

# The double inversion technique in the Somé Blaise Abbo method applied to Schrödinger-type fractional-order evolution equations

## **Abstract :**

In this **article** we have used the new double inversion technique in the Somé Blaise Abbo(SBA) method to solve some fractional order partial differential equations (edp) in the Caputo sense of the Schrödinger type by integrating all boundary conditions.

**Keywords :** SOME BLAISE ABBO method ; double inversion ; fractional Caputo derivative ; fractional Riemann Liouville integral ; linear Schrödinger edp ; Neumann condition

**AMS classification codes :** 65Nxx, 65Lxx, 65Mxx, 65Qxx

## **1 Introduction**

In this **manuscript** we are interested in solving Schrödinger problems with initial and Neumann conditions. These types of problems model quantum, probabilistic and many other phenomena. Indeed, some reference works present in the literature for example in 2016 S.O. Edeki, G.O. Akinlabi and S.A. Adeosun in [1] used the modified differential transformation method to determine solutions of linear Schrödinger equations of fractional order. In 2007 S. Wang and M. Xu in [20] made a generalized fractional study of the Schrödinger equations. In 2004 C. BESSE in [8] applied numerical methods and artificial boundary conditions for linear and

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nonlinear Schrödinger equations and modeling irregularities in terrestrial ionospheric plasma. This work is organized in four sections, the second of which presents some fractional tools, the third describes the solution strategy, the fourth determines the analytical results of fractional Schrödinger-type models and the last section gives the conclusion.

## 2 Preliminaries

In this section, we define the fractional derivative and the fractional integral. For definitions of the Mittag Leffler, Gamma and Bêta functions, we refer to the documents [15] et [17].

### 2.1 Caputo derivatives

**Definition 2.1** Caputo's fractional derivatives on the right denoted by  $c_{D_{d^+}^\alpha}$  and on the left symbolized by  $c_{D_{s^-}^\alpha}$  are established respectively by the following formulas

$$c_{D_{d^+}^\alpha} p(x) = \frac{1}{\Gamma(j - \alpha)} \int_d^x (x - u)^{j-\alpha-1} p^{(j)}(u) du, \alpha > 0 \quad (2.1)$$

$$c_{D_{s^-}^\alpha} p(x) = \frac{1}{\Gamma(j - \alpha)} \int_x^s (u - x)^{j-\alpha-1} p^{(j)}(u) du, \alpha > 0 \quad (2.2)$$

where  $j = [\alpha] + 1$ ,  $[\alpha]$  defines the integer part of  $\alpha$ .

### 2.2 Integral in the Riemann-Liouville sense

**Definition 2.2** If  $p$  is defined in  $C([0, +\infty])$  then that of order  $\alpha > 0$  of the function  $p$  is defined by

$$I^\alpha(p)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x - u)^{\alpha-1} p(u) du, t > 0 \quad (2.3)$$

$$I^0(p)(x) = p(x) \quad (2.4)$$

This defines the left-hand integral in the Riemann-Liouville sense.

## 3 Description of the technique

The description of this strategy is based on the approach taken by the inventors in [4], and the technique that will be adapted to our fractional problem is defined in a Banach space denoted B, and is formulated as follows :

$$\begin{cases} D_t^\alpha u = L_{xx}(u) + f(x, t), t \geq 0, 0 < \alpha \leq 1, 0 \leq x \leq a \\ u^{(i)}(x, 0) = w_i, i = 0, 1, \dots, j - 1 \\ \frac{\partial u(0, t)}{\partial x} = h(t) \\ \frac{\partial u(a, t)}{\partial x} = h(t) \end{cases} \quad (3.1)$$

### 3 Description of the technique

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Where

$D_t^\alpha u$  presents the fractional derivative of order  $\alpha$  in Caputo's sense.

$u \in B$  solution of the problem.

