An Empirical Study on Volatility Forecasting Ability of Various Symmetric and Asymmetric GARCH Models

Abstract

This article investigates the volatility dynamics of major global stock indexes, including the FTSE 100, Hang Seng Index, NIKKEI 225, and S&P 500, using a range of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. The analysis covers a comprehensive 20-year period from January 1, 2004, to December 31, 2023, incorporating various market conditions such as bull and bear markets, the 2008 financial crisis, and the COVID-19 pandemic. Preprocessing involves calculating daily log returns, performing descriptive statistics, and conducting stationarity and ARCH effect tests to ensure data suitability. The study evaluates several GARCH models, including the GARCH, EGARCH, NGARCH, APARCH, GJR-GARCH, and TGARCH models, to forecast volatility and address both symmetric and asymmetric effects.

The TGARCH model exhibits strong performance in capturing leverage effects and asymmetries, particularly for the FTSE 100 and Hang Seng Index. The APARCH model performs well for the S&P 500, showing sensitivity to past shocks. Overall, the study underscores the importance of advanced GARCH models in accurately capturing and predicting volatility in global financial markets. The findings highlight the TGARCH model's effectiveness in addressing asymmetries and offer insights into selecting appropriate models for enhanced financial analysis and risk management.

KeyWords: GARCH Models, Global Stock Indexes, Leverage Effect, Symmetry & Asymmetry Effects, Predictability and Volatility

1. Introduction

The ability to forecast the volatility of these global stock indexes is of paramount importance for various stakeholders, including institutional investors, portfolio managers, and policymakers. GARCH models, introduced by Bollerslev (1986), have become the cornerstone of volatility modelling due to their ability to model time-varying volatility and capture the persistence and clustering of volatility in financial time series. The basic GARCH(1,1) model posits that the current conditional variance of a time series is a function of past squared observations and past conditional variances. This model effectively captures the "volatility clustering" phenomenon, where periods of high volatility tend to be followed by high volatility, and periods of low volatility tend to be followed by low volatility. Despite its simplicity, the GARCH(1,1) model has proven to be remarkably effective in a wide range of applications.

Over the years, various extensions of the GARCH model have been developed to address its limitations and enhance its forecasting performance. The Exponential GARCH (EGARCH) model, proposed by Nelson (1991), accounts for asymmetries in the impact of positive and negative shocks on volatility. This feature is particularly relevant in financial markets, where negative news often has a more significant impact on volatility than positive news of the same magnitude. Glosten, Jagannathan, and Runkle (1993) developed the GJR model to capture the leverage effect, where negative shocks have a greater impact on volatility than positive shocks. Bera and Higgins (1993) proposed the NGARCH model, which introduces a more general functional form for the conditional variance, allowing for more complex dynamics in volatility. Ding, Granger, and Engle (1993) introduced this model to account for asymmetries and varying powers in the volatility process, offering a more flexible structure for modelling heteroskedasticity.

Zakoian (1994) proposed the TGARCH model, which incorporates threshold effects to model the different responses of volatility to positive and negative innovations. Bollerslev and Ghysels (1996) developed the PGARCH model to address the periodicity observed in some financial time series, allowing for seasonal effects in volatility modelling. The Component GARCH (CGARCH) model, developed by Engle and Lee (1999), decomposes volatility into permanent and transitory components, providing insights into the long-term and short-term factors driving market volatility. This decomposition is particularly useful for distinguishing between structural changes in the market and temporary fluctuations. The GARCH-MIDAS model, proposed by Engle, Ghysels, and Sohn (2013), integrates mixed data sampling to incorporate macroeconomic variables and other low-frequency information into the volatility forecasting process. This approach enhances the predictive power of GARCH models by leveraging additional information beyond daily financial returns.

The global financial markets are inherently volatile, characterized by significant fluctuations in asset prices driven by myriad factors ranging from economic indicators and geopolitical events to investor sentiment and market speculation. Accurate forecasting of this volatility is crucial for risk management, investment strategies, and policy formulation. Among the numerous econometric models developed to analyze and predict market volatility, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) of models has gained prominence due to its flexibility and robustness in capturing the dynamic nature of financial time series. Global stock indexes serve as benchmarks for the performance of equity markets and are crucial indicators of economic health and investor sentiment.

Four major indexes – FTSE 100, Hang Seng, NIKKEI 225, and S&P 500 – represent diverse geographical regions and provide a comprehensive view of global market dynamics. The Financial Times Stock Exchange 100 Index, commonly known as the FTSE 100, comprises the 100 largest companies listed on the London Stock Exchange. It is a key indicator of the performance of major British companies and their exposure to international markets. The Hang Seng Index is a major stock market index in Hong Kong, representing the largest and most liquid companies listed on the Hong Kong Stock Exchange. It reflects the economic and financial environment of Hong Kong and its interconnectedness with mainland China. The Nikkei 225, also known as the Nikkei Stock Average, is a stock market index for the Tokyo Stock Exchange. It includes 225 of the largest publicly traded companies in Japan, providing insights into the Japanese economy and its sensitivity to global economic changes. The Standard & Poor's 500 Index, commonly known as the S&P 500, includes 500 of the largest publicly traded companies in the United States. It is widely regarded as a barometer of the U.S. stock market and, by extension, the global economy.

This empirical study aims to evaluate the forecasting ability of various GARCH models in predicting the volatility of these major global stock indexes. By comparing the performance of different models, including GARCH, EGARCH, TGARCH, and other variants, this study seeks to identify the most effective approaches for volatility forecasting across diverse market conditions. Accurate volatility forecasting is essential for managing financial risk, optimizing investment portfolios, and developing effective regulatory policies. By providing a comprehensive analysis of the strengths and limitations of different GARCH models, this study contributes to the literature on volatility modelling and offers practical insights for market participants. The dynamic nature of global stock markets necessitates robust volatility forecasting models, and the GARCH of models, with their diverse extensions, offers powerful tools for capturing the complexities of financial volatility. This study endeavours to assess the empirical performance of these models in forecasting the volatility of key global stock indexes, thereby enhancing our understanding of market dynamics, and improving decision-making in financial markets.

2. Literature Review

The study of financial market volatility has been a cornerstone of financial econometrics, particularly through the development and application of various autoregressive conditional heteroskedasticity (ARCH) models. Bollerslev (1986) pioneered this area by introducing the generalized autoregressive conditional heteroskedasticity (GARCH) model, extending the basic ARCH framework to incorporate past conditional variances. This model set the stage for a plethora of GARCH-type models that aim to capture the complex dynamics of financial time series data. Among the early extensions of the GARCH model, Nelson (1991) introduced the exponential GARCH (EGARCH) model, which accommodates the asymmetric effects of shocks on volatility. The EGARCH model, along with others like the GJR model by Glosten et al. (1993) and the NGARCH model by Bera and Higgins (1993), have been instrumental in understanding financial volatility. These models address the need to capture volatility asymmetry, a common feature in financial markets where negative shocks often have a larger impact on volatility than positive ones.

Further advancements include the Asymmetric Power GARCH model by Ding et al. (1993) and the Threshold GARCH model by Zakoian (1994), which refine the ability to model asymmetries and threshold effects in volatility. Bollerslev and Ghysels (1996) introduced the periodic GARCH (PGARCH) model, which accounts for seasonal volatility patterns in high-frequency asset returns. These models have significantly improved our understanding of volatility dynamics, providing better tools for risk management and forecasting. Empirical studies across different markets and time periods have demonstrated the utility of these models. For instance, Rossetti et al. (2017) analysed fixed income market volatility across 11 countries from 2000 to 2011, finding that asymmetric GARCH processes, particularly EGARCH, effectively capture volatility influenced more by internal macroeconomic events than external factors. Similarly, Gupta (2023) compared symmetric and asymmetric GARCH models in forecasting stock market volatility in emerging nations, concluding that the EGARCH model best captures the asymmetric effects of shocks on volatility.Duppati et al. (2017) examines the ability of intraday data to predict long-term memory in volatility for five Asian equity indices, using GARCH-based models and realized volatility approaches.

Innovative modelling approaches have also been explored. Paul et al. (2017) introduced a Realized GARCH-EVT model for quantile forecasting, which generally outperformed traditional models in forecasting Value-at-Risk and expected shortfall for European stock indices. This highlights the potential of integrating high-frequency data and extreme value theory in volatility modelling. In the context of emerging markets, Sharma et al. (2021) found that the GARCH(1,1) model outperforms non-linear models in forecasting volatility for major emerging markets, such as China, India, Indonesia, Brazil, and Mexico. This indicates that while advanced models provide theoretical improvements, simpler models like GARCH(1,1) can sometimes offer better practical performance due to fewer estimation errors. Jiang (2012) introduces GARCH, E-GARCH, and GJR-GARCH models to predict the conditional variance of returns in five global stock markets using normal and student-t distributions for error terms. Kumar et al. (2012) explored the volatility of stock indices from the PIIGS economies using wavelet techniques and various GARCH models, revealing long-range dependence and support for the Taylor effect in volatility proxies. Their findings highlight the importance of considering both asymmetry and long memory in volatility modelling.

