An appraisal of theDesign Sustainability of Solar Water Heating Systemsvia Cosine Similarity Index

Abstract

This study aimed to assess the sustainability of solar water heating systems (SWHSs) to identify the sustainability indicators necessary for improvement. It involves the development of a fuzzified multi-criteria decision model (MCDM) using cosine similarity index for the sustainability assessment of SWHSs namely; integrated collector storage (ICS), thermosyphon (TS), active open-loop (AOL) and active closed-loop (ACL). The article was able to identify the indicators necessary for improvement following the sustainability assessment of SWHSs in domestic and industrial applications. The framework for the sustainability assessment of the SWHSs included the traditional sustainability indicators such as economic (EC), environmental (EN), and social (SO), and peculiar sustainability indicators namely manufacturing (MA), maintenance (MN), reliability (RE), and life-cycle (LC).This study employed conceptual design developed based on working principle of SWHSs and responses of design experts for sustainability data. For future research, practical evaluation, and more sustainability indicators is recommended for increased analysis depth on the sustainability of the systems.The establishment of sustainability indicators and sub-indicators towards the assessment of SWHSs using MCDM considering the indicators identified for the benefit of energy planning from the manufacturer to the end user.This research focus is on sustainability using the computational strength of the fuzzified MCDM method to assess the sustainability of SWHSs using traditional and peculiar sustainability indicators. The highest economic indicator was in ACL at 53% and the lowest in TS at 42%. This shows that it is important for sustainable economic ease and growth, across the domestic and importantly the industrial sector that the rate of heating using SWHSs should be improved for economic sustainability. The highest environmental indicator was in ICS at 54% and the lowest in TS at 39%. The social indicator was highest in ICS with 43% and joint lowest in TS and AOL at 30%.

Keywords: Sustainability assessment, Sustainability indicators, Solar water heating systems, fuzzified multi-criteria decision model, Cosine similarity measure, Euclidean distance.

1. Introduction

Hot water is essential in human daily living. It is requiredfor hygienic purposes domestically, and product processing industrially (Ogueke *et al.*, 2009). Though, there are dissatisfying situations regarding high cost of energy required to produce hot water. These situations includeinadequate power generation, distribution, and environmental concernsthat has driven the need for sustainable alternatives. Solar energy application for water heating in residential and commercial buildings have become more feasible technically and economically (Rutz *et al.*, 2019).It can operate independently or serve as a pre-heater for gas or electric heaters. Sustainable development requires improvement in socio-technological systems and technological innovations; therefore, it is essential to investigate the sustainability of solar water heating systems.This research aims to identify the sustainability indicators necessary for improvement, to assist domestic and industrial end users, in the process of adopting aSolar Water Heating System (SWHS), to ensure better energy planning decisions

Sustainability assessment can be used to provide decision-makers with a process to operationalize sustainability foruse within decision-making processes for sustainable development, with an evaluation of international to rural integrated nature-society systems in short and long-term visions to determine actions that should or not be taken to make a system or society sustainable (Lindfors, 2021; Halla *et al.*, 2020; Ness *et al.*, 2007). The sustainability assessment of a system is best achieved classifying the system into four stages namely pre-manufacturing, manufacturing, usage, and post-usage (Olabanji and Mpofu, 2020b; Badurdeen*et al.*, 2018). It involves the establishment of indicators that identifies active situations and prioritize sustainable development (Ramos, 2019). The sustainability indicators are the measuring instruments for sustainability assessment (Batalhao*et al.*, 2019). They eliminate complexity, improve the quality, and assist in making better and efficient decisions due to its un-ambiguous data compilationand easy accessibility (Hai *et al.*, 2014; Fredericks, 2012).The establishment of indicators can be complicated, becausethere are no generally accepted sets, making it challenging to identify indicators to include or exclude(Batalhao*et al.*, 2019; Parris and Kates, 2003). Indicators can be selected based on similar area of application as stated by Lindfors, (2021), expert input on importantareas for the alternative'ssustainability, or approach which involves categorizing indicators into economic, environment, and social as done by Yi *et al.*, (2019); Junior *et al.*, (2018); Wang *et al.*, (2018).

Olabanji and Mpofu, (2020b); Harik *et al.*, (2015) employed the classification of indicators into traditional and peculiar, with the traditional comprising economic, environment and social, while the peculiar focused on technical issues of the system being assessed, includingpotential sustainability understudied, or overlooked issues, due to uncertainty about performance.