In posing

$$\begin{cases} L_t(\cdot) = D_t^\alpha u \\ L_t^{-1}(\cdot) = I^\alpha(\cdot) \\ L_x(\cdot) = \frac{\partial(\cdot)}{\partial x^a} \\ L_x^{-1}(\cdot) = \int_0^a ds \end{cases} \quad (3.2)$$

Where  $L^{-1}$  is an invertible operator in the Adomian sense.

The equation (3.1) will give us the following two equations :

$$L_t u = L_x(L_x u) + f(x, t) \quad (3.3)$$

where

$$L_x(L_x u) = L_t(u) - f(x, t) \quad (3.4)$$

Then the canonical form associated with (3.3) is

$$L_t^{-1} L_t u = L_t^{-1} L_x(L_x u) + L_t^{-1} f(x, t) \quad (3.5)$$

$$L_t^{-1} L_t u = u(t) - \sum_{i=0}^{j-1} \frac{u^{(i)}(0)}{i!} t^i, \quad z = \sum_{i=0}^{j-1} \frac{u^{(i)}(0)}{i!} t^i$$

$j = [\alpha] + 1$ ,  $[\alpha]$  defines the integer part of  $\alpha$ . So the equation (3.5) becomes

$$u(x, t) = z + L_t^{-1} L_x(L_x u) + L_t^{-1} f(x, t) \quad (3.6)$$

By composing  $L_x^{-1}$  with equation (3.4) we obtain

$$0 = (-h(t) + g(t)) + L_x^{-1} (L_t u + f(x, t)) \quad (3.7)$$

By adding (3.6) and (3.7) we get

$$u(x, t) = z + L_t^{-1} L_x(L_x u) + L_t^{-1} f(x, t) + (-h(t) + g(t)) a + L_x^{-1} L_t u + L_x^{-1} f(x, t) \quad (3.8)$$

Successive approximations applied to (3.8) give

$$u^k(x, t) = z + (-h(t) + g(t)) a + L_x^{-1} f(x, t) + L_t^{-1} f(x, t) + L_t^{-1} L_x(L_x u^k) + L_x^{-1} L_t u^k, \quad k \geq 1 \quad (3.9)$$

Applying the SBA algorithm to (3.9) we have

$$\begin{cases} u_0^1 = z + (-h(t) + g(t)) a + L_x^{-1} f(x, t) + L_t^{-1} f(x, t) \\ u_1^1 = L_t^{-1} L_x(L_x u_0^1) + L_x^{-1} L_t u_0^1 \\ u_2^1 = L_t^{-1} L_x(L_x u_1^1) + L_x^{-1} L_t u_1^1 \\ \cdot \\ \cdot \\ \cdot \\ u_k^1 = L_t^{-1} L_x(L_x u_{k-1}^1) + L_x^{-1} L_t u_{k-1}^1 \end{cases} \quad (3.10)$$

Then, if the analytical solution  $u$  exists, we obtain

$$u = \sum_{k=0}^{+\infty} u_k^1$$

## 4 Applications

**Example 4.1** Consider the following equation

$$\begin{cases} D_t^\alpha H(x, t) + i \frac{\partial^2 H(x, t)}{\partial x^2} = 0, 0 < x \leq \pi, 0 < \alpha \leq 1, t \geq 0 \\ H(x, 0) = e^{2ix} \\ \frac{\partial H(0, t)}{\partial x} = 0 \\ \frac{\partial H(\pi, t)}{\partial x} = 0 \end{cases} \quad (4.1)$$

Taking  $H(x, t) = u(x, t) + iv(x, t)$  and replacing it in equation (4.1) we get

$$D_t^\alpha (u(x, t) + iv(x, t)) + i \frac{\partial^2 (u(x, t) + iv(x, t))}{\partial x^2} = 0 \quad (4.2)$$

