The double inversion technique in the Somé Blaise Abbo method applied to Schrödinger-type fractional-order evolution equations

Abstract :
In this paper we have used the new double inversion technique in the Somé Blaise Abbo(SBA) method to solve some fractional order partial differential equations (edp) in the Caputo sense of the Schrödinger type by integrating all boundary conditions.

Keywords : SOME BLAISE ABBO method ; double inversion ; fractional Caputo derivative ; fractional Riemmann Liouville integral ; linear Schrödinger edp ; Neumann condition

AMS classification codes : 65Nxx, 65Lxx, 65Mxx, 65Qxx

1 Introduction

In this project we are interested in solving Schrödinger problems with initial and Neumann conditions. These types of problems model quantum, probabilistic and many other phenomena.Indeed, some reference works present in the literature for example in 2016 S.O. Edeki, G.O. Akinlabi and S.A. Adeosun in [1] used the modified differential transformation method to determine solutions of linear Schrödinger equations of fractional order. In 2007 S. Wang and M. Xu in [20] made a generalized fractional study of the Schrödinger equations. In 2004 C. BESSE in [8] applied numerical methods and artificial boundary conditions for linear and nonlinear Schrödinger equations and modeling irregularities in terrestrial ionospheric plasma. This work is organized in four sections, the second of which presents some fractional tools, the third describes the solution strategy, the fourth determines the analytical results of fractional Schrödinger-type models and the last section gives the conclusion.

2 Preliminaries

In this section, we define the fractional derivative and the fractional integral. For definitions of the Mittag Leffler, Gamma and Bêta functions, we refer to the documents [15] et [17].

2.1 Caputo derivatives

Definition 2.1 *Caputo's fractional derivatives on the right denoted by* ${}^{c}D^{\alpha}_{d+}$ *and on the left symbolized by* ${}^{c}D^{\alpha}_{c-}$ *are established respectively by the following formulas :*

$$ {}^{c}D^{\alpha}_{d+}p(x) = \frac{1}{\Gamma(j-\alpha)}\int_{d}^{x}(x-u)^{j-\alpha-1}p^{(j)}(u)du,\ \alpha>0 \tag{2.1}$$

$$ {}^{c}D^{\alpha}_{s-}p(x) = \frac{1}{\Gamma(j-\alpha)}\int_{x}^{s}(u-x)^{j-\alpha-1}p^{(j)}(u)du,\ \alpha>0 \tag{2.2}$$

where $j=[\alpha]+1$, $[\alpha]$ defines the integer part of α

2.2 Integral in the Riemann-Liouville sense

Definition 2.2 *If* p *is defined in* $C([0,+\infty])$ *then that of order* $\alpha>0$ *of the function* p *is defined by :*

$$ I^{\alpha}(p)(x) = \frac{1}{\Gamma(\alpha)}\int_{0}^{x}(x-u)^{\alpha-1}p(u)du,\ t>0 \tag{2.3}$$

$$ I^{0}(p)(x) = p(x) \tag{2.4}$$

This defines the left-hand integral in the Riemmann-Liouville sense.

3 Description of the technique

The description of this strategy is based on the approach taken by the inventors in [4], and the technique that will be adapted to our fractional problem is defined in a Banach space denoted B, and is formulated as follows :

$$\begin{cases} D_t^{\alpha}u = L_{xx}(u) + f(x,t),\ t\geq 0,\ 0<\alpha\leq 1,\ 0\leq x\leq a \\ u^{(i)}(x,0) = w_i,\ \ i=0,1,...,j-1 \\ \dfrac{\partial u(0,t)}{\partial x} = h(t) \\ \dfrac{\partial u(a,t)}{\partial x} = h(t) \end{cases} \tag{3.1}$$

With :
$D_t^\alpha u$ presents the fractional derivative of order α in Caputo's sense ;
$u \in B$ solution of the problem.
In posing :

$$\begin{cases} L_t(.) = D_t^\alpha u \\ L_t^{-1}(.) = I^\alpha(.) \\ L_x(.) = \dfrac{\partial(.)}{\partial x} \\ L_x^{-1}(.) = \displaystyle\int_0^a ds \end{cases} \tag{3.2}$$