The abundance of the energy of the sun calls for its utilization in various operations in order to provide a renewable energy system (Makinde and Anjorin, 2018; Makinde and Obikoya, 2024). Thesolar water heating system operation begins by absorbing sunlight, collecting and transferring heat with the aid of an array of solar collectors into the water, which then is transported into the receiver by natural or forced convection. And after the heated fluid has been discharged from the receiver, coolerwater is immediately supplied to fill up the receiver and continue the cycle (Kakaza and Folly, 2015; Ijumba *et al.*, 2009). Basically, SWHSs can be classified into passive and active systems. Passive systems rely on gravitational force and natural convection to circulate water through the system. The heat exchanging fluid is water. The integratedcollector storage andthermosyphon are types of passive systems (Jamar *et al*., 2016). The integrated collector storage is also known as the batch system, with one or more black-painted receivers or tubes inside an insulated box with a glass cover, in which the solar collector and receiver are integrated (Jamar *et al*., 2016; Hudon, 2014). The receivers are painted black to increase the absorptivity. In the thermosyphon system, the receiver is separate from the solar collector. Cold water flows from the receiver downward to the bottom of the solar collector, where it gains heat, expands and become less dense than the cool water in the receiver. Due to density difference created by temperature gradients, the heated water rises in the collector and into the receiver, while the cold water falls to the bottom of the collector, and the cycle continues. There are various applications of thermosyphon depending on the collector such as Budiharjo*et al*., (2004) with a water-in-glass evacuated tube, or Arekete, (2013) with a flat-plate collector. However, the solar collector must be installed below the receiver for the thermosyphon effect to perform efficiently (Arekete, 2013). The active systems use an electrical pump to circulate fluid through the system (Arekete, 2013). A fluid that vaporizes at low temperature such as antifreeze or water is used as the working fluid. It can be installed at a low or high altitude with the pump generating the pressure required for effective transportation of the heated water. They are categorized into active open-loop, and active closed-loop. The active open loop is a system wherea pump is used to circulate water through the solar collector and into the receiver (Ogueke *et al.*, 2009). There are variousapplications of active open-loop depending on climate conditions. It can be used in colder climates but must be drained in winter,using an air glazed collector as suggested by Hudon, (2014), or concentration of water and propylene glycol as working fluid by Memon *et al*., (2020) to prevent the water in the pipes from freezing. The active close loop is also known as indirect where a solution or antifreeze such as propylene glycol serves as the working fluid (Hudon, 2014). The working fluid flows through the sealed piping of a heat exchanger, after gaining heat it changes phase, becomes lighter and rises into the receiver, where it loses the heat gained to the cold water, returns to its liquid phase and falls to the bottom of the collector for the cycle to continue. The principle for active closed-loop is as illustrated in Figure1. This can be used in climates where freezing occurs (Ayompe and Duffy, 2013). There are various applications of active closed-loop depending on the heat exchanging device such as solar coil, or heat-pipe evacuated tube (Jamar *et al*., 2016).

The Multi-Criteria Decision Model (MCDM) is a decision aid tool used to integrate multiple criteria for evaluation in the decision process. It has the ability to handle inherent complexity and broad scope of a sustainability assessment, including qualitative and quantitative data, including when expressed in fuzzy membership function (Olabanji, 2024; Olabanji and Mpofu, 2023; Olabanji, 2020).Further managing uncertainty related to data input (human limitations and linguistic fuzziness) and quality (Olabanji, 2022; Lindfors, 2021; Olabanji and Mpofu, 2022). MCDM guides in the design stages to make sustainable design rigour free and inexpensive, while facilitating the setting of design goals, and its evaluation by the creation of a model (Balcomb and Curtner, 2000).Most times, two or more MCDM models are hybridized in a decision process in order to enhance the decision process and increase the computational integrity of the decision process (Olabanji and Mpofu, 2019a; Olabanji and Mpofu, 2019b; Olabanji and Mpofu, 2020c Olabanji and Mpofu, 2020d) .It has proved to be one of the better tools for efficient energy planning (Kumar *et al*., 2017). Generally, the MCDM can be classified into multi-objective decision model (MODM) and Multiattribute decision model (MADM). Analytic Hierarchy Process (AHP), Weighted Decision Matrix (WDM), Grey Relational Analysis (GRA), COmplex PRoportional ASsessment (COPRAS), and VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) among others (Olabanji and Mpofu, 2021a).VIKOR is a distance-to-ideal methodto ascertain best and worst values, and rank based on its performance index. It is readily applied in engineering by Olabanji and Mpofu, (2021b), business management by Ouenniche*et al*., (2021), including health by Chang, (2014) among others.Sustainable Similarity Measure (SSM) is a method used to obtain the improvement level required in sustainability indicators. It is expected that the higher the similarity measure, the better the performance of the system relative to the indicator under consideration, as it depicts the closeness of the system to the optimum sustainability measure with respect to the indicator under consideration (Olabanji and Mpofu. 2020b).