Similarly, Manera et al (2002) compares the forecasting performance of nonlinear GARCH models (VS-GARCH, GJR-GARCH, Q-GARCH) against the standard GARCH(1,1) model using ten European stock price indexes. The results show that nonlinear models generally offer better forecasts with smaller errors and biases. Lim et al. (2013) examined Malaysia's stock market volatility, finding that symmetric GARCH models perform better during normal periods, while asymmetric models are more effective during crises.Raji et al. (2018) investigated the dynamic relationship between the Nigeria-US exchange rate and crude oil prices, using various GARCH models.Ugurlu et al. (2014) evaluated GARCH-type models for stock market volatility in four European emerging countries and Turkey, finding persistent volatility shocks and significant impacts from old news.

Abdul Manap et al. (2011) examined the long memory property and asymmetric effects in Malaysian equity market volatility, finding that the FIAPARCH model effectively captures both asymmetry and long memory. Rajvanshi et al. (2017) compared GARCH, GJR-GARCH, and EGARCH models with their implied volatility (IV) augmented counterparts, concluding that the GARCH IV model excels in predicting volatility. Conversely, Ederington et al. (2004) determined that while the GARCH(1,1) model generally outperforms historical and exponentially weighted models, a novel non-linear least squares model based on absolute return deviations provides superior forecast accuracy. Similarly, Sharma et al. (2015) found that the standard GARCH model surpasses more advanced GARCH models in forecasting accuracy.

Ghorbel et al. (2013), Akinlaso et al. (2021)and Ghorbel et al. (2021) provided insights into volatility spillovers and dynamic conditional correlations between different markets. The former study highlighted the contagion effect of the 2008 oil shock and US financial crisis on GCC and BRIC stock markets, while the latter focused on the volatility spillover during the COVID-19 pandemic, revealing strong volatility spikes with gold acting as a safe haven. Volatility dynamics in specific sectors have also been explored. Hassan (2023) examined the response of NASDAQ clean energy stock returns volatility to external energy security elements, finding significant impacts from natural gas prices, carbon prices, and green information technology stocks. This study emphasizes the importance of external factors in assessing risks associated with clean energy stocks. Bhargava et al. (2023) highlights an asymmetric spillover effect for the Australian dollar and suggests future analysis with MV-GARCH models and different market perspectives.

The utility of GARCH models in risk management has been well-documented. Walther (2017) analysed the conditional volatility of Vietnamese stock indices, suggesting that long memory GARCH models combined with skewed Student's t-distribution are best for forecasting VaR and ES. Similarly, Liu et al. (2010) found that EGARCH provides the most accurate volatility forecasts during the 2008 financial crisis, highlighting the effectiveness of GARCH models in risk management. Even, Mușetescu et al. (2022) finds that the EGARCH(1,1) model best captures volatility asymmetry, with Brent Crude Oil responding negatively to over 90% of market shocks. But,AL-Najjar (2016)finds symmetric ARCH/GARCH models effectively capture volatility clustering and leptokurtosis, while EGARCH does not support leverage effects. Other notable contributions include studies by Priya et al. (2023) and Aggarwal et al. (2023), which examined the impact of COVID-19 protocols on sectoral volatility and the day-of-the-week effect in the Indian stock market, respectively. These studies underscore the importance of considering external shocks and market inefficiencies in volatility modelling.

Other side, Hentschel (1995) introduces a parametric type of GARCH models that includes both symmetric and asymmetric variants, allowing for nested tests of different asymmetries and functional forms. Analysis of daily U.S. stock return data reveals that standard GARCH models are rejected in favour of a model where the conditional standard deviation depends on the absolute value of past shocks raised to the power of 1.5 and past standard deviations. Smolović et al. (2016) finds that none of the eight GARCH models tested passed the Kupiec test at a 95% confidence level, though some models passed the Christoffersen test at 95%. The results highlight challenges in accurately capturing volatility clustering and VaR in emerging markets.

Finally, works like Wang et al. (2024) and Radha et al. (2024) explore the impact of economic uncertainty and forecast short-term interest rates using GARCH-based models. These studies highlight the relevance of GARCH models in capturing the nuances of financial time series data and their application in various economic contexts. In summary, the evolution of GARCH-type models has been pivotal in advancing our understanding of financial market volatility. From the basic GARCH model introduced by Bollerslev (1986) to sophisticated variants like EGARCH, TGARCH, and PGARCH, these models have provided powerful tools for analysing and forecasting volatility. Empirical studies across different markets and time periods demonstrate their effectiveness in capturing the complex dynamics of financial time series, offering valuable insights for risk management, investment strategies, and policy-making.

3. Methodology

The study will use daily closing prices of the FTSE 100, Hang Seng, NIKKEI 225, and S&P 500 indexes. These indexes are chosen due to their representation of major global economies and their significance in financial markets. The data will be obtained from reliable financial databases from Yahoo Finance, covering a period of twenty years (01-01-2004 to 31-12-2023) to ensure a comprehensive analysis. The time span should include various market conditions, such as bull and bear markets, 2007/2008 financial crises, periods of economic stability and, COVID-19 effect to capture the full spectrum of volatility dynamics.

3.1. Preprocessing

Before applying the GARCH models, the collected data will be pre-processed to ensure its suitability for analysis. This includes:

3.1.1. Log Returns Calculation: The daily log returns will be computed from the closing prices as they are more stationary and suitable for volatility modelling. The log return is calculated using the formula:

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \text{--}(1)$$

Where p_t and p_{t-1} are the closing prices at time t and t-1

3.1.2. Descriptive Statistics: Basic descriptive statistics, including mean, standard deviation, skewness, and kurtosis, will be calculated to understand the characteristics of the log returns.

3.1.3. Stationarity Check and: The stationarity of the log returns will be tested using the Augmented Dickey-Fuller (ADF) test, Augmented Dickey-Fuller Generalized Least Squares (ADF-GLS) test and The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. If the ADF test and ADF-GLS test statistic is less than the critical value at a given confidence level, and the p-value is below the chosen significance level, can reject the null hypothesis of a unit root, suggesting that the series is stationary. The KPSS test is designed to test the null hypothesis of stationarity against the alternative hypothesis of non-stationarity. It can be applied to time series data to determine whether the series is stationary around a trend (trend-stationary) or around a mean (level-stationary).

3.1.4. ARCH Effect: The ARCH-LM (Autoregressive Conditional Heteroskedasticity - Lagrange Multiplier) test is a statistical test used to detect the presence of autoregressive conditional heteroskedasticity (ARCH) effects in a time series. This test is crucial for identifying whether the variance of the residuals from a regression model is dependent on past error terms, which is a common characteristic of financial time series. The test statistic is compared with critical values from the chi-squared distribution. If the test statistic exceeds the critical value, the null hypothesis of no ARCH effects is rejected.

3.1.5. Visual Analysis: Time series plots of the log returns will be generated to visually inspect the data for patterns, anomalies, and ARCH effect.

3.2. Model Specification

The study will employ various GARCH models to forecast the volatility of the selected stock indexes by using the *t* distribution. The Symmetric and Asymmetric models to be considered include, ARCH Model, GARCH Model, Taylor/Schwert GARCH Model, EGARCH Model, APARCH Model, NGARCH Model, and TGARCH Model.

3.2.1. ARCH Model

In traditional econometrics, it is often assumed that the variance of a random variable remains constant over time. However, financial time series typically exhibit heteroscedasticity, meaning they are stable over the long term but show instability in the short term. To account for this time-varying volatility, Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model, which is used to model both the mean and variance of time series data. The general form of the ARCH model is expressed as follows:

$$y_t = \phi x_t + \mu_t \text{--}(2)$$

$$\sigma_t^2 = E(\mu_t^2|\mu_{t-1}, \mu_{t-2}......) = \alpha_0 + \alpha_1\mu_{t-1}^2 + ... + \alpha_p\mu_{t-p}^2 = \sum_{i=1}^{p} \alpha_i\mu_{t-i}^2 \text{-----------}(3)$$

ϕ is a non-zero parameter to be estimated, x_t represents the independent variable observed at time t, and μ_t is a random error term, which is typically assumed to follow a normal distribution in the standard model. The fundamental concept of the ARCH model is that the variance of the residuals μ_t at time t depends on the squared error terms from previous periods. Specifically, the model posits that the variance of the error term at time t is a linear function of the squared error terms from the previous p periods.