By identifying the real and imaginary parts, we have

$$\begin{cases} D_t^\alpha u(x, t) = \frac{\partial^2 v(x, t)}{\partial x^2} \\ D_t^\alpha v(x, t) = -\frac{\partial^2 u(x, t)}{\partial x^2} \\ u(x, 0) = \cos 2x \\ \frac{\partial u(0, t)}{\partial x} = 0 \\ \frac{\partial u(\pi, t)}{\partial x} = 0 \\ v(x, 0) = \sin 2x \\ \frac{\partial v(0, t)}{\partial x} = 0 \\ \frac{\partial v(\pi, t)}{\partial x} = 0 \end{cases} \quad (4.3)$$

The 2 equations of the system (4.3) can be rewritten as follows

$$\begin{cases} L_t(u(x, t)) = L_{xx}^2(v) \\ L_t(v(x, t)) = -L_{xx}^2(u) \end{cases} \quad (4.4)$$

where

$$\begin{cases} L_t(\cdot) = D_t^\alpha(\cdot) \\ L_t^{-1}(\cdot) = I^\alpha(\cdot) \\ L_{xx}^2(\cdot) = \frac{\partial^2}{\partial x^2}(\cdot) \\ L_{xx}^{-2}(\cdot) = \int_0^\pi \int_0^\pi (\cdot) dz dz \end{cases} \quad (4.5)$$

Applying the inverse of  $L_t(\cdot)$  to equation (4.4) we get

$$\begin{cases} u(x, t) = u(x, 0) + L_t^{-1}(L_{xx}^2(v)) \\ v(x, t) = v(x, 0) + L_t^{-1}(-L_{xx}^2(u)) \end{cases} \quad (4.6)$$

By adding the inverse of  $L_x(\cdot)$  to equation (4.4), we obtain

$$\begin{cases} 0 = L_x^{-1}(L_t(u)) \\ 0 = L_x^{-1}(L_t(v)) \end{cases} \quad (4.7)$$

By adding member by member (4.6) and (4.7)

$$\begin{cases} u(x, t) = u(x, 0) + (L_{xx}^{-2}(L_t(v)) + L_t^{-1}(L_{xx}^2(v))) \\ v(x, t) = v(x, 0) + (L_{xx}^{-2}(L_t(u)) + L_t^{-1}(-L_{xx}^2(u))) \end{cases} \quad (4.8)$$

The successive approximation applied to (4.8) gives us

$$\begin{cases} u^k(x, t) = u^k(x, 0) + (L_{xx}^{-2}(L_t(v^k)) + L_t^{-1}(L_{xx}^2(v^k))), k \geq 1 \\ v^k(x, t) = v^k(x, 0) + (L_{xx}^{-2}(L_t(u^k)) + L_t^{-1}(-L_{xx}^2(u^k))), k \geq 1 \end{cases} \quad (4.9)$$

Then the SBA algorithm will be

$$\begin{cases} u_0^1(x, t) = u^1(x, 0) \\ u_1^1(x, t) = (-L_x^{-1}(L_t(v_0^1)) + L_t^{-1}(L_{xx}^2(v_0^1))) \\ \cdot \\ \cdot \\ u_n^k(x, t) = (-L_x^{-1}(L_t(v_{n-1}^k)) + L_t^{-1}(L_{xx}^2(v_{n-1}^k))) \end{cases} \quad (4.10)$$

$$\begin{cases} v_0^1(x, t) = v^1(x, 0) \\ v_1^1(x, t) = L_x^{-1}(L_t(u_0^1)) + L_t^{-1}(-L_{xx}^2(u_0^1)) \\ \cdot \\ \cdot \\ v_n^k(x, t) = (L_x^{-1}(L_t(u_{n-1}^k)) + L_t^{-1}(-L_{xx}^2(u_{n-1}^k))) \end{cases} \quad (4.11)$$