2. Methodology

A model for the application of the methodology is shown in Figure 1 while the framework for the sustainability model, sustainability indicators and sub-indicators for the solar water heating systems is asillustrated in Figure 2.

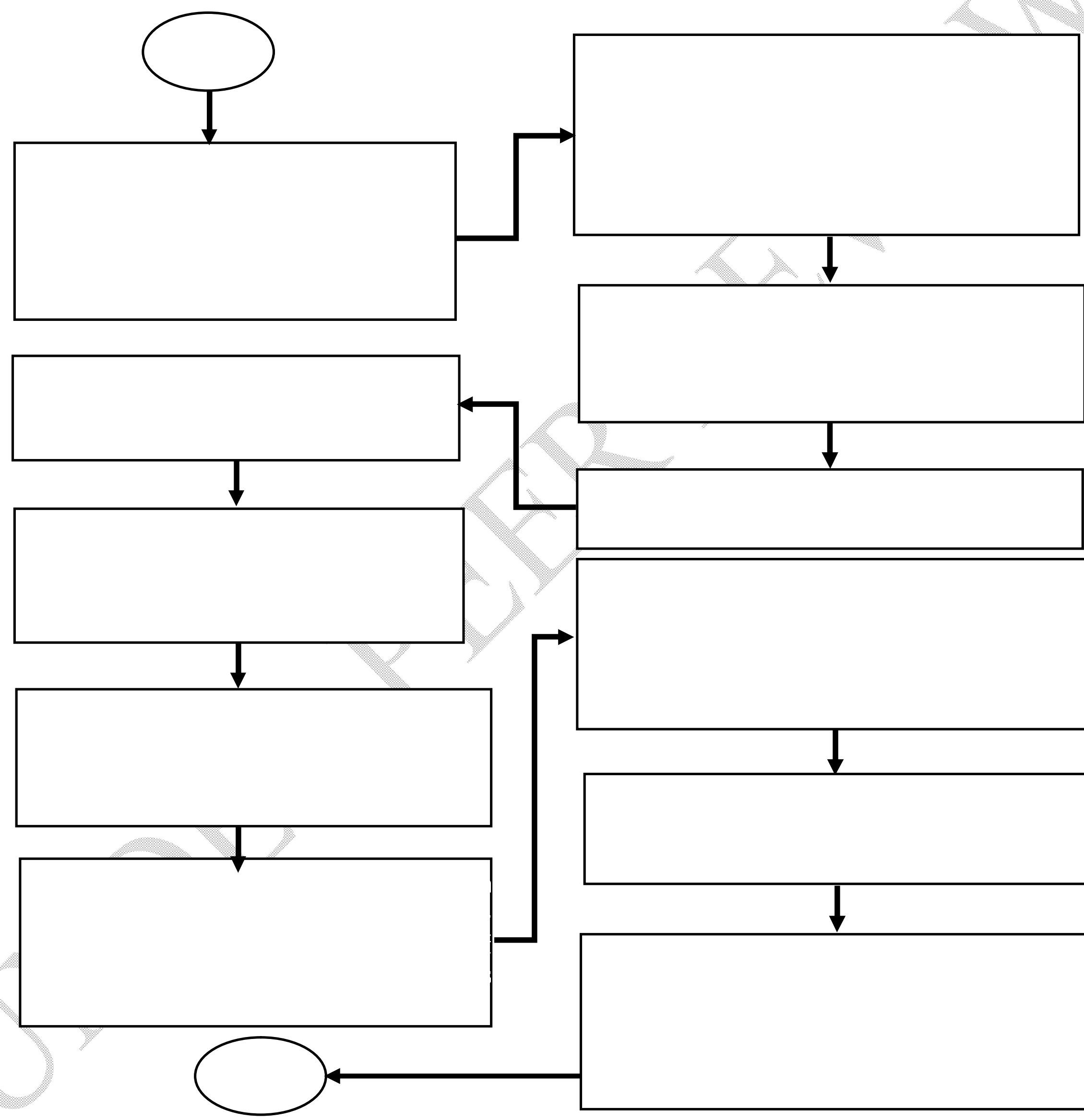

Figure 1: Framework for methodology.