Where L^{-1} is an invertible operator in the Adomian sense.
The equation (3.1) will give us the following two equations :

$$L_t u = L_x(L_x u) + f(x,t), \tag{3.3}$$

where

$$L_x(L_x u) = L_t(u) - f(x,t). \tag{3.4}$$

Then the canonical form associated with (3.3) is :

$$L_t^{-1} L_t u = L_t^{-1} L_x(L_x u) + L_t^{-1} f(x,t) \tag{3.5}$$

$L_t^{-1} L_t u = u(t) - \displaystyle\sum_{i=0}^{j-1} \frac{u^{(i)}(0)}{i!} t^i$, $z = \displaystyle\sum_{i=0}^{j-1} \frac{u^{(i)}(0)}{i!} t^i$
$j = [\alpha] + 1$, $[\alpha]$ defines the integer part of α. So the equation (3.5) becomes :

$$u(x,t) = z + L_t^{-1} L_x(L_x u) + L_t^{-1} f(x,t) \tag{3.6}$$

By composing L_x^{-1} with equation (3.4) we obtain :

$$0 = (-h(t) + g(t)) + L_x^{-1}\left(L_t u + f(x,t)\right) \tag{3.7}$$

By adding (3.6) and (3.7) we get :

$$u(x,t) = z + L_t^{-1} L_x(L_x u) + L_t^{-1} f(x,t) + (-h(t) + g(t))\, a + L_x^{-1} L_t u + L_x^{-1} f(x,t) \tag{3.8}$$

Successive approximations applied to (3.8) give :

$$u^k(x,t) = z + (-h(t) + g(t))\, a + L_x^{-1} f(x,t) + L_t^{-1} f(x,t) + L_t^{-1} L_x(L_x u^k) + L_x^{-1} L_t u^k,\ k \geq 1 \tag{3.9}$$

Applying the SBA algorithm to (3.9) we have :

$$\begin{cases} u_0^1 = z + (-h(t) + g(t)) + L_x^{-1} f(x,t) + L_t^{-1} f(x,t) \\ u_1^1 = L_t^{-1} L_x(L_x u_0^1) + L_x^{-1} L_t u_0^1 \\ u_2^1 = L_t^{-1} L_x(L_x u_1^1) + L_x^{-1} L_t u_1^1 \\ . \\ . \\ . \\ u_k^1 = L_t^{-1} L_x(L_x u_{k-1}^1) + L_x^{-1} L_t u_{k-1}^1 \end{cases} \tag{3.10}$$

Then, if the analytical solution u exists, we obtain :

$$u = \sum_{k=0}^{+\infty} u_k^1$$

4 Applications

Example 4.1 *Consider the following equation :*

$$\begin{cases} D_t^\alpha H(x,t) + i\dfrac{\partial^2 H(x,t)}{\partial x^2} = 0, 0 < x \leq \pi, 0 < \alpha \leq 1, t \geq 0 \\ H(x,0) = e^{2ix} \\ \dfrac{\partial H(0,t)}{\partial x} = 0 \\ \dfrac{\partial H(\pi,t)}{\partial x} = 0 \end{cases} \tag{4.1}$$

Taking $H(x,t) = u(x,t) + iv(x,t)$ and replacing it in equation (4.1) we get :

$$D_t^\alpha (u(x,t) + iv(x,t)) + i\frac{\partial^2 (u(x,t) + iv(x,t))}{\partial x^2} = 0 \tag{4.2}$$

By identifying the real and imaginary parts, we have :

$$\begin{cases} D_t^\alpha u(x,t) = \dfrac{\partial^2 v(x,t)}{\partial x^2} \\ D_t^\alpha v(x,t) = -\dfrac{\partial^2 u(x,t)}{\partial x^2} \\ u(x,0) = \cos 2x \\ \dfrac{\partial u(0,t)}{\partial x} = 0 \\ \dfrac{\partial u(\pi,t)}{\partial x} = 0 \\ v(x,0) = \sin 2x \\ \dfrac{\partial v(0,t)}{\partial x} = 0 \\ \dfrac{\partial v(\pi,t)}{\partial x} = 0 \end{cases} \tag{4.3}$$

The 2 equations of the system (4.3) can be rewritten as follows :

$$\begin{cases} L_t(u(x,t)) = L_{xx}^2(v) \\ L_t(v(x,t)) = -L_{xx}^2(u) \end{cases} \tag{4.4}$$

with :

$$\begin{cases} L_t(.) = D_t^\alpha(.) \\ L_t^{-1}(.) = I^\alpha(.) \\ L_{xx}^2(.) = \dfrac{\partial^2}{\partial x^2}(.) \\ L_{xx}^{-2}(.) = \displaystyle\int_0^\pi \int_0^\pi (.)dzdz \end{cases} \tag{4.5}$$