Determining solutions

$$\begin{aligned} u_0^1(x, t) &= (\cos 2x) \\ v_0^1(x, t) &= \sin 2x \\ u_1^1(x, t) &= (L_x^{-1}(L_t(v_0^1)) + L_t^{-1}(L_{xx}^2(v_0^1))) \end{aligned}$$

$$L_t^{-1}(L_{xx}^2(v_0^1)) = \frac{-4 \sin 2x}{\Gamma(\alpha + 1)} t^\alpha$$

$$L_x^{-1}(L_t(v_0^1)) = \int_0^\pi (D_t^\alpha(v_0^1(z, t))) dz dz$$

$$D_t^\alpha(v_0^1(z, t)) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - s)^{-\alpha} \frac{\partial v_0^1(x, s)}{\partial s} ds$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} (0) ds$$

$$= 0.$$

So

$$u_1^1(x, t) = \frac{-4 \sin 2x}{\Gamma(\alpha+1)} t^\alpha$$

$$v_1^1(x, t) = (L_x^{-1}(L_t(u_0^1)) + L_t^{-1}(-L_{xx}^2(u_0^1)))$$

$$L_t^{-1}(-L_{xx}^2(u_0^1)) = I^\alpha \left( -\frac{\partial^2 u_0^1(x, t)}{\partial x^2} \right) = \frac{4 \cos 2x}{\Gamma(\alpha+1)} t^\alpha$$

$$L_x^{-1}(L_t(u_0^1)) = \int_0^\pi (D_t^\alpha(u_0^1(z, t))) dz dz$$

$$D_t^\alpha(u_0^1(z, t)) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{\partial u_0^1(x, s)}{\partial s} ds$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} (0) ds$$

$$= 0.$$

So

$$v_1^1(x, t) = \frac{4 \cos 2x}{\Gamma(\alpha+1)} t^\alpha.$$

$$u_2^1(x, t) = (-L_{xx}^{-2}(L_t(v_1^1)) + L_t^{-1}(L_{xx}^2(v_1^1)))$$

$$L_t^{-1}(L_{xx}^2(v_1^1)) = I^\alpha \left( \frac{\partial^2 v_1^1(x, t)}{\partial x^2} \right) = \frac{-4^2 \sin 2x}{\Gamma(2\alpha+1)} t^{2\alpha}$$

$$L_x^{-1}(L_t(v_1^1)) = \int_0^\pi (D_t^\alpha(v_1^1(z, t))) dz dz$$

$$D_t^\alpha(v_1^1(z, t)) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{\partial v_1^1(x, s)}{\partial s} ds$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \left[ \frac{4\alpha \cos 2x}{\Gamma(\alpha+1)} s^{\alpha-1} \right] ds$$

$$D_t^\alpha(v_1^1(z, t)) = \frac{4\alpha \cos 2x}{\Gamma(1-\alpha)\Gamma(\alpha+1)} \int_0^t (t-s)^{-\alpha} s^{\alpha-1} ds$$

Let's say  $s = yt$ ,  $t = \frac{s}{y}$  and  $ds = t dy$ .

So it becomes

$$D_t^\alpha(v_1^1(z, t)) = \frac{4\alpha \cos 2z}{\Gamma(1-\alpha)\Gamma(\alpha+1)} \int_0^1 y^{\alpha-1} (1-y)^{-\alpha} ds$$

$$D_t^\alpha(v_1^1(z, t)) = \frac{4\alpha \cos 2z}{\Gamma(1-\alpha)\Gamma(\alpha+1)} B(1-\alpha, 2-\alpha)$$

$$D_t^\alpha(v_1^1(z, t)) = \frac{(4\alpha \cos 2z)(\Gamma(1 - \alpha))(\Gamma(2 - \alpha))}{(\Gamma(1 - \alpha))(\Gamma(\alpha + 1))(\Gamma(3 - 2\alpha))}$$

$$D_t^\alpha(v_1^1(z, t)) = \frac{4\alpha(\Gamma(2 - \alpha))}{(\Gamma(\alpha + 1))(\Gamma(3 - 2\alpha))} \cos 2z.$$