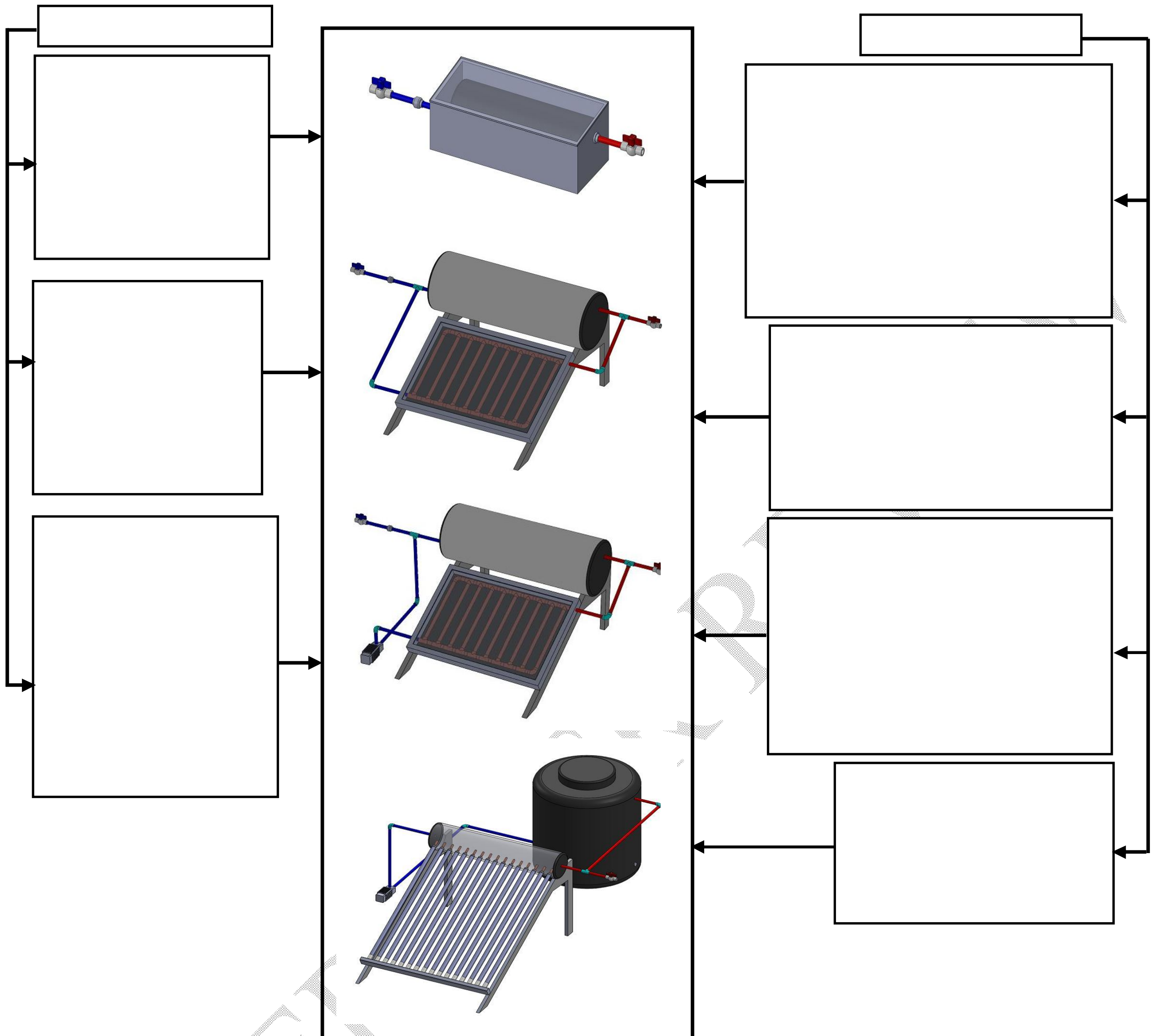

Figure 2: Framework for sustainability indicators for SWHSs.

2.1 Fuzzy membership function and decision matrix

For ease of analysis, consider '*n*' number of design concepts(DC_n) of Solar Water Heating Systems (SWHSs). Using '*m*' number of sustainability indicators (S_m) that are characterized by '*k*' number of sub-indicators (S_{mk}). To measure the relative significance of the sustainability indicators and sub-indicators in the SWHSs, it is essential to assign Triangular Fuzzy Numbers (TFNs) to the elements of the matrices using linguistic terms. The terms and distribution for the TFN is as presented in Table 1.

Table 1:TFNS for rating and ranking sustainability indicators and Linguistic terms adopted

Linguistic terms for rating of relative significance of sustainability indicators and sub-indicators of the SWHSs	Triangular fuzzy scale membership function	Crisp Value of Ranking and Rating
Disagree	1/2 1 3/2	1
Somewhat Disagree	1 3/2 2	2
Neither Agree nor Disagree	3/2 2 5/2	3
Somewhat Agree	2 5/2 3	4
Agree	5/2 3 7/2	5

The membership function $\mu_m\ (y)$of the TFNsis contained in [0 1] and can be defined as;

$$\mu_m(y)=\begin{cases} 0 & a<1 \\ \dfrac{1}{b-a}y-\dfrac{I}{b-a} & y\in[a,b], \\ \dfrac{1}{b-c}y-\dfrac{w}{b-c} & y\in[a,b], \\ 0 & b>c \end{cases} \tag{1}$$

Where $a\ \leq b\ \leq c$ represents the lower, modal and upper values of the fuzzy number M respectively. (Olabanji and Mpofu, 2021b).