However, the ARCH model assumes that positive and negative shocks have the same impact on volatility, making it unsuitable for analysing series with asymmetric effects.

3.2.2. GARCH Model

Bollerslev (1986) introduced a significant enhancement to the ARCH model, termed the GARCH model, which better captures the phenomenon of volatility clustering commonly observed in financial time series. This approach considers the conditional variance as a GARCH process to effectively estimate volatility that changes over time. The defining equations of the model are as follows:

$$y_t = \phi x_t + \mu_t,\ \mu \sim N(0, \sigma_t^2) \text{---}(4)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i\mu_{t-i}^2 + \sum_{i=1}^{p} \beta_i\sigma_{t-i}^2 \text{---}(5)$$

In this model, μ_{t-i}^2 represents the ARCH parameter, while σ_{t-i}^2 is the GARCH parameter. The coefficients associated with the ARCH and GARCH terms are indicated by α and β, respectively, and p and q indicate the lag order of the model. Therefore, the ARCH model can be seen as a specific case within the broader GARCH framework. In this study, primarily utilize the GARCH(1,1) model, which includes one lag, to estimate the sample series. The strength of the GARCH model lies in its ability to reflect and interpret heteroscedasticity. However, it still falls short in capturing asymmetry in financial time series.

3.2.3. EGARCH Model

In 1991, Nelson introduced the EGARCH model, which formulates the variance equation in logarithmic form. This approach simplifies the estimation of the parameters for σ_t^2, as it removes the need for any constraints on the model's parameters.

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \left| \frac{u_{t-i}}{\sigma_{t-i}} - E\left(\frac{u_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j}^2) + \sum_{k=1}^{r} \gamma_k \frac{u_{t-k}}{\sigma_{t-k}} \text{-------------}(6)$$

The variance σ_t^2 remains positive irrespective of the sign of the coefficients on the right side of Equation. Unlike the GARCH model, the logarithmic conditional variance on the left side permits negative coefficients, enhancing the flexibility of the solution process. The presence of asymmetrical terms $\gamma \neq 0$ in the EGARCH model's equation means that their impact is measured in an exponential form rather than a quadratic one.

3.2.4. APARCH Model

Taylor (1986) and Schwert (1989) introduced a variant of the GARCH model designed to model the standard deviation rather than the variance. This modification aims to mitigate the influence of large shocks on the conditional variance. Following this, Ding et al. (1993)

advanced the model further by developing the Asymmetric Power Autoregressive Conditional Heteroscedasticity (APARCH) model. The APARCH model generalizes the standard deviation GARCH approach by incorporating asymmetric effects and varying powers, as specified in the following variance equation:

$$\sigma_t^{\delta} = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta} + \sum_{i=1}^{p} \alpha_i (|u_{t-i}| - \gamma_i u_{t-i})^{\delta} \text{--------------------------------(7)}$$

In the APARCH model, δ represents the power parameter applied to the estimated standard deviation, and it typically evaluates the impact on the conditional variance, with δ >. The parameter γ denotes the asymmetric coefficient, which captures the asymmetric effects up to the order r. For i=1,2…,r, $|\gamma_i| \leq 1$, and for i > r, $\gamma_i = 0$, with $r \leq p$. Unlike the traditional GARCH model, the APARCH model removes the restriction of non-negative parameters. While the GARCH model assumes a symmetric response of the conditional variance to positive and negative shocks, it fails to account for the observed negative correlation between financial returns and return volatility. The APARCH model, by incorporating asymmetric power effects, addresses this limitation and offers a more nuanced representation of volatility dynamics.

3.2.5. GJR GARCH Model

The GJR-GARCH model, introduced by Glosten, Jagannathan, and Runkle (1993), is an extension of the standard GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model that specifically addresses the asymmetry observed in financial time series. This model accounts for the "leverage effect," which suggests that negative shocks to asset prices have a more significant impact on volatility than positive shocks of the same magnitude. The GJR-GARCH model aims to capture asymmetries in volatility responses to positive and negative shocks. Traditional GARCH models assume that positive and negative shocks have the same impact on volatility, which may not align with real-world observations where negative shocks often lead to more significant increases in volatility compared to positive shocks.

The GJR-GARCH model modifies the GARCH model to include an asymmetric term that differentiates the effects of positive and negative shocks. The variance equation for the GJR-GARCH model is specified as follows:

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1})^2 + \gamma\, I(\varepsilon_{t-1} < 0)(\varepsilon_{t-1})^2 + \beta_1 \sigma_{t-1}^2 \text{--------------------------------(8)}$$

In this equation, σ_t^2 represents the conditional variance at time t, ω is a constant term,α is the coefficient for the lagged squared residuals $(\varepsilon_{t-1})^2$, γ is the coefficient for the asymmetric term that captures the impact of negative shocks $I(\varepsilon_{t-1} < 0)$is an indicator variable that equals 1 if ε_{t-1} is negative, and 0 otherwise, and β is the coefficient for the lagged conditional variance σ_{t-1}^2. The GJR-GARCH model is a valuable tool for modelling volatility in financial time series, particularly when asymmetries are present. By including an asymmetric term to capture the leverage effect, this model provides a more nuanced understanding of how different types of shocks affect volatility. Its ability to differentiate between positive and negative shocks makes it suitable for analysing financial data where such asymmetries are evident.

3.2.6. NGARCH Model

The NGARCH (Nonlinear GARCH) model, introduced by Bera and Higgins in 1993, extends the traditional GARCH framework by incorporating nonlinear elements into the conditional variance equation. This model is designed to capture more complex dynamics in financial time series that are not adequately addressed by linear GARCH models. In the NGARCH model, the conditional variance is specified as:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\,\varepsilon_{t-2}^2 + \ldots + \alpha_p\varepsilon_{t-p}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\,\sigma_{t-2}^2 + \ldots + \beta_q\sigma_{t-q}^2 \text{----------(9)}$$

Here, σ_t^2 represents the conditional variance at time t, ε_{t-i}^2 are past squared residuals, and σ_{t-j}^2 are past conditional variances. ω is a constant term, α_i are coefficients for the lagged squared residuals, and $\beta_j$are coefficients for the lagged conditional variances.The key feature of the NGARCH model is the incorporation of nonlinear transformations in the conditional variance equation. This can involve modelling the conditional variance as a function of the absolute value of past shocks or employing more complex functional forms that allow for varying degrees of nonlinear effects. This flexibility enables the NGARCH model to capture volatility clustering and asymmetries more effectively than linear models.

3.2.7. TGARCH Model

In 1994, Zakoian introduced the TGARCH (Threshold GARCH) model to address the asymmetry in volatility observed in financial markets. By incorporating threshold variables into the conditional variance equation, the TGARCH model allows for different responses to positive and negative shocks. The model is specified as follows:

$$\sigma_t^2 = \omega+\alpha_1\varepsilon_{t-1}^2+\alpha_2\,\varepsilon_{t-2}^2+\ldots+\alpha_p\varepsilon_{t-p}^2+\beta_1\sigma_{t-1}^2+\beta_2\,\sigma_{t-2}^2+\ldots+\beta_q\sigma_{t-q}^2+\gamma\varepsilon_{t-1}^2.\ \mathrm{II}(\varepsilon_{t-1}<0)\text{----(10)}$$

In this equation, σ_t^2 represents the conditional variance at time t, ε_{t-i}^2 are past squared residuals, σ_{t-j}^2 are past conditional variances, ω is a constant term, α_i are coefficients for the lagged squared residuals, $\beta_j$are coefficients for the lagged conditional variances. γ is a parameter capturing the asymmetric effect and $\mathrm{II}(\varepsilon_{t-1}<0)$ is an indicator function that equals 1 if the lagged residual ε_{t-1} is negative and 0 otherwise.By incorporating the threshold variable, the TGARCH model allows for different impacts of positive and negative shocks on the volatility, providing a more nuanced view of how volatility responds to varying market conditions. This model is particularly useful in capturing the leverage effect, where negative shocks have a greater impact on volatility than positive shocks of the same magnitude.

3.3. Diagnostic Tests:

Conducted diagnostic tests to evaluate the adequacy and predictability of the models:

The Akaike Information Criterion (AIC), The Bayesian Information Criterion (BIC) and The Hannan-Quinn Criterion (HQC) used to compare the goodness of fit of different models. These metrics help in balancing model fit with complexity, guiding the selection of the most appropriate model for volatility forecasting. While AIC tends to favour models with more parameters, BIC and HQC offer more stringent penalties, promoting model simplicity. By minimizing AIC, BIC and HQC, can select models that are both accurate and parsimonious.