Applying the inverse of $L_t(.)$ to equation (4.4) we get :

$$\begin{cases} u(x,t) = u(x,0) + L_t^{-1}(L_{xx}^2(v)) \\ v(x,t) = v(x,0) + L_t^{-1}(-L_{xx}^2(u)) \end{cases} \tag{4.6}$$

By adding the inverse of $L_x(.)$ to equation (4.4), we obtain :

$$\begin{cases} 0 = L_x^{-1}(L_t(u)) \\ 0 = L_x^{-1}(L_t(v)) \end{cases} \tag{4.7}$$

By adding member by member (4.6) and (4.7)

$$\begin{cases} u(x,t) = u(x,0) + \left(L_{xx}^{-2}(L_t(v)) + L_t^{-1}(L_{xx}^2(v))\right) \\ v(x,t) = v(x,0) + \left(L_{xx}^{-2}(L_t(u)) + L_t^{-1}(-L_{xx}^2(u))\right) \end{cases} \tag{4.8}$$

The successive approximation applied to (4.8) gives us :

$$\begin{cases} u^k(x,t) = u^k(x,0) + \left(L_{xx}^{-2}(L_t(v^k)) + L_t^{-1}(L_{xx}^2(v^k))\right), k \geq 1 \\ v^k(x,t) = v^k(x,0) + \left(L_{xx}^{-2}(L_t(u^k)) + L_t^{-1}(-L_{xx}^2(u^k))\right), k \geq 1 \end{cases} \tag{4.9}$$

Then the SBA algorithm will be :

$$\begin{cases} u_0^1(x,t) = u^1(x,0) \\ u_1^1(x,t) = \left(-L_x^{-1}(L_t(v_0^1)) + L_t^{-1}(L_{xx}^2(v_0^1))\right) \\ . \\ . \\ . \\ u_n^k(x,t) = \left(-L_x^{-1}(L_t(v_{n-1}^k)) + L_t^{-1}(L_{xx}^2(v_{n-1}^k))\right) \end{cases} \tag{4.10}$$

$$\begin{cases} v_0^1(x,t) = v^1(x,0) \\ v_1^1(x,t) = L_x^{-1}(L_t(u_0^1)) + L_t^{-1}(-L_{xx}^2(u_0^1)) \\ . \\ . \\ . \\ v_n^k(x,t) = \left(L_x^{-1}(L_t(u_{n-1}^k)) + L_t^{-1}(-L_x^1(u_{n-1}^k))\right) \end{cases} \tag{4.11}$$

Determining solutions :

$$u_0^1(x,t) = (\cos 2x)$$

$$v_0^1(x,t) = \sin 2x$$

$$u_1^1(x,t) = \left(L_x^{-1}(L_t(v_0^1)) + L_t^{-1}(L_{xx}^2(v_0^1))\right)$$

$$L_t^{-1}(L_{xx}^2(v_0^1)) = \frac{-4\sin 2x}{\Gamma(\alpha+1)}t^{\alpha}$$

$$
\begin{aligned}
L_x^{-1}(L_t(v_0^1)) &= \int_0^{\pi} \left(D_t^{\alpha}(v_0^1(z,t))\right) dzdz \\
D_t^{\alpha}(v_0^1(z,t)) &= \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{\partial v_0^1(x,s)}{\partial s} ds \\
&= \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} (0) ds \\
&= 0.
\end{aligned}
$$

So

$$
\begin{aligned}
u_1^1(x,t) &= \frac{-4\sin 2x}{\Gamma(\alpha+1)} t^{\alpha} \\
v_1^1(x,t) &= \left(L_x^{-1}(L_t(u_0^1)) + L_t^{-1}(-L_{xx}^2(u_0^1))\right) \\
L_t^{-1}(-L_{xx}^2(u_0^1)) &= I^{\alpha}\left(-\frac{\partial^2 u_0^1(x,t)}{\partial x^2}\right) = \frac{4\cos 2x}{\Gamma(\alpha+1)} t^{\alpha} \\
L_x^{-1}(L_t(u_0^1)) &= \int_0^{\pi} \left(D_t^{\alpha}(u_0^1(z,t))\right) dzdz \\
D_t^{\alpha}(u_0^1(z,t)) &= \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{\partial u_0^1(x,s)}{\partial s} ds \\
&= \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} (0) ds \\
&= 0.
\end{aligned}
$$