To develop a decision matrix for the assessment of the sustainability indicators in the SWHSs, it is necessary to determine the weight of the sub-indicators of each indicator. For ease of analysis DE represents the aggregation of extracted TFNs responses of design experts for the k^{th} sub-indicator, for the m^{th} sustainability indicator. The responses of (y)design experts assigned to the TFNs are used to develop matrices for the weights of sub-indicators as presented in Equation (2).

$$
\begin{array}{cccccccc}
 & & DC_1 & DC_2 & DC_3 & \cdots & DC_n & \\
 & S_{11} & DE^{11} & DE^{12} & DE^{13} & \cdots & DE^{1n} & \left[\left(\sum_{n=1}^{n=n} DE^{1n}\right)\Big/ y\right] \\
S_m & S_{12} & DE^{21} & DE^{22} & DE^{23} & \cdots & DE^{2n} & \left[\left(\sum_{n=1}^{n=n} DE^{2n}\right)\Big/ y\right] \\
 & S_{13} & DE^{31} & DE^{32} & DE^{33} & \cdots & DE^{3n} & \left[\left(\sum_{n=1}^{n=n} DE^{3n}\right)\Big/ y\right] \\
 & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 & S_{1k} & DE^{k1} & DE^{k2} & DE^{k3} & \cdots & DE^{kn} & \left[\left(\sum_{n=1}^{n=n} DE^{kn}\right)\Big/ y\right]
\end{array}
\tag{2}
$$

The aggregation will be replicated for all sustainability indicators considering their sub-indicators. In Equation (2) DE^{k} represents the Triangular Fuzzy Number (TFN) decision of design experts for the k^{th} sub-indicator, and the m^{th} sustainability indicator. Equation (2) will be replicated for all indicators considering their sub-indicators.

To determine the decision matrix, it is necessary to establish the performance of the Solar Water Heating Systems (SWHSs) considering the responses of the design experts as presented in Equation (3).

$$
\begin{array}{cccccc}
 & & & S_{mk} & & \\
 & \sum DE_{m1} & \sum DE_{m1} & \sum DE_{m1} & \cdots & \sum DE_{mk} \\
DC_1 & DE_{av}^{11} & DE_{av}^{12} & DE_{av}^{13} & \cdots & DE_{av}^{1k} \\
DC_2 & DE_{av}^{21} & DE_{av}^{22} & DE_{av}^{23} & \cdots & DE_{av}^{2k} \\
DC_3 & DE_{av}^{31} & DE_{av}^{32} & DE_{av}^{33} & \cdots & DE_{av}^{3k} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
DC_n & DE_{av}^{n1} & DE_{av}^{n2} & DE_{av}^{n3} & \cdots & DE_{av}^{nk}
\end{array}
\tag{3}
$$

In Equation (3) DE_{av}^{nk} represents the average TFNs decision of the k^{th} design experts for 'n' designs of SWHSs considering the m^{th} sustainability indicator. Equation (3) will be replicated for all indicators considering their sub-indicators.

The sub-weighted decision matrix for the SWHSs can be developed from Equation (3) as described in Equation (4)

$$
\begin{array}{c|c}
 & S_{mk} \\
\begin{array}{ccccccc}
DC_1 & \sum DE_{m1}*DE_{av}^{11} & \sum DE_{m2}*DE_{av}^{12} & \sum DE_{m3}*DE_{av}^{13} & \cdots & \sum DE_{m1}*DE_{av}^{1k} \\
DC_2 & \sum DE_{m1}*DE_{av}^{21} & \sum DE_{m2}*DE_{av}^{22} & \sum DE_{m3}*DE_{av}^{23} & \cdots & \sum DE_{m1}*DE_{av}^{2k} \\
DC_3 & \sum DE_{m1}*DE_{av}^{31} & \sum DE_{m2}*DE_{av}^{23} & \sum DE_{m3}*DE_{av}^{33} & \cdots & \sum DE_{m1}*DE_{av}^{3k} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
DC_n & \sum DE_{m1}*DE_{av}^{n1} & \sum DE_{m2}*DE_{av}^{n2} & \sum DE_{m3}*DE_{av}^{n3} & \cdots & \sum DE_{mk}*DE_{av}^{nk}
\end{array}
&
\begin{array}{c}
\sum_{k=1}^{k=k}\left(\sum DE_{m1}*DE_{av}^{1k}\right) \\
\sum_{k=1}^{k=k}\left(\sum DE_{m1}*DE_{av}^{2k}\right) \\
\sum_{k=1}^{k=k}\left(\sum DE_{m1}*DE_{av}^{3k}\right) \\
\vdots \\
\sum_{k=1}^{k=k}\left(\sum DE_{mk}*DE_{av}^{nk}\right)
\end{array}
\end{array}
\qquad (4)
$$