4. Results Analysis

The results analysis of the empirical study on volatility predictability using various symmetric and asymmetric GARCH models reveals insightful findings across the FTSE 100, Hang Seng, NIKKEI 225, and S&P 500 indexes. Under the t-distribution, these models capture the distinct volatility dynamics of these major global stock indexes, reflecting their responses to economic events and market conditions over the past two decades. In particular, the TGARCH model consistently demonstrates superior performance in accounting for asymmetries and leverage effects, as evidenced by lower AIC, BIC, and HQC values. Additionally, utilizing the t-distribution for model estimation enhances the robustness of the volatility forecasts by accommodating the heavy tails and extreme movements observed in financial time series data.

Table 1: Descriptive statistics of Four International Indexes during 2004 to 2023 period.

Variable	FTSE 100	HANGSENG	NIKKEI225	S&P500
Mean	0.010824	0.0061719	0.023056	0.02893
Median	0.05645	0.046503	0.064472	0.06967
Minimum	-11.512	-13.582	-12.111	-12.765
Maximum	9.3842	13.407	13.235	10.957
Std. Dev.	1.1127	1.4624	1.4216	1.2072
C.V.	102.8	236.94	61.656	41.723
Skewness	-0.43601	0.058254	-0.44337	-0.5249
Ex. kurtosis	10.537	7.894	7.7374	13.161
5% Perc.	-1.6955	-2.303	-2.1938	-1.8047
95% Perc.	1.6097	2.1204	2.0817	1.6344
IQ range	1.022	1.4046	1.4091	0.97875

(Source: Statistical calculations)

The descriptive statistics for the FTSE 100, Hang Seng, Nikkei 225, and S&P 500 indexes from 2004 to 2023 reveal distinct characteristics in terms of central tendency, dispersion, and distribution shape. The S&P 500 has the highest mean return (0.02893) and median return (0.06967), indicating a generally upward trend in returns compared to the other indexes. In contrast, the Hang Seng index shows the lowest mean return (0.0061719) and median return (0.046503), suggesting relatively lower performance. The minimum and maximum values illustrate the range of returns, with the Hang Seng showing the most extreme values (-13.582 minimum and 13.407 maximum), reflecting its higher volatility. The standard deviation, a measure of return variability, further confirms this, with the Hang Seng at 1.4624, followed by the Nikkei 225 (1.4216), indicating these indexes experienced more pronounced fluctuations than the FTSE 100 (1.1127) and S&P 500 (1.2072).

Analysing the coefficients of variation (C.V.) provides additional insights into relative volatility. The Hang Seng has the highest C.V. at 236.94, indicating it has the highest relative volatility compared to its mean return. The Nikkei 225, despite having a higher standard deviation, has a lower C.V. (61.656) due to its relatively higher mean return. The skewness values indicate the asymmetry of return distributions, with the FTSE 100 (-0.43601), Nikkei 225 (-0.44337), and S&P 500 (-0.5249) all displaying negative skewness, suggesting a tendency for more frequent small gains and occasional large losses. The Hang Seng, with a skewness of 0.058254, is nearly symmetric. Kurtosis values reveal the 'tailedness' of distributions, with the S&P 500 (13.161) and FTSE 100 (10.537) having the highest excess kurtosis, indicating a higher likelihood of extreme returns compared to a normal distribution. The interquartile range (IQ range) shows that the Nikkei 225 (1.4091) and Hang Seng

(1.4046) have wider ranges between the 25th and 75th percentiles, reflecting greater spread in their middle 50% of returns compared to the FTSE 100 (1.022) and S&P 500 (0.97875).

Table 2: Unit Root Testsand ARCH Effect of Four International Indexes Returns

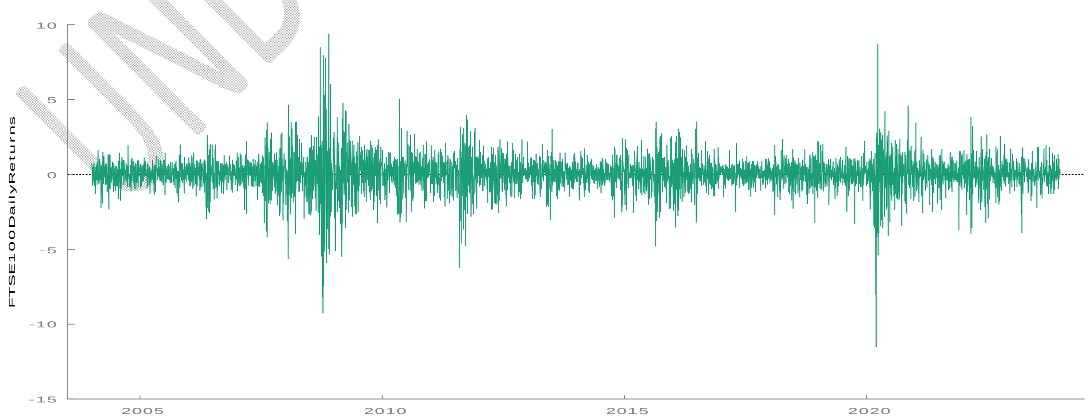
Indexes	ADF Test (12 lag)	ADF GLS Test (12 lag)	KPSS Test (10 lag)	ARCH-LMTest (5 lag)
FTSE 100	-21.0416* (0.0000)	-66.6225* (0.0000)	0.022720* (>0.1000)	965.428* (0.0000)
HANG SENG	-23.5606* (0.0000)	-54.4850* (0.0000)	0.027693* (>0.1000)	1083.88* (0.0000)
NIKKEI 225	-72.2617*(0.0000)	-49.5165*(0.0000)	0.050675*(>0.1000)	1071.17*(0.0000)
S&P 500	-23.8751*(0.0000)	-40.4292*(0.0000)	0.035506*(>0.1000)	1455.22*(0.0000)

(Source: Statistical calculations)(* 5 percent level of significance) (Probabilities in parenthesis)

The table presents the results of unit root tests (ADF and ADF GLS) and the KPSS test on the returns of four popular global stock market indexes: FTSE 100, Hang Seng, Nikkei 225, and S&P 500. The ADF and ADF GLS tests show significantly negative values with p-values of 0.0000, indicating strong rejection of the null hypothesis of a unit root for all four indexes. This suggests that the returns of these indexes are stationary. The KPSS test, which tests for stationarity, also supports this conclusion as the test statistics are very low, with p-values greater than 0.1, indicating failure to reject the null hypothesis of stationarity. Therefore, all three tests consistently show that the returns of these indexes are stationary.

In addition to stationarity, the ARCH-LM test results highlight the presence of time-varying volatility in all four indexes. The high values and p-values of 0.0000 from the ARCH-LM test indicate strong evidence of ARCH effects. This means that the volatility of returns for FTSE 100, Hang Seng, Nikkei 225, and S&P 500 changes over time and is not constant. These findings are crucial for financial modelling and forecasting, as they imply the need for models that can account for changing volatility, such as GARCH models. Understanding the stationary nature of returns and the presence of ARCH effects helps in building more accurate and reliable financial models.

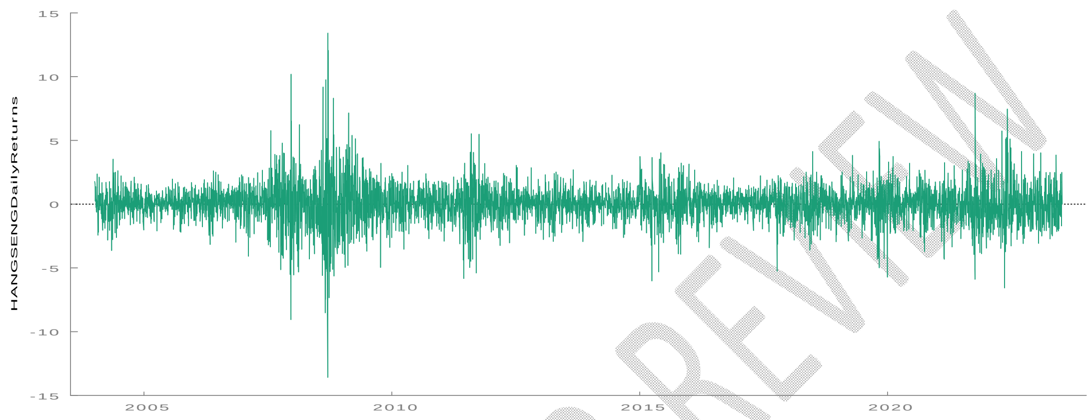
Chart 1: FTSE 100 daily returns from 2004 to 2023



(Source: Statistical calculations)

The chart 1 illustrates daily returns of a FTSE 100 index from around 2004 to 2023, highlighting significant periods of volatility. The most notable spike in volatility occurs during the 2008-2009 financial crisis, where returns exhibit large fluctuations. There are also smaller spikes around other market events, but overall, the returns show a pattern of volatility clustering, where high-volatility periods are followed by more stable ones. This time-varying nature of volatility, evident from the chart, supports the presence of ARCH effects as indicated in the earlier table. This behaviour underscores the importance of employing models like GARCH that can account for changing volatility in financial time series.