So

$$
v_1^1(x,t) = \frac{4\cos 2x}{\Gamma(\alpha+1)} t^{\alpha}.
$$

$$
\begin{aligned}
u_2^1(x,t) &= \left(-L_{xx}^{-2}(L_t(v_1^1)) + L_t^{-1}(L_{xx}^2(v_1^1))\right) \\
L_t^{-1}(L_{xx}^2(v_1^1)) &= I^{\alpha}\left(\frac{\partial^2 v_1^1(x,t)}{\partial x^2}\right) = \frac{-4^2\sin 2x}{\Gamma(2\alpha+1)} t^{2\alpha} \\
L_x^{-1}(L_t(v_1^1)) &= \int_0^{\pi} \left(D_t^{\alpha}(v_1^1(z,t))\right) dzdz \\
D_t^{\alpha}(v_1^1(z,t)) &= \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{\partial v_1^1(x,s)}{\partial s} ds \\
&= \frac{1}{\Gamma(1-\alpha)} \int^t (t-s)^{-\alpha} \left[\frac{4\alpha\cos 2x}{\Gamma(\alpha+1)} s^{\alpha-1}\right] ds \\
D_t^{\alpha}(v_1^1(z,t)) &= \frac{4\alpha\cos 2x}{\Gamma(1-\alpha)\Gamma(\alpha+1)} \int_0^t (t-s)^{-\alpha} s^{\alpha-1} ds
\end{aligned}
$$

Let's ask $s = yt$, $t = \frac{s}{y}$ and ds=tdy.

So it becomes :

$$D_t^\alpha(v_1^1(z,t)) = \frac{4\alpha\cos 2z}{\Gamma(1-\alpha)\Gamma(\alpha+1)}\int_0^1 y^{\alpha-1}(1-y)^{-\alpha}ds$$

$$D_t^\alpha(v_1^1(z,t)) = \frac{4\alpha\cos 2z}{\Gamma(1-\alpha)\Gamma(\alpha+1)}B(1-\alpha,2-\alpha)$$

$$D_t^\alpha(v_1^1(z,t)) = \frac{(4\alpha\cos 2z)(\Gamma(1-\alpha))(\Gamma(2-\alpha))}{(\Gamma(1-\alpha))(\Gamma(\alpha+1))(\Gamma(3-2\alpha))}$$

$$D_t^\alpha(v_1^1(z,t)) = \frac{4\alpha(\Gamma(2-\alpha))}{(\Gamma(\alpha+1))(\Gamma(3-2\alpha))}\cos 2z.$$

So

$$L_x^{-1}(L_t(v_1^1)) = \frac{4\alpha\Gamma(2-\alpha)}{(\Gamma(\alpha+1))(\Gamma(3-2\alpha))}\int_0^\pi \cos 2z dz$$

$$L_x^{-1}(L_t(v_1)) = 0, because\ the\ integral\ of\ cos2x\ on\ [0,\pi]\ is\ zero.$$

Hence

$$u_2^1(x,t) = \frac{-4^2\cos 2x}{\Gamma(2\alpha+1)}t^{2\alpha}$$

$$v_2^1(x,t) = \left(L_{xx}^{-2}(L_t(u_1^1)) + L_t^{-1}(-L_{xx}^2(u_1^1))\right)$$

$$L_t^{-1}(-L_{xx}^2(u_1^1)) = I^\alpha\left(-\frac{\partial^2 u_1^1(x,t)}{\partial x^2}\right) = \frac{-4^2\sin 2x}{\Gamma(2\alpha+1)}t^\alpha$$

$$L_x^{-1}(L_t(u_1^1)) = \int_0^\pi \left(D_t^\alpha(u_1^1(z,t))\right)dzdz$$

$$D_t^\alpha(u_1^1(z,t)) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-s)^{-\alpha}\frac{\partial u_1^1(x,s)}{\partial s}ds$$

$$= \frac{1}{\Gamma(1-\alpha)}\int^t (t-s)^{-\alpha}\left[\frac{-4\alpha\sin 2x}{\Gamma(\alpha+1)}s^{\alpha-1}\right]ds$$

$$D_t^\alpha(u_1^1(z,t)) = -\frac{4\alpha\sin 2x}{2\Gamma(1-\alpha)\Gamma(\alpha+1)}\int_0^t (t-s)^{-\alpha}s^{\alpha-1}ds.$$