In Equation (4) $\left(\sum DE_{mk} * DE_{av}^{nk}\right)$ represents the sub-weighted Triangular Fuzzy Numbers (TFNs) for n^{th} design of Solar Water Heating Systems (SWHSs) considering all the k^{th} sub-indicators in the sustainability indicators. Equation (4) will be replicated for allindicators considering their sub-indicators. From Equation (2), the overall weight of the sustainability indicator can be derived by determining the aggregate of the weight of all the sub-indicator, hence,

$$
S_m^w = \sum_{k=1}^{k=k}\left[\left(\sum_{n=1}^{n=n} DE^{kn}\right)\Big/ y\right] \qquad (5)
$$

In Equation 5 S_m^w represents the overall weight of the m^{th} sustainability indicator. Therefore, the decision matrix for the SWHSs can be developed considering the k^{th} number of sub-indicators in m^{th} sustainability indicator as described in using Equation (6).

$$
\begin{array}{cc}
 & S_m^w \\
DC_n &
\begin{array}{ccccc}
\sum_{k=1}^{k=k}\left(\sum DE_{1k}*DE_{av}^{1k}\right)/k & \sum_{k=1}^{k=k}\left(\sum DE_{2k}*DE_{av}^{1k}\right)/k & \sum_{k=1}^{k=k}\left(\sum DE_{3k}*DE_{av}^{1k}\right)/k & \cdots & \sum_{k=1}^{k=k}\left(\sum DE_{mk}*DE_{av}^{1k}\right)/k \\
\sum_{k=1}^{k=k}\left(\sum DE_{1k}*DE_{av}^{2k}\right)/k & \sum_{k=1}^{k=k}\left(\sum DE_{2k}*DE_{av}^{2k}\right)/k & \sum_{k=1}^{k=k}\left(\sum DE_{3k}*DE_{av}^{2k}\right)/k & \cdots & \sum_{k=1}^{k=k}\left(\sum DE_{mk}*DE_{av}^{2k}\right)/k \\
\sum_{k=1}^{k=k}\left(\sum DE_{1k}*DE_{av}^{3k}\right)/k & \sum_{k=1}^{k=k}\left(\sum DE_{2k}*DE_{av}^{3k}\right)/k & \sum_{k=1}^{k=k}\left(\sum DE_{3k}*DE_{av}^{3k}\right)/k & \cdots & \sum_{k=1}^{k=k}\left(\sum DE_{mk}*DE_{av}^{3k}\right)/k \\
\cdots & \cdots & \cdots & \ddots & \vdots \\
\sum_{k=1}^{k=k}\left(\sum DE_{1k}*DE_{av}^{nk}\right)/k & \sum_{k=1}^{k=k}\left(\sum DE_{2k}*DE_{av}^{nk}\right)/k & \sum_{k=1}^{k=k}\left(\sum DE_{3k}*DE_{av}^{nk}\right)/k & \cdots & \sum_{k=1}^{k=k}\left(\sum DE_{mk}*DE_{av}^{nk}\right)/k
\end{array}
\end{array}
\qquad (6)
$$

It is important to normalize the TFNs of the decision matrix to ensure they are contained within [0,1], as they represent the weights of the m^{th} sustainability indicators and k^{th} sub-indicators. To normalize the TFNs, consider a fuzzy number

$y_{ij} = (a_{ij} b_{ij} c_{ij})$ For (i =1……n, j=1……m) as defined in Equation (1) (Olabanji and Mpofu, 2021b;Moktariam and Hadi-Vencheh, 2012).

$$(y_{ij})_N = N = \left[(a_{ij})_N (b_{ij})_N (c_{ij})_N\right] \tag{7}$$

$$N = \left[\frac{a_{ij} - a_j^{\min}}{\square_{\min}^{\max}}, \frac{b_{ij} - a_j^{\min}}{\square_{\min}^{\max}}, \frac{c_{ij} - a_j^{\min}}{\square_{\min}^{\max}}\right] \tag{8}$$

In Equation (7) and (8), $a_j^{\min} = Min\ a_{ij}$ and $c_j^{\max} = Max\ c_{ij}$; $\square_{\min}^{\max} = c_j^{\max} - a_j^{\min}$.