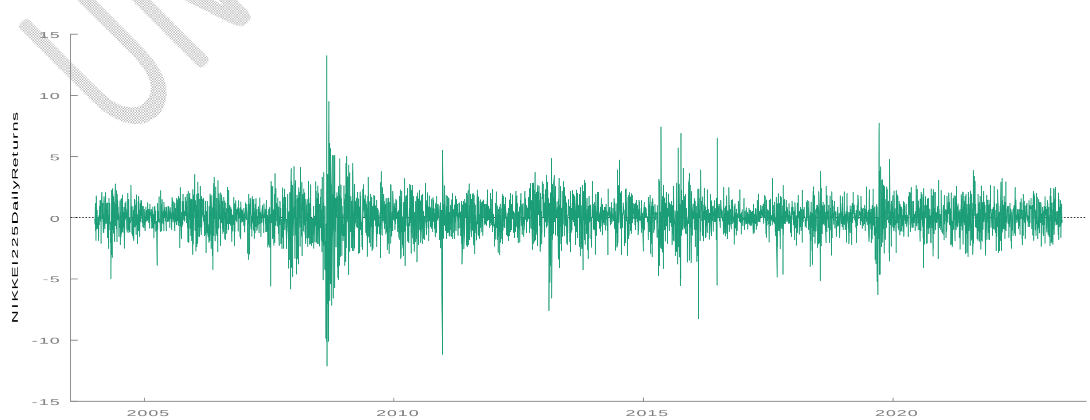
Chart 2: HANG SENG Index daily returns from 2004 to 2023



(Source: Statistical calculations)

The chart 2 displays the daily returns of the Hang Seng index from 2004 to 2023, highlighting significant volatility phases. The 2008-2009 financial crisis stands out with dramatic return fluctuations, reflecting market turmoil. Post-crisis, volatility decreases but occasional spikes occur, such as around 2011 due to the European debt crisis and in 2020 during the COVID-19 pandemic. The returns exhibit a pattern of volatility clustering, where periods of high volatility are followed by calmer phases. This pattern is indicative of ARCH effects, suggesting that the volatility of returns changes over time in response to market conditions. This visual evidence supports the need for advanced econometric models like GARCH to effectively capture and predict these dynamics, essential for risk management and financial decision-making.

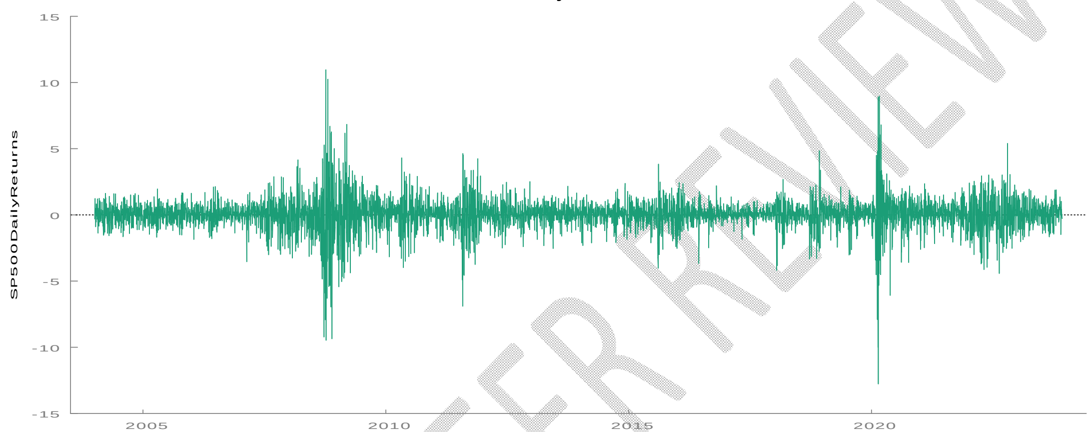
Chart 3: NIKKEI 225 Index daily returns from 2004 to 2023



(Source: Statistical calculations)

The chart 3 shows the daily returns of the Nikkei 225 index from 2004 to 2023, exhibiting significant periods of volatility. The most notable spikes occur during the 2008-2009 financial crisis, where extreme positive and negative returns highlight the market instability. Another period of increased volatility is seen around 2020, corresponding to the COVID-19 pandemic, which similarly caused sharp market fluctuations. Outside these major events, the returns display lower but still noticeable volatility, with occasional spikes likely linked to various economic and geopolitical factors. The pattern of volatility clustering—high volatility followed by periods of relative calm—indicates the presence of ARCH effects. This suggests that the volatility of the Nikkei 225 returns is time-varying, with periods of high volatility clustering together, necessitating the use of econometric models like GARCH to accurately capture and predict these dynamics in financial analysis.

Chart 4: S&P 500 Index daily returns from 2004 to 2023



(Source: Statistical calculations)

The chart 4 illustrates the daily returns of the S&P 500 index from 2004 to 2023, displaying notable periods of volatility, particularly during the 2008-2009 financial crisis and the 2020 COVID-19 pandemic. These periods are characterized by sharp spikes and drops, indicating significant market turbulence. After the financial crisis, while overall volatility decreases, there are still occasional bursts, reflecting reactions to various economic and geopolitical events. The pattern of volatility clustering—high volatility periods followed by calmer ones—suggests the presence of ARCH effects, where the volatility of returns changes over time. This time-varying nature of volatility is consistent with the need for GARCH models, which extend ARCH by modelling the persistence of volatility over time, to effectively capture and forecast these dynamics. Employing such models is crucial for accurate risk management and financial decision-making, as they account for the changing volatility inherent in financial markets.

The analysis of the daily returns for the FTSE 100, Hang Seng, Nikkei 225, and S&P 500 indexes from 2004 to 2023 reveals a consistent pattern of volatility clustering, particularly pronounced during the 2008-2009 financial crisis and the 2020 COVID-19 pandemic. Each index shows sharp spikes and drops during these periods, reflecting significant market instability. Post-crisis, all indexes exhibit reduced but still noticeable volatility with occasional bursts linked to various economic and geopolitical events. The observed time-varying nature of volatility, characteristic of ARCH effects, underscores the necessity of using GARCH models to account for the persistence and changing volatility over time. This pattern indicates that advanced econometric models like GARCH are essential for accurate financial analysis, risk management, and decision-making, as they effectively capture and predict the dynamic nature of market volatility inherent across these global indexes.

Table 3: GARCH Variants Parameter coefficients of FTSE 100 Index Returns

Model/ Parameter	**GARCH (1,1)**	**T/S GARCH (1,1)**	**GJR (1,1)**	**TGARCH (1,1)**	**NGARCH (1,1)**	**APARCH (0,1)**	**EGARCH (1,1)**
Constant	0.0505* (0.0000)	0.0499 * (0.0000)	0.0242* (0.0202)	**0.0167* (0.0000)**	0.0507* (0.0000)	0.0427* (0.0001)	0.0204* (0.0497)
Omega (ω)	0.0242* (0.0000)	0.0268* (0.0000)	0.0256* (0.0000)	**0.0260* (0.0000)**	0.0252* (0.0000)	0.8725* (0.0000)	-0.1128* (0.0000)
Alpha (α)	0.1217* (0.0000)	0.1282* (0.0000)	0.0495* (0.0000)	**0.0813* (0.0000)**	0.1275* (0.0000)	0.4061* (0.0000)	0.1368* (0.0000)
Gamma (γ)	-	-	1.0200* (0.0000)	**1.0459* (0.0000)**	-	0.2346* (0.0000)	-0.1433* (0.0000)
Beta (β)	0.8596* (0.0000)	0.8789* (0.0000)	0.8717* (0.0000)	**0.9129* (0.0000)**	0.8633* (0.0000)	-	0.9790* (0.0000)
Delta (δ)	-	-	-	-	1.7630* (0.0000)	1.3229* (0.0000)	-
AIC	13116.41	13128.79	12964.31	**12925.75**	13117.59	13734.40	12945.35
BIC	13149.04	13161.43	13003.52	**12964.91**	13156.76	13773.56	12984.51
HQC	13127.84	13140.23	12978.01	**12939.47**	13131.31	13748.12	12959.07

(Source: Statistical calculations)(* 5 percent level of significance) (Probabilities in parenthesis)

The table summarizes the parameter coefficients and fit statistics for several GARCH variants applied to the FTSE 100 Index returns, each offering a unique approach to modelling volatility. The GARCH (1,1) model presents a constant of 0.0505, Omega (ω) of 0.0242, Alpha (α) of 0.1217, and Beta (β) of 0.8596. These parameters reflect the standard approach of the GARCH model, capturing a general volatility persistence where the current volatility is influenced by past squared returns and past volatility. The T/S GARCH (1,1) model shows similar trends but with a slightly lower constant (0.0499) and Omega (0.0268), alongside a higher Alpha (0.1282) and Beta (0.8789). These variations suggest that while T/S GARCH aligns closely with the standard GARCH model, it adjusts for different volatility clustering and persistence dynamics, evidenced by the marginally higher Alpha and Beta values.

