations of exogenous variable, such as Eq. (4-14). Its influence on volatility is observed and its degree of influence is discussed. For example, the rate of relative change for supply quantity of pre-sale homes is included in this paper, take absolute value as $|\Delta\chi_t|$. If coefficient of this exogenous variable θ>0, it means that the influence of current, σ_t^2 on volatility increases. On the contrary, if θ<0, then influence degree of current σ_t^2 on current volatility decreases.

Residual equation of GARCH(1,0) is $\sigma_t^2=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\beta_1\sigma_{t-1}^2+\theta|\Delta\chi_t|_t$ Eq. (4-13)

Put coefficients in Table 16 into Eq. (4-13)

σ_t^2=−0.008745−0.101085ε_{t-1}^2+1.032349σ_{t-1}^2+0.00012|$\Delta\chi_t$|t ... Eq (4-14)式

The volatility of short-term and long-term impact of apartment price and influence degree of exogenous variables are discussed based on the empirical results. Apartment price is non-stationary data and transferred into to stationary data through first difference. ARIMA(2,1,0)-GARCH(1,1) is the optimal model. In ARIMA(2,1,0) model, the mean equation for apartment price is related to solution by iteration for the second lagging period. MA(0) denotes that no correction is needed. Eq. (4-13)ε_{t-1}^2 means taking squared value of the prediction error of last period as residuals variance. It denotes short-term accidental impact and its influence on volatility. If accidental impact is large, then its coefficient α_1 is large. In order to measure size of short-term impact, short-term impact of this model $\alpha_1\varepsilon_{t-1}^2$=0. According to Eq. 6-1, conditional variance formula of $\sigma_{t-1}^2, \sigma_{t-2}^2, \sigma_{t-3}^2$.... can be accumulated and impact is comparatively longer. Volatility decrease of continuous periods can be expressed by the absolute value of β_1 and β_2, which is <1; otherwise, volatility increases progressively. Eq. 6-2 is the influence of impact on volatility persistence. In Table 16, the coefficient of GARCH(-1) $|\beta_1|$ =0.958039***<1, marginal influence of volatility aroused by impact decreases progressively. If the value is small, the decrease is fast. Thus, for the residual equation model of GARCH(1, 0) p=1,q=0, the volatility of apartment price has no short-term impact effect but has obvious influence of volatility persistence of one deferred lagging period impact. Intercept term and trend term of optimal model in this paper means supply and demand conditions remain the same, only supply and demand quantity tends to convergent. Market participants can refer to price of last period directly.

Volatility clustering of housing price is mentioned in research literature, which is the presentation of unique attribute of residual volatility. $|\beta_1|$ is coefficient of conditional variance σ_t^2 and $|\beta_1|$=0.958039***<1 in this research, which reaches significance level. The marginal influence of volatility aroused by impact decreases progressively. In some studies, factors related to housing price, such as exchange rate, interest rate, and policies are exogenous variables. This method is also adopted to measure impact in impact period. In Eq.(4-14), supply increment absolute value of pre-sale homes $|\Delta\chi|$ is exogenous variable and its coefficient θ=0.00017*>0, which reaches a significance level. The rate of supply change makes the housing price fluctuating more sharply. It is found that supply increment of pre-sale homes makes no contribution to the stabilization of apartment price volatility.

Based on the empirical results, the findings are as follows:

1.Stationarity of Apartment Prices: Apartment prices are non-stationary data, which, after first differencing, become stationary. The most suitable model is identified as ARIMA(2,1,0)-GARCH(1,1). The ARIMA(2,1,0) model's mean equation for apartment prices, after first differencing, involves AR(2) with two-period lagged recursive calculation (solution by iteration). MA(0) implies no need for adjustment**.2.Short-term Shock**: In equation (4-13), ε_{t-1}^{2} represents the previous period's forecast error squared, indicating the variance of the residual value. This measures the impact of short-term unexpected shocks on volatility. If the unexpected shock is significant, its coefficient α_1=$-$0.101085***is large, indicating the magnitude of short-term risk. In this model, the value for the short-term unexpected shock's impact isα_1=$-$0.101085***.**3.Persistence of Volatility**: The persistence of the impact of an unexpected shock on volatility depends on the sum of α_1=$-$0.101085***and β_1=1.032349***. The absolute value of $|\beta_1|$ being less than 1 indicates decreasing volatility over successive periods, whereas a value greater than 1 indicates increasing volatility. In Table 16, the GARCH(-1) coefficient ｜$|\beta_1|$ =1.032349***>1， shows increasing marginal impact of shocks on volatility. In comparison, the GARCH(-1) coefficient before including the pre-sale housing volume is $|\beta_1|$ =0.957313***<1 indicating decreasing volatility over successive periods. This suggests that including the pre-sale housing volume induces increasing volatility.**4.Impact of Pre-sale Housing Volume**: In equation (4-14), the pre-sale housing volume$|\Delta\chi|$is an exogenous variable with a coefficient =0.00012>0 and a p-value of 0.1041, indicating a 10.41% probability. This shows that the pre-sale housing volume does not effectively reduce the volatility in apartment transaction prices.

5.Conclusion

The study period spans from the 2018 financial crisis to the end of COVID-19 in 2023, during which global monetary policy shifted from quantitative easing to tightening, and interest rates moved from cuts to hikes. These changes led to rising housing prices, causing incomes to lag behind. In response, the Taiwanese government introduced policies to curb housing prices, such as the integrated housing and land tax, the Equalization of Land Rights Act 2.0, and incentives for urban renewal and the reconstruction of hazardous old buildings.

The research focuses on the transaction prices of apartment buildings in Taichung City. After various tests, the ARIMA(2,1,0)-GARCH(1,1) model was identified as the most suitable for housing price volatility. The GARCH(-1) volatility coefficient without the volume of pre-sale housing projects is $|\beta_1| = 0.957913 < 1$, indicating converging volatility. However, when the volume of pre-sale housing projects is included, the GARCH(-1) volatility coefficient is $|\beta_1| = 1.032349 > 1$, indicating diverging volatility. The coefficient for the pre-sale housing projects volume, $|\Delta\chi|$, as an exogenous variable is $\theta = 0.000120 > 0$, which is not significant at the 5% level, suggesting that the volume of pre-sale housing projects cannot effectively reduce volatility.

The volatility clustering of apartment prices is comparable to the real estate markets in Hong Kong, Finland (15 regions), and half of the U.S. states, all of which exhibit volatility clustering. Apartments, due to their low floor area ratio, are not mainstream in metropolitan areas. Table 1 shows that most transactions involve apartments over 30 years old, with few transactions for new or recently built apartments, similar to the volatility clustering observed in related literature.

Kaulihowa and Kamari (2019) demonstrated that macroeconomic factors support volatility persistence. Wang et al. (2022) built housing price models using price indices from Taipei City, New Taipei City, Taichung City, and Kaohsiung City, showing that the pre-sale housing market often overheats in the short term. Therefore, macroeconomic impacts or pre-sale housing project volumes do not help in converging housing price volatility.

This study's results are similar to those of Wang et al. (2022) on housing price volatility in Taiwan's four metropolitan areas. The comparisons are as follows: (1) **Different Stances**: Wang et al. (2022) investigated the impact of existing home market demand shifts on the pre-sale housing market, while this study examines the impact of pre-sale housing project volumes on the existing home market, focusing on apartments within multi-unit dwellings. (2) **Methodological Differences**: Wang et al. (2022) used cobweb models and forward contract pricing models, while this study employs the GARCH housing price volatility model. (3) **Different Scopes**: Wang et al. (2022) studied housing price index volatility across four metropolitan areas in Taiwan, while this study focuses on the volatility of apartment transaction prices in Taichung City. (4) **Price Trend Differences**: Wang et al. (2022) found that demand shifts in the existing home market led to a rise in the pre-sale housing market, resulting in non-converging price volatility. This study found that adding the volume of pre-sale housing projects caused originally declining apartment prices to diverge, suggesting a business opportunity in rebuilding old apartments. Repeatedly preferential purchases lead to more substandard apartments, causing prices to decline.