To understand the range of the sustainability indicators to the best and worst performance in the SWHSs, it is necessary to obtain the ideal fuzzy best and worst performance of the indicators in the SWHSs. The fuzzy best (F^*) and worst (F^-) values can be derived from the normalized decision matrix for all the indicators using Equations (9) and (10) (Olabanji and Mpofu, 2020a; Musani and Jemain, 2015; Shemshadi*et al.*, 2011).

$$F^* = Max\left[\sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_n^{\bar{N}}\right] \tag{9}$$

$$F^- = Min\left[\sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_n^{\bar{N}}\right] \tag{10}$$

Sustainability Separation Measures

There is no certainty that a SWHSs will perform perfectly in all sustainability indicators. However, it is expected that the systems should have a satisfactory performance in all indicators. The overall sustainability index (S_I) is a function of sustainability indices of the indicators, this implies that the overall sustainability index for any of the SWHSs ($\tilde{S}_I^{SWHSs}$) can be obtained using Equation (11) by summing the aggregates of the sustainability indicators in the normalized decision matrix (Olabanji and Mpofu, 2020b; Olabanji, 2018)

$$\tilde{S}_I^{SWHSs} = \sum_{m=1}^{m=m}\left(\sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_n^{\bar{N}}\right) \tag{11}$$

In Equation (11), $\sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_n^{\bar{N}}$ is the normalized value of an element in the decision matrix corresponding to the n^{th} Solar Water Heating Systems (SWHSs) with the m^{th} sustainability index.

The resulting elements of the normalized decision matrix using equation (11) represents the performance of the SWHSs considering their indicators. Therefore, the current performance of the elements in the normalized decision matrix must be compared to an ideal situation in order to measure the extent of the performances, for continuous improvements in the SWHSs to have a sustainable performance in all the indicators. Considering the TFNs of the current performance of the SWHSs in (11), using an ideal TFNs to compare the current performance, to obtain the level of improvement required in the sustainability indicators is as presented by the cosine similarity measure in Equation (12) (Olabanji and Mpofu, 2020b; Chen, 1996).

$$CS\left[\sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_{n}^{\bar{N}*}, \sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_{n}^{\bar{N}}\right] = \left[1-\left(\frac{\left|a_n^* - a_n\right| + \left|b_n^* - b_n\right| + \left|c_n^* - c_n\right|}{3}\right)\right]_{N} \tag{12}$$

In Equation (12), $CS\left[\sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_{n}^{\bar{N}*}, \sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_{n}^{\bar{N}}\right]$ is the cosine similarity measure between the ideal performance and current performance. While, $\sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_{n}^{\bar{N}*}$ is the ideal performance which can be defined as;

$$\sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_{n}^{\bar{N}*} = \left[a_n^*, b_n^*, c_n^*\right] = [1,1,1] \tag{13}$$

In order to identify the indicators to improve, it is necessary to find the Euclidean distance, to reveal how close or far the performance of the indicator is to the ideal. It is necessary to develop an analysis of the Euclidean distances of the indicators relative to the highest Euclidean distance that the indicators must approach. Euclidean distance d is obtainable from Equation (14) (Olabanji and Mpofu. 2020b; Ye, 2011a; Ye, 2011b).

$$d = \left(2\left(1 - CS\left[\sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_{n}^{\bar{N}*}, \sum_{k=1}^{k=k}\left(\sum DE_{mk} * DE_{av}^{nk}\right)/k\Big|_{n}^{\bar{N}}\right]\right)\right)^{\frac{1}{2}} \tag{14}$$

Hence, from Equation (14), a value of zero and the square root of two indicates the best and worst sustainable performance, respectively, with 'd_{best}' and 'd_{worst}' representing the best and worst values of the Euclidean distance, respectively.

Considering Equation (12), a higher value of the similarity measure is preferable for the sustainability indicator because it depicts the closeness of the SWHSs to the optimum sustainability measure (S_{opt}^{m}) with respect to the indicator under consideration. The optimum sustainability measure for any indicator is obtained as the similarity measure equals ideal. It is practically impossible to achieve optimum sustainability for all the indicators, because of the adverse effects that will occur in the design process, in order to ensure that all the indicators have satisfactory sustainable performance. The optimum sustainability for any of the indicators can be obtained from Equation (15).

$$S_{opt}^{m} = Max\left[CS\left[\sum_{k=1}^{k=k} \left(\sum DE_{mk} * DE_{av}^{nk} \right) / k \Big|_{n}^{\bar{N}*}, \sum_{k=1}^{k=k} \left(\sum DE_{mk} * DE_{av}^{nk} \right) / k \Big|_{n}^{\bar{N}} \right] \right] = 1 \qquad (15)$$

It is expected that the higher the similarity measure, the better the performance of the Solar Water Heating Systems (SWHSs) relative to indicator under consideration. Therefore, the Sustainable Similarity Measure (SSM) of SWHSs can be obtained by summing all the similarity measures for all the indicators, as presented in Equation (16).