The GJR (1,1) model introduces an asymmetry effect with a significant Gamma (γ) of 1.0200, indicating that negative shocks have a greater impact on volatility than positive shocks of the same magnitude. This model also has a high Beta (0.8717), suggesting substantial persistence of volatility. The TGARCH (1,1) model exhibits a higher Gamma (γ) of 1.0459 and a higher Beta (0.9129), further emphasizing its ability to capture leverage effects, where negative returns amplify volatility more than positive returns. The NGARCH (1,1) model's high Delta (δ) of 1.7630 indicates a pronounced impact of past shocks on current volatility, while the APARCH (0,1) model's very high Omega (0.8725) and Delta (1.3229) highlight its significant asymmetric response. The EGARCH (1,1) model, with its negative Omega (-0.1128) and high Beta (0.9790), captures asymmetric effects and a unique approach to volatility modelling. Among these models, the TGARCH (1,1) achieves the lowest AIC (12925.75) and BIC (12964.91), suggesting it provides the most efficient fit for the data by balancing model complexity and goodness of fit.

Table 4: GARCH Variants Parameter coefficients of HANG SENG Index Returns

Model/ Parameter	**GARCH (1,1)**	**T/S GARCH (1,1)**	**GJR (1,1)**	**TGARCH (1,1)**	**NGARCH (1,1)**	**APARCH (1,1)**	**EGARCH (1,1)**
Constant	0.0520* (0.0004)	0.0520* (0.0004)	0.0337* (0.0253)	**0.0307* (0.0399)**	0.0520* (0.0004)	0.0316* (0.0339)	0.0331* (0.0000)
Omega (ω)	0.0117* (0.0028)	0.0175* (0.0002)	0.0175* (0.0004)	**0.0272* (0.0000)**	0.0117* (0.0064)	0.0255* (0.0000)	-0.0896* (0.0000)
Alpha (α)	0.0582* (0.0000)	0.0705* (0.0000)	0.0510* (0.0000)	**0.0653* (0.0000)**	0.0582* (0.0000)	0.0640* (0.0000)	0.1236* (0.0000)
Gamma (γ)	-	-	0.3656* (0.0000)	**0.5693* (0.0000)**	-	0.5359* (0.0000)	-0.0636* (0.0000)
Beta (β)	0.9369* (0.0000)	0.9376* (0.0000)	0.9314* (0.0000)	**0.9351* (0.0000)**	0.9369* (0.0000)	0.9344* (0.0000)	0.9870* (0.0000)
Delta (δ)	-	-	-	-	2.0011* (0.0000)	1.1608* (0.0000)	-
AIC	15904.63	15915.72	15853.57	**15843.89**	15906.63	15845.14	15850.69
BIC	15937.15	15948.23	15892.58	**15882.91**	15945.65	15890.66	15889.70
HQC	15916.04	15927.12	15867.25	**15857.58**	15920.32	15861.10	15864.37

(Source: Statistical calculations)(* 5 percent level of significance) (Probabilities in parenthesis)

The table provides the parameter coefficients for various GARCH variants applied to the Hang Seng Index returns, revealing significant differences across models. The Constant term (C) is significant for all models, with values ranging from 0.0307 in TGARCH (1,1) to 0.0520 in both GARCH (1,1) and NGARCH (1,1), indicating the presence of a consistent base level of volatility. The Omega (ω) parameter, which represents the constant part of the conditional variance, varies from 0.0117 in GARCH (1,1) and NGARCH (1,1) to 0.0272 in TGARCH (1,1). Interestingly, EGARCH (1,1) shows a negative value of -0.0896, suggesting a different approach in capturing the volatility dynamics compared to other models. The Alpha (α) parameter, measuring the impact of past squared returns on current volatility, is significant across all models, with the highest value in EGARCH (1,1) at 0.1236 and the lowest in GJR (1,1) at 0.0510, indicating varying sensitivity to past shocks.

The Gamma (γ) parameter, which captures the asymmetry in volatility, is present and significant in GJR (1,1), TGARCH (1,1), APARCH (1,1), and EGARCH (1,1). TGARCH (1,1) exhibits the highest Gamma value at 0.5693, indicating strong asymmetry in the volatility response to negative shocks, whereas EGARCH (1,1) shows a negative Gamma at -0.0636, highlighting the leverage effects. The Beta (β) parameter, indicative of volatility persistence, is high across all models, with values ranging from 0.9314 in GJR (1,1) to 0.9870 in EGARCH (1,1), signifying strong persistence in volatility. Delta (δ), specific to NARCH

(1,1) and APARCH (1,1), shows significant values of 2.0011 and 1.1608, respectively. The model fit criteria (AIC, BIC, and HQC) suggest that TGARCH (1,1) and GJR (1,1) models provide a better fit, with TGARCH (1,1) having the lowest AIC at 15843.89 and HQC at 15857.58, indicating these models' effectiveness in capturing the Hang Seng Index's volatility dynamics.

Table 5: GARCH Variants Parameter coefficients of NIKKEI 225 Index Returns

Model/ Parameter	**GARCH (1,1)**	**T/S GARCH (1,1)**	**GJR (1,1)**	**TGARCH (1,1)**	**NGARCH (1,1)**	**APARCH (1,1)**	**EGARCH (1,1)**
Constant	0.0771* (0.0000)	0.0747* (0.0000)	0.0520* (0.0005)	**0.0418* (0.0048)**	0.0762* (0.0000)	0.0416* (0.0047)	0.0460* (0.0019)
Omega (ω)	0.0391* (0.0002)	0.0454* (0.0002)	0.0623* (0.0004)	**0.0748* (0.0000)**	0.0425* (0.0000)	0.0750* (0.0000)	-0.1312* (0.0000)
Alpha (α)	0.1007* (0.0000)	0.1078* (0.0000)	0.0800* (0.0000)	**0.1030* (0.0000)**	0.1093* (0.0000)	0.1032* (0.0000)	0.1881* (0.0000)
Gamma (γ)	-	-	0.5425* (0.0000)	**0.7434* (0.0000)**	-	0.7450* (0.0000)	-0.1253* (0.0000)
Beta (β)	0.8806* (0.0000)	0.8936* (0.0000)	0.8591* (0.0000)	**0.8790* (0.0000)**	0.8885* (0.0000)	0.8792* (0.0000)	0.9605* (0.0000)
Delta (δ)	-	-	-	-	1.4095* (0.0000)	0.9857* (0.0000)	-
AIC	15794.34	15792.39	15688.69	**15647.02**	15790.03	15649.00	15662.37
BIC	15826.82	15824.87	15727.67	**15685.99**	15829.00	15694.47	15701.35
HQC	15805.73	15803.78	15702.37	**15660.69**	15803.70	15664.95	15676.05

(Source: Statistical calculations)(* 5 percent level of significance) (Probabilities in parenthesis)

The table 5 presents the parameter coefficients for various GARCH variants applied to NIKKEI 225 index returns, demonstrating how each model captures the volatility dynamics of the index. The Constant term is significant across all models, with the highest value in GARCH (1,1) at 0.0771 and the lowest in TGARCH (1,1) at 0.0418. The Omega (ω) parameter, representing the Constant volatility component, ranges from 0.0391 in GARCH (1,1) to 0.0748 in TGARCH (1,1), with APARCH (1,1) showing a slightly higher value at 0.0750, indicating a notable base level of volatility across these models. Alpha (α), the coefficient for past squared returns, is significant across all models, with values ranging from 0.0800 in GJR (1,1) to 0.1881 in EGARCH (1,1), highlighting a strong sensitivity to past shocks, particularly in the EGARCH model.