This paper contributes in the following ways: (1) **Providing an Alternative Risk Measurement Method**: By assessing the persistence of price volatility and predicting whether price trends converge, quantitative data can inform risk-averse choices. (2) **Validation Results**: It shows that not all housing price trends rise during economic prosperity; the trend for apartment transactions in Taichung City indicates a decline. (3) **Expanding Perspectives on Old Housing Issues**: Future research could explore whether the volatility of multi-unit buildings and apartment buildings is similar and investigate the relationship between new and old housing in terms of co-integration.

6. References

Wang, C. T., Liu, H. Y., Chen, W., Lung, F. C. (2007), *"Stabilizing effect of the pre-sale market on spot house prices",* Journal of Tsinghua University (Science and Technology) 47(6): 781-784.

Lee, J. H., Yang, C. H. (2010) , *"The Relationship of Price Difference with Pre-sale and Existing House-take Taipei City and Taipei County as an Example",.* Journal of Architecture and Planning, 11(1), 1-14.

Chiang, Y. H., Chu, C. Y., Chang, C. O. (2019) ,*"A Mass Appraisal Modeling of the Pre-Sale Housing".* Journal of Housing Studies, 28(2),17 -36.

Wu, J. L., Huang, H. C., Wang, C. N., Wu, R. W. (2012), *"Revisiting to Taiwan's Foreign Exchange Rate Policies",* Taiwan Economic Review,40(2), P273-284.

Chen, S. S. (2020) , *"Applied Time-Series Econometrics for Macroeconomics and Finance",.* Dong Hua Books, 2nd ed. 4th prim., Taipei City; P319-328.

Yang, Y. N. (2017), *"Time Series Analysis in Economics and Finance".* 3nd ed., Yeh Yeh Book Gallery, P164-171; P341-345. Time Series: Economic and Financial Applications

Chi, C. X., Wang, S. (2010) , *"Research on Influence of Pre-sale House Price on Price of Spot House Taking Real Estate of Yunnan Province as an Example", China Economic Review* (10): 94-95

Josephine Dufitinema (2020) ,*"Volatility clustering, risk-return relationship and asymmetric adjustment in the Finnish housing market ",* The current issue and full text archive of this journal is available on Emerald Insight at: https://www.emerald.com/insight/1753-8270.htm.

Josephine Dufitinema (2021),*"Forecasting the Finnish house price returns and volatility: a comparison of timeseries models",* International Journal of Housing Markets and Analysis Vol. 15 No. 1, 2022 pp. 165-187

Klaus Adam, Oliver Pfauti, Timo Reinelt (2021),,*"Falling Natural Rates, Rising Housing Volatility," and the Optimal,* Inlow risen Target June 21

Teresia Kaulihowa, Katrina Kamari (2019), "D*eterminants of house price volatility in Namibia",* International Journal of Housing Markets and Analysis.ISSN:1753-8270.

Ting Fan, Asadullah Khaskheli, Syed Ali Raza and Nida Shah (2022), *"The role of economic policy uncertainty in forecasting house prices volatility in developed economies: evidence* *from a GARCH-MIDAS approach",* International Journal of Housing Markets and Analysis Issue(s) available: 74 – From Volume: 1 Issue: 1, to Volume: 16 Issue: 7

Shizhen Wang, David Hartzel1 (2020), *"What influences real estate volatility in Hong Kong? An ARMA-GARCH approac",* The current issue and full text archive of this journal is available on Emerald Insight at: https://www.emerald.com/insight/1753-8270.htm

Willian Miles (2020), *"Volatility Clustering in U.S. Home Prices, Journal of Real Estate Research",* Pages 73-90 | Published online: 17 Jun 2020.

Wen-Kai Wang, Che-Chun Lin, I-Chun Tsai (2022), *"Long- and short-term price behaviors in pre-sale housing",* Economic Analysis and Policy 74 350–364journal homepage: www.elsevier.com/locate/eap.

Wong, S. K., C. Y. Yiu & K. S. Tse (2006), "*Do the Forward Sales of Real Estate Stabilize Spot Prices? "* The Journal of Real Estate Financial and Economics. 32(3): 289-304.