Using the same procedure as above for $D_t^\alpha(v_1(z,t))$, we obtain :

$$D_t^\alpha(u_1^1(z,t)) = -\frac{4\alpha(\Gamma(2-\alpha))}{(\Gamma(\alpha+1))(\Gamma(3-2\alpha))}\sin 2z$$

$$L_x^{-1}(L_t(u_1^1)) = -\frac{4\alpha\Gamma(2-\alpha)}{(\Gamma(\alpha+1))(\Gamma(3-2\alpha))}\int_0^\pi \sin 2zdzdz$$

$$D_t^\alpha(u_1^1(z,t)) = 0, because\ the\ integrale\ of\ sin2x\ in\ [0,\pi]\ is\ zero.$$

Hence

$$v_2^1(x,t) = \frac{-4^2 \sin 2x}{\Gamma(2\alpha+1)} t^{2\alpha}$$

So, step by step, we can deduce that :

$$\begin{cases} L_x^{-1}(L_t(u_{n-1}^1)) = 0, n \geq 1 \\ L_x^{-1}(L_t(v_{n-1}^1)) = 0, n \geq 1 \end{cases}$$

$$\begin{cases} H^1 = \sum_{n=0}^{+\infty}(u_n^1 + iv_n^1) \\ = (\cos 2x + i\sin 2x) + 4i(\cos 2x + i\sin 2x)\frac{t^\alpha}{\Gamma(\alpha+1)} + (4i)^2(\cos 2x + i\sin 2x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \\ +(4i)^3(\cos 2x + i\sin 2x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + ... + \\ = e^{2ix}[1 + \frac{[4it^\alpha]^1}{\Gamma(\alpha+1)} + \frac{[4it^\alpha]^2}{\Gamma(2\alpha+1)} + \frac{[4it^\alpha]^3}{\Gamma(3\alpha+1)} + ... +] \\ = e^{2ix}\sum_{n=0}^{+\infty}\frac{[4it^\alpha]^n}{\Gamma(n\alpha+1)} \\ H^1 = e^{2ix}E_\alpha(4it^\alpha) \end{cases}$$

As by hypothesis we have $N_j(u,v) = 0, j = 1,2$, therefore the solution of the problem is :

$$H(x,t) = e^{2ix}E_\alpha(2it^\alpha)$$

Example 4.2 *Consider the following equation :*

$$\begin{cases} D_t^\alpha H(x,t) + i\frac{\partial^2 H(x,t)}{\partial x^2} = 0 \\ H(x,0) = 1 + cosh2x \\ \frac{\partial H(0,t)}{\partial x} = 0 \\ \frac{\partial H(\pi,t)}{\partial x} = 0 \end{cases} \tag{4.12}$$

Proceeding in the same way as the previous example, we get :

$$\begin{cases} u_0^1(x,t) = u^1(x,0) = 1 + cosh2x \\ u_1^1(x,t) = 0 \\ u_2^1(x,t) = \frac{-4^2 cosh2x}{\Gamma(2\alpha+1)}t^{2\alpha} \\ u_3^1(x,t) = 0 \\ . \\ . \\ . \\ u_{2n}^1(x,t) = \frac{(-1)^{n-1}cosh2x}{\Gamma(3\alpha+1)}(4t^\alpha)^n \end{cases} \quad and \quad \begin{cases} v_0^1(x,t) = v^1(x,0) = 0 \\ v_1^1(x,t) = \frac{4cosh2x}{\Gamma(\alpha+1)}t^\alpha \\ v_2^1(x,t) = 0 \\ v_3^1(x,t) = \frac{-4^3 cosh2x}{\Gamma(3\alpha+1)}t^{3\alpha} \\ . \\ . \\ . \\ v_{2n+1}^1(x,t) = \frac{-4^3 cosh2x}{\Gamma(3\alpha+1)}t^{3\alpha} \end{cases}$$

Then the approximate solution at the first iteration is :

$$\begin{cases} H_1 = \sum_{n=0}^{+\infty}(u_n^1 + iv_n^1) \\ = (1+\cosh 2x + i(0)) + (0 + 4i\cosh 3x)\dfrac{t^\alpha}{\Gamma(\alpha+1)} + (-4^2\cosh 2x + i(0))\dfrac{t^{2\alpha}}{\Gamma(2\alpha+1)} \\ +(0 - 4^3 i\cosh 2x)\dfrac{t^{3\alpha}}{\Gamma(3\alpha+1)} \\ = 1 + \cosh 2x + \cosh 2x\dfrac{[4it^\alpha]^1}{\Gamma(\alpha+1)} + \cosh 2x\dfrac{[4it^\alpha]^2}{\Gamma(2\alpha+1)} + \cosh 2x\dfrac{[4it^\alpha]^3}{\Gamma(3\alpha+1)} + ... \\ = [1 + \cosh 2x \sum_{n=0}^{+\infty}\dfrac{[4it^\alpha]^n}{\Gamma(n\alpha+1)}] \\ H_1 = 1 + \cosh 2x E_\alpha(4it^\alpha) \end{cases}$$