Gamma (γ), which captures the asymmetry in volatility, shows significant variation among the models where it is applicable. GJR (1,1) and TGARCH (1,1) have values of 0.5425 and 0.7434, respectively, while APARCH (1,1) and EGARCH (1,1) exhibit values of 0.7450 and -0.1253, respectively, indicating the presence of leverage effects. Beta (β), representing the persistence of volatility, is consistently high and significant across all models, with the highest value in EGARCH (1,1) at 0.9605, suggesting strong volatility persistence. Delta (δ), specific to NGARCH (1,1) and APARCH (1,1), shows significant values of 1.4095 and

0.9857, respectively. The model fit criteria (AIC, BIC, and HQC) suggest that TGARCH (1,1) and APARCH (1,1) models provide a better fit with lower values, with TGARCH (1,1) having the lowest AIC and HQC at 15647.02 and 15660.69, respectively, indicating their superior performance in capturing the volatility dynamics of the NIKKEI 225 index returns.

Table 6: GARCH Variants Parameter coefficients of S&P 500 Index Returns

Model/Parameter	GARCH (1,1)	T/S GARCH (1,1)	GJR (1,1)	TGARCH (1,1)	NGARCH (1,1)	APARCH (0,1)	EGARCH (1,1)
Constant	0.0818* (0.0000)	0.0814* (0.0000)	0.0550* (0.0000)	**0.0460* (0.0000)**	0.0818* (0.0000)	0.0728* (0.0000)	0.0534* (0.0000)
Omega (ω)	0.0165* (0.0000)	0.0244* (0.0000)	0.0196* (0.0000)	**0.0297* (0.0000)**	0.0160* (0.0000)	1.0794* (0.0000)	-0.1250* (0.0000)
Alpha (α)	0.1312* (0.0000)	0.1383* (0.0000)	0.0549* (0.0000)	**0.0933* (0.0000)**	0.1293* (0.0000)	0.4606* (0.0000)	0.1516* (0.0000)
Gamma (γ)	-	-	1.0049* (0.0000)	**1.1391* (0.0000)**	-	0.2327* (0.0003)	-0.1693* (0.0000)
Beta (β)	0.8631* (0.0000)	0.8777* (0.0000)	0.8713* (0.0000)	**0.9036* (0.0000)**	0.8620* (0.0000)	-	0.9789* (0.0000)
Delta (δ)	-	-	-	-	2.0709* (0.0000)	1.1832* (0.0000)	-
AIC	13087.93	13117.41	12921.62	**12870.59**	13089.85	14010.28	12899.18
BIC	13120.55	13150.03	12960.77	**12909.74**	13128.99	14049.43	12938.32
HQC	13099.35	13128.84	12935.34	**12884.31**	13103.56	14024.00	12912.89

(Source: Statistical calculations)(* 5 percent level of significance) (Probabilities in parenthesis)

The table 6 outlines the parameter coefficients for various GARCH variants applied to the S&P 500 index returns, revealing insights into the volatility dynamics of the index. Across all models, the Constant term is significant, with values ranging from 0.0460 in TGARCH (1,1) to 0.0818 in GARCH (1,1) and NGARCH (1,1), indicating consistent base-level volatility. The Omega (ω) parameter, which represents the Constant component of the conditional variance, ranges from 0.0160 in NGARCH (1,1) to 0.0297 in TGARCH (1,1), except for APARCH (0,1) where it is notably higher at 1.0794, and in EGARCH (1,1) where it is negative at -0.1250. The Alpha (α) parameter, which measures the impact of past squared returns on current volatility, is significant across all models, with the highest value in APARCH (0,1) at 0.4606 and the lowest in GJR (1,1) at 0.0549, indicating varying degrees of sensitivity to past shocks across models.

Gamma (γ), which captures asymmetry in volatility, is present and significant in GJR (1,1), TGARCH (1,1), APARCH (0,1), and EGARCH (1,1). The highest Gamma value is observed in TGARCH (1,1) at 1.1391, indicating strong asymmetry, while the lowest is in EGARCH (1,1) at -0.1693, suggesting the presence of leverage effects. The Beta (β) parameter, indicating volatility persistence, is high across all models, with values ranging from 0.8620 in NARCH (1,1) to 0.9789 in EGARCH (1,1), signifying strong persistence in volatility. Delta (δ), specific to NARCH (1,1) and APARCH (0,1), shows significant values of 2.0709 and 1.1832, respectively. The model fit criteria (AIC, BIC, and HQC) indicate that TGARCH

(1,1) and GJR (1,1) models provide a better fit, with TGARCH (1,1) having the lowest AIC at 12870.59 and HQC at 12884.31, suggesting these models' superiority in capturing the S&P 500's volatility dynamics effectively.

The above comparative analysis of various GARCH model variants applied to different stock indices, highlighting their distinct approaches to capturing volatility dynamics. For the FTSE 100 Index, the standard GARCH (1,1) model offers a balanced fit with a Constant of 0.0505, Omega (ω) of 0.0242, Alpha (α) of 0.1217, and Beta (β) of 0.8596. The T/S GARCH (1,1) model shows similar results but with slightly lower Constant and Omega values, coupled with marginally higher Alpha and Beta, indicating refined adjustments for volatility clustering. The GJR (1,1) and TGARCH (1,1) models incorporate asymmetry effects, with the TGARCH (1,1) model displaying the best fit metrics (lowest AIC and BIC), capturing leverage effects effectively through a higher Gamma (γ) of 1.0459 and substantial Beta (β) of 0.9129.

In contrast, models applied to the Hang Seng Index and NIKKEI 225 Index show varying parameter values and fit statistics. For the Hang Seng Index, the TGARCH (1,1) and GJR (1,1) models provide a better fit with low AIC and HQC values, highlighting their effectiveness in capturing volatility dynamics. Similarly, the NIKKEI 225 Index results reveal TGARCH (1,1) and APARCH (1,1) as superior models, with TGARCH (1,1) again showing the lowest AIC and HQC. For the S&P 500 Index, the APARCH (0,1) model stands out with the highest Alpha (α) of 0.4606, reflecting strong sensitivity to past shocks. Overall, TGARCH (1,1) consistently demonstrates strong performance across indices, balancing asymmetry, volatility persistence, and model fit criteria(Wang et al., 2022).

5. Conclusion

This study comprehensively examines the volatility dynamics of major global stock indexes—FTSE 100, Hang Seng Index, NIKKEI 225, and S&P 500—over a 20-year period using various GARCH models. The analysis reveals significant volatility clustering across all indexes, with pronounced spikes during critical market events such as the 2008 financial crisis and the COVID-19 pandemic. Among the models evaluated, the TGARCH (Threshold GARCH) model consistently demonstrates superior performance in capturing asymmetries and leverage effects, particularly in the FTSE 100 and Hang Seng Index(Wang et al., 2022). This model's ability to differentiate between positive and negative shocks provides a more nuanced understanding of volatility dynamics. The APARCH (Asymmetric Power ARCH) model also shows notable effectiveness, especially for the S&P 500 Index, by addressing asymmetries in volatility response.

The study highlights the limitations of traditional GARCH models in explaining the asymmetric nature of financial volatility. Advanced models like TGARCH and APARCH offer significant improvements by incorporating asymmetric effects, thereby enhancing the accuracy of volatility forecasts. In conclusion, the findings underscore the necessity of employing sophisticated econometric models to capture the complex volatility patterns in global financial markets. The TGARCH and APARCH models, in particular, offer valuable insights for financial analysis, risk management, and decision-making, by providing a more accurate representation of market volatility and its response to various shocks.

References

Abdul Manap, T. A., & Kassim, S. H. (2011). Long memory properties and asymmetric effects of emerging equity market: Evidence from Malaysia. Journal of Risk Finance, 12(5), 356–370. https://doi.org/10.1108/15265941111176109.

Aggarwal, K., & Jha, M. K. (2023). Day-of-the-week effect and volatility in stock returns: evidence from the Indian stock market. Managerial Finance, 49(9), 1438–1452. https://doi.org/10.1108/MF-01-2023-0010.

Akinlaso, M. I., Robbana, A., & Mohamed, N. (2021). Volatility transmission within financial markets during the COVID-19 pandemic: are faith-based investors well off in Tunisia? *Journal of Islamic Accounting and Business Research*, *13*(1), 98–113. https://doi.org/10.1108/JIABR-12-2020-0388

AL-Najjar, D. M. (2016). Modelling and Estimation of Volatility Using ARCH/GARCH Models in Jordan's Stock Market. Asian Journal of Finance & Accounting, 8(1), 152. https://doi.org/10.5296/ajfa.v8i1.9129.