So

$$L_x^{-1}(L_t(v_1^1)) = \frac{4\alpha\Gamma(2 - \alpha)}{(\Gamma(\alpha + 1))(\Gamma(3 - 2\alpha))} \int_0^\pi \cos 2z dz$$

$$L_x^{-1}(L_t(v_1)) = 0$$

. Hence

$$u_2^1(x, t) = \frac{-4^2 \cos 2x}{\Gamma(2\alpha + 1)} t^{2\alpha}$$

$$v_2^1(x, t) = (L_{xx}^{-2}(L_t(u_1^1)) + L_t^{-1}(-L_{xx}^2(u_1^1)))$$

$$L_t^{-1}(-L_{xx}^2(u_1^1)) = I^\alpha \left( -\frac{\partial^2 u_1^1(x, t)}{\partial x^2} \right) = \frac{-4^2 \sin 2x}{\Gamma(2\alpha + 1)} t^\alpha$$

$$L_x^{-1}(L_t(u_1^1)) = \int_0^\pi (D_t^\alpha(u_1^1(z, t))) dz dz$$

$$D_t^\alpha(u_1^1(z, t)) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - s)^{-\alpha} \frac{\partial u_1^1(x, s)}{\partial s} ds$$

$$= \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - s)^{-\alpha} \left[ \frac{-4\alpha \sin 2x}{\Gamma(\alpha + 1)} s^{\alpha-1} \right] ds$$

$$D_t^\alpha(u_1^1(z, t)) = -\frac{4\alpha \sin 2x}{2\Gamma(1 - \alpha)\Gamma(\alpha + 1)} \int_0^t (t - s)^{-\alpha} s^{\alpha-1} ds.$$

Using the same procedure as above for  $D_t^\alpha(v_1(z, t))$ , we obtain

$$D_t^\alpha(u_1^1(z, t)) = -\frac{4\alpha(\Gamma(2 - \alpha))}{(\Gamma(\alpha + 1))(\Gamma(3 - 2\alpha))} \sin 2z$$

$$L_x^{-1}(L_t(u_1^1)) = -\frac{4\alpha\Gamma(2 - \alpha)}{(\Gamma(\alpha + 1))(\Gamma(3 - 2\alpha))} \int_0^\pi \sin 2z dz dz$$

$$D_t^\alpha(u_1^1(z, t)) = 0.$$

Hence

$$v_2^1(x, t) = \frac{-4^2 \sin 2x}{\Gamma(2\alpha + 1)} t^{2\alpha}$$

So, step by step, we can deduce that

$$\begin{cases} L_x^{-1}(L_t(u_{n-1}^1)) = 0, n \geq 1 \\ L_x^{-1}(L_t(v_{n-1}^1)) = 0, n \geq 1 \end{cases}$$

$$\left\{ \begin{aligned} H^1 &= \sum_{n=0}^{+\infty} (u_n^1 + iv_n^1) \\ &= (\cos 2x + i \sin 2x) + 4i(\cos 2x + i \sin 2x) \frac{t^\alpha}{\Gamma(\alpha + 1)} + (4i)^2(\cos 2x + i \sin 2x) \frac{t^{2\alpha}}{\Gamma(2\alpha + 1)} \\ &\quad + (4i)^3(\cos 2x + i \sin 2x) \frac{t^{3\alpha}}{\Gamma(3\alpha + 1)} + \dots + \\ &= e^{2ix} \left[ 1 + \frac{[4it^\alpha]^1}{\Gamma(\alpha + 1)} + \frac{[4it^\alpha]^2}{\Gamma(2\alpha + 1)} + \frac{[4it^\alpha]^3}{\Gamma(3\alpha + 1)} + \dots + \right] \\ &= e^{2ix} \sum_{n=0}^{+\infty} \frac{[4it^\alpha]^n}{\Gamma(n\alpha + 1)} \\ H^1 &= e^{2ix} E_\alpha(4it^\alpha) \end{aligned} \right.$$