Hence the analytical solution :

$$H(x,t) = 1 + \cosh 2x E_\alpha(4it^\alpha)$$

5 Conclusion

In this article we have used the gymnastics of double inversion resulting from that of SBA to determine the solutions of fractional-order Schrödinger equations in the sense of Caputo with initial and Neumann conditions. The results thus obtained converge with those determined by other analytical methods. This strategy is well suited to fractional-order models.

Références

[1] **S.O. Edeki, G.O. Akinlabi and S.A. Adeosun.** *Analytic and Numerical Solutions of Time-Fractional Linear Schrödinger Equation . Published by RGN Publications, Vol. 7, No. 1, pp. 1–10, 2016.*

[2] **Blaise SOME.** *SBA method for solving environmental mathematical models in the environmental . European University Publihing, 2018.*

[3] **BRAHIM Tellab.** *Solving fractional differential equations . Thesis defended on 11/01/2018. Constantine-1 Mentouri Brothers University .*

[4] **B.Abbo** *New numerical algorithm for solving ordinary differential equations (ODE) and non linear partial differential equations(PDE). Doctorat thesis unique . University of Ouagadougou, January 2007, UFR/ SEA, Mathematics and Cumputer Science Departement(Burkina Faso).*

[5] **B.ABBO, O. SO, G. BARRO & B. SOME** *A new Numerical algorithm for solving nonlinear partial differential equations with initial and Boundary conditions. Far East J. Appl. Math. 28 (1) (2007), 37-52.*

[6] **Fatemeh NOROUZI and Gaston M. N' GUÉRÉKATA** *A new study of fractional order financial system via homotopy analysis. Analele University Oradea. Fasc. Matematica, Tom XXVII(2020), Issue No. 1, 141-152.*

[7] **Christophe BESSE** *Méthodes numériques et conditions aux limites artificielles pour les équations de Schrdinger linéaires et non linéaires et modélisation d'irrégularités du plasma ionosphérique terrestre .Thesis defended on 08/12/2004. Université Paul Sabatier - Toulouse III, France.*

[8] **Germain KABORE, Bakari Abbo, Windjiré SOME, Ousséni SO and Blaise SOME,** *Solving a fractional evolution equation in the sense of caputo-hadamard with cauchy and boundary conditions by sba method, Advances in Differential Equations and Control Processes 2024 http ://dx.doi.org/10.17654/0974324324001 Volume 31, Number 1, 2024, Pages 1-14.*

[9] **Germain KABORE, KÉRÉ Moumini, Windjiré SOME, Ousséni SO and Blaise SOME,** *Solving some fractional equations, in the sense of Riemann-Liouville, of Navier-Stokes by the numerical method SBA plus, International Journal of Numerical Methods and Applications 23(2) (2023), 209-228. http ://dx.doi.org/10.17654/0975045223012 Published Online : August 24, 2023 .*

[10] **Germain KABORE, Windjiré SOME, Moumini KÉRÉ, Ousséni SO, Blaise SOME.** *Solving Some Fractional Ordinary Differential Equations by SBA Method. Journal of Mathematics Research; Vol. 15, No. 1; Pages 47-56; February 2023. doi :10.5539/jmr.v15n1pxx.*

[11] KEHAILI ABDELKADER (2021), *Numerical resolution of a certain type of partial equations of any order .* Doctoral thesis . University of DJILLALI LIABES, ALGERIA.

[12] BIEREBO Piercy Fare (2023), *Conttribution to the solution of some partial integro differential equations of usual and fractional order by numerical methods ADM, SBA and its variants.* Memory presented in view of obtaining a Master's degree . University of MARIEN NGOUABI.

[13] NSIMBA NDAMBA Higelin (2023), *Numerical solution Generation for certain types linear and nonlinear equations of fraction order*. Dissertation presented to obtaining a Master's is degree. University of MARIEN NGOUAI , CONGO.

[14] **S. Wang and M. Xu.** *Generalized fractional Schrödinger equation with space-time fractional derivatives, J. Math. Phys. 48 (2007), 043502.*