Bera, A. K., and Higgins, M. L. (1993), "ARCH Models: Properties, Estimation and Testing," Journal of Economic Surveys, Vol. 7, No. 4, 307-366. https://doi.org/10.1111/j.1467-6419.1993.tb00170.x

Bhargava, V., & Konku, D. (2023). Impact of exchange rate fluctuations on US stock market returns. Managerial Finance, 49(10), 1535–1557. https://doi.org/10.1108/MF-08-2022-0387.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 234, 25–37. https://doi.org/10.1016/j.jeconom.2023.02.001.

Bollerslev, T., & Ghysels, E. (2014). *Autoregressive Heteroscedasticity*. *14*(2), 139–151.

Ding, Z., Granger, C. W. J., & Engle, R. F. (1993). A long memory property of stock market returns and a new model. Journal of Empirical Finance, 1(1), 83–106. https://doi.org/10.1016/0927-5398(93)90006-D.

Duppati, G., Kumar, A. S., Scrimgeour, F., & Li, L. (2017). Long memory volatility in Asian stock markets. *Pacific Accounting Review*, *29*(3), 423–442. https://doi.org/10.1108/par-02-2016-0009.

Ederington, L. H., & Guan, W. (2005). Forecasting volatility. *Journal of Futures Markets*, *25*(5), 465–490. https://doi.org/10.1002/fut.20146.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50(4), 987-1007. https://doi.org/10.2307/1912773.

Engle, R. F., & Lee, G. G. J. (1999). A long-run and short-run component model of stock return volatility. Oxford Bulletin of Economics and Statistics, 61(2), 215-239. https://doi.org/10.1111/1468-0084.0610s1215.

Engle, R. F., Ghysels, E., &Sohn, B. (2013). Stock market volatility and macroeconomic fundamentals. Review of Economics and Statistics, 95(3), 776-797. https://doi.org/10.1162/REST_a_00200.

Ghorbel, A., &Boujelbene, Y. (2013). Contagion effect of the oil shock and US financial crisis on the GCC and BRIC countries. International Journal of Energy Sector Management, 7(4), 430–447. https://doi.org/10.1108/IJESM-04-2012-0002.

Ghorbel, A., &Jeribi, A. (2021). Contagion of COVID-19 pandemic between oil and financial assets: the evidence of multivariate Markov switching GARCH models. Journal of Investment Compliance, 22(2), 151–169. https://doi.org/10.1108/joic-01-2021-0001

GLOSTEN, L. R., JAGANNATHAN, R., & RUNKLE, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. The Journal of Finance, 48(5), 1779–1801. https://doi.org/10.1111/j.1540-6261.1993.tb05128.x.

Granger, C. W. J., & Engle, F. (1995). A long memory property of stock market returns and a new model. Journal of Empirical Finance, 2(1), 98. https://doi.org/10.1016/0927-5398(95)90049-7.

Gupta, H. (2023). Analysing volatility patterns in emerging markets: symmetric or asymmetric models? Journal of Economic and Administrative Sciences. https://doi.org/10.1108/JEAS-07-2023-0186

Hassan, A. (2023). External energy security elements and the riskiness of clean energy stocks: a volatility analysis. Sustainability Accounting, Management and Policy Journal, 14(2), 396–419. https://doi.org/10.1108/SAMPJ-09-2022-0484.

Hentschel, L. (1995). All in the family Nesting symmetric and asymmetric GARCH models. Journal of Financial Economics, 39(1), 71–104. https://doi.org/10.1016/0304-405X(94)00821-H

Jiang, W. (2012). Using the GARCH model to analyze and predict the different stock markets. 1–28. file:///C:/Users/acer/Documents/PhD/Reference/WEI JIANG.pdf%0Ahttp://www.diva-portal.org/smash/get/diva2:576215/FULLTEXT02.

Kumar, D., & Maheswaran, S. (2013). Asymmetric long memory volatility in the PIIGS economies. Review of Accounting and Finance, 12(1), 23–43. https://doi.org/10.1108/14757701311295818.

Lim, C. M., & Sek, S. K. (2013). Comparing the Performances of GARCH-type Models in Capturing the Stock Market Volatility in Malaysia. *Procedia Economics and Finance*, *5*(13), 478–487. https://doi.org/10.1016/s2212-5671(13)00056-7

Liu, H. C., & Hung, J. C. (2010). Forecasting volatility and capturing downside risk of the Taiwanese futures markets under the financial tsunami. Managerial Finance, 36(10), 860–875. https://doi.org/10.1108/03074351011070233

Manera, M., & Forte, G. (2005). Forecasting Volatility in European Stock Markets with Non-Linear GARCH Models. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.351606

Mușetescu, R. C., Grigore, G. E., & Nicolae, S. (2022). The Use of GARCH Autoregressive Models in Estimating and Forecasting the Crude Oil Volatility. European Journal of Interdisciplinary Studies, 14(1–6), 13–38. https://doi.org/10.24818/ejis.2022.02.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59(2), 347–370. https://doi.org/10.2307/2938260.

Paul, S., & Sharma, P. (2018). Quantile forecasts using the Realized GARCH-EVT approach. Studies in Economics and Finance, 35(4), 481–504. https://doi.org/10.1108/SEF-09-2016-0236.

Priya, P., & Sharma, C. (2023). COVID-19 related stringencies and financial market volatility: sectoral evidence from India. Journal of Financial Economic Policy, 15(1), 16–34. https://doi.org/10.1108/JFEP-05-2022-0136.

Radha, S., & M. Thenmozhi. (2024). Forecasting short term interest rates using ARMA, ARMA-GARCH and ARMA-EGARCH models, Indian Institute of Technology Madras, Chennai. (n.d.). 1–14.

Rajvanshi, V., Santra, A., & Basu, S. (2019). Implied Volatility and Predictability of GARCH Models. SSRN Electronic Journal, 797. https://doi.org/10.2139/ssrn.3341695.

Raji, J. O., Idowu Abdulkadir, R., & Badru, B. O. (2018). Dynamic relationship between Nigeria-US exchange rate and crude oil price. African Journal of Economic and Management Studies, 9(2), 213–230. https://doi.org/10.1108/AJEMS-06-2017-0124.

Rossetti, N., Nagano, M. S., & Meirelles, J. L. F. (2017). A behavioral analysis of the volatility of interbank interest rates in developed and emerging countries. *Journal of Economics, Finance and Administrative Science*, *22*(42), 99–128. https://doi.org/10.1108/JEFAS-02-2017-0033.

Schwert, G. W. (1989). Why does stock market volatility change over time? The Journal of Finance, 44(5), 1115-1153. https://doi.org/10.1111/j.1540-6261.1989.tb02647.x.

Sharma, P., & Vipul. (2015). Forecasting stock index volatility with GARCH models: international evidence. Studies in Economics and Finance, 32(4), 445–463. https://doi.org/10.1108/SEF-11-2014-0212.

Sharma, S., Aggarwal, V., &Yadav, M. P. (2021). Comparison of linear and non-linear GARCH models for forecasting volatility of select emerging countries. Journal of Advances in Management Research, 18(4), 526–547. https://doi.org/10.1108/JAMR-07-2020-0152

Smolović, J. C., Božović, M. L., & Vujošević, S. (2017). GARCH models in value at risk estimation: Empirical evidence from the Montenegrin stock exchange. *Economic Research-EkonomskaIstrazivanja*, *30*(1), 477–498. https://doi.org/10.1080/1331677X.2017.1305773.

Taylor, S. J. (1986). Modelling financial time series. Wiley.

Ugurlu, E., Thalassinos, E., &Muratoglu, Y. (2014). Modeling Volatility in the Stock Markets using GARCH Models: European Emerging Economies and Turkey. International Journal of Economics and Business Administration, II(Issue 3), 72–87. https://doi.org/10.35808/ijeba/49.

Walther, T. (2017). Expected shortfall in the presence of asymmetry and long memory. Pacific Accounting Review, 29(2), 132–151. https://doi.org/10.1108/par-06-2016-0063.

Wang, Z., Xing, T., & Wang, X. (2024). Economic uncertainty and stock market asymmetric volatility: analysis based on the asymmetric GARCH-MIDAS model. International Journal of Emerging Markets. https://doi.org/10.1108/IJOEM-05-2023-0841.

Wang, Y., Xiang, Y., Lei, X., & Zhou, Y. (2022). Volatility analysis based on GARCH-type models: Evidence from the Chinese stock market. Economic Research-EkonomskaIstrazivanja , 35(1), 2530–2554. https://doi.org/10.1080/1331677X.2021.1967771.

Zakoian, J.-M. (1994). Threshold Heteroskedastic models. Journal of Economic Dynamics and Control, 18(5), 931–955. https://doi.org/10.1016/0165-1889(94)90039-6.