As by hypothesis we have  $N_j(u, v) = 0, j = 1, 2$ , therefore the solution of the problem is

$$H(x, t) = e^{2ix} E_\alpha(2it^\alpha).$$

**Example 4.2** Consider the following equation

$$\left\{ \begin{aligned} D_t^\alpha H(x, t) + i \frac{\partial^2 H(x, t)}{\partial x^2} &= 0 \\ H(x, 0) &= 1 + \cosh 2x \\ \frac{\partial H(0, t)}{\partial x} &= 0 \\ \frac{\partial H(\pi, t)}{\partial x} &= 0 \end{aligned} \right. \quad (4.12)$$

Proceeding in the same way as the previous example, we get

$$\left\{ \begin{aligned} u_0^1(x, t) &= u^1(x, 0) = 1 + \cosh 2x \\ u_1^1(x, t) &= 0 \\ u_2^1(x, t) &= \frac{-4^2 \cosh 2x}{\Gamma(2\alpha + 1)} t^{2\alpha} \\ u_3^1(x, t) &= 0 \\ &\vdots \\ &\vdots \\ &\vdots \\ u_{2n}^1(x, t) &= \frac{(-1)^{n-1} \cosh 2x}{\Gamma(3\alpha + 1)} (4t^\alpha)^n \end{aligned} \right. \quad \text{and} \quad \left\{ \begin{aligned} v_0^1(x, t) &= v^1(x, 0) = 0 \\ v_1^1(x, t) &= \frac{4 \cosh 2x}{\Gamma(\alpha + 1)} t^\alpha \\ v_2^1(x, t) &= 0 \\ v_3^1(x, t) &= \frac{-4^3 \cosh 2x}{\Gamma(3\alpha + 1)} t^{3\alpha} \\ &\vdots \\ &\vdots \\ &\vdots \\ v_{2n+1}^1(x, t) &= \frac{-4^3 \cosh 2x}{\Gamma(3\alpha + 1)} t^{3\alpha} \end{aligned} \right.$$

Then the approximate solution at the first iteration is

$$\left\{ \begin{aligned} H_1 &= \sum_{n=0}^{+\infty} (u_n^1 + iv_n^1) \\ &= (1 + \cosh 2x + i(0)) + (0 + 4i \cosh 3x) \frac{t^\alpha}{\Gamma(\alpha + 1)} + (-4^2 \cosh 2x + i(0)) \frac{t^{2\alpha}}{\Gamma(2\alpha + 1)} \\ &\quad + (0 - 4^3 i \cosh 2x) \frac{t^{3\alpha}}{\Gamma(3\alpha + 1)} \\ &= 1 + \cosh 2x + \cosh 2x \frac{[4it^\alpha]^1}{\Gamma(\alpha + 1)} + \cosh 2x \frac{[4it^\alpha]^2}{\Gamma(2\alpha + 1)} + \cosh 2x \frac{[4it^\alpha]^3}{\Gamma(3\alpha + 1)} + \dots \\ &= [1 + \cosh 2x \sum_{n=0}^{+\infty} \frac{[4it^\alpha]^n}{\Gamma(n\alpha + 1)}] \\ H_1 &= 1 + \cosh 2x E_\alpha(4it^\alpha) \end{aligned} \right.$$

Hence the analytical solution

$$H(x, t) = 1 + \cosh 2x E_\alpha(4it^\alpha).$$

## 5 Conclusion

In this article we have used the gymnastics of double inversion resulting from that of SBA to determine the solutions of fractional-order Schrödinger equations in the sense of Caputo with initial and Neumann conditions. The results thus obtained converge with those determined by other analytical methods [1]. This strategy is well suited to fractional-order models.

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