$$SSM = \sum_{m=1}^{m=m} CS\left[\sum_{k=1}^{k=k} \left(\sum DE_{mk} * DE_{av}^{nk} \right) / k \Big|_{n}^{\bar{N}*}, \sum_{k=1}^{k=k} \left(\sum DE_{mk} * DE_{av}^{nk} \right) / k \Big|_{n}^{\bar{N}} \right] \qquad (16)$$

It is practically difficult for the SSM for a SWHS to be equal to the overall optimum sustainability measure, but a significant performance is expected from the SWHSs, as the SSM will give a more definite performance assessment of the SWHSs compared to the sustainability index obtained from Equation (11) because it shows their sustainable measure relative to the overall optimum sustainability. Hence, the overall Optimum Sustainability (OS_{opt}) is expected to be equal to the number of indicators, as presented in Equation (17).

$$OS_{opt} = \sum_{m=1}^{m=m} Max\left[CS\left[\sum_{k=1}^{k=k} \left(\sum DE_{mk} * DE_{av}^{nk} \right) / k \Big|_{n}^{\bar{N}*}, \sum_{k=1}^{k=k} \left(\sum DE_{mk} * DE_{av}^{nk} \right) / k \Big|_{n}^{\bar{N}} \right] \right] \qquad (17)$$

For the analysis, as the SSM of the indicator approaches zero in (16), the Euclidean distance tends to a maximum value, which translates to the worst sustainable performance for the indicator. Also, as the SSM increases, Euclidean distance approaches a minimum value of zero, which depicts the best sustainable performance for the indicator.

Results

The result obtained from the sustainability assessment of SWHSs based on the working principles starts from the determination of weights for the sub-indicators of the sustainability indicators to the determination of sub-decision matrices for the SWHSs considering all the indicators. The weighted sub-decision matrix is determined by considering the weights of the sub-indicators and the performance of the SWHSs considering the sub-indicators. The overall decision matrix is a function of the aggregate performance of the SWHSs considering the sub-indicators, these aggregates are harnessed to form the performance of the SWHSs in the overall decision matrix. Similarly, the weight of the sustainability indicator is the average weight of all the sub-indicators, contributing to the performance.

For ease of analysis, the determination of weights, sub decision matrices and weighted sub decision matrices for the performances of the SWHSs considering the indicators is presented in APPENDIX I to XIX. Considering economic indicator, the weight of the economic indicator is obtained from the aggregate of the average weight of the sub-indicators of economic considering all the Solar Water Heating Systems (SWHSs) as presented in Table 2. Also, from Table 2 the weight of the sub-indicators for the economic indicator is the average weight of the relevance of the sub-indicator in the SWHSs. Table 2 was replicated for the performances of the SWHSs considering the environmental, social, manufacturing, maintenance, reliability and life-cycle indicator respectively. Further, it is necessary to consider the performances of the SWHSs in all the indicators considering the responses of the design experts. The average of the responses for the SWHSs for the sub-indicators of economic is presented in Table 3. Table 3 was replicated to presents the average of the responses for the SWHSs for the sub-indicators of environment, social, manufacturing, maintenance, reliability and life-cycle respectively.

In essence the weighted sub-decision matrix is a function of the weights of the sub-indicators and the performance of the SWHS in the sub-indicators. For instance, considering economic indicator, the average weight of sub-indicator obtained from Table 2 and the performance of the SWHS in Table 3 are used to determine the weighted sub-decision matrix for the SWHSs considering economic indicator as presented in Table 4. In the same manner, the weighted sub-decision matrices for the SWHSs considering environmental, social, manufacturing, maintenance, reliability and life-cycle were derived. The average weights of the performances of the SWHSs from Table 4 and others are harnessed to determine the fuzzified decision matrix considering the weights of the indicators as presented in Table 5. Also, it is necessary to normalize the fuzzified decision matrix in Table 5 in order to ensure that the membership functions of the fuzzy elements are contained in [0,1] as described in Equation 1. Applying Equations (9) and (10), the fuzzy best and fuzzy worst value can be obtained from the normalized fuzzy decision matrix as presented in Table 6.

The overall sustainability index was derived using Equation (11). Cosine Similarity (CS) measure for all the indicators of sustainability for the SWHSs was derived using Equation (12). From the result obtained from CS, the Euclideandistance to determine the distances of all the sustainability indicators for the SWHSs to the worst and best sustainable performance was derived using Equation (14). Applying Equation (15), (16) and (17) the optimum sustainability, sustainable similarity measure, and the overall optimum sustainability respectively were derived andis as presented in Table 7.

Table 2: Determination of weights for sub-indicators of economic indicator

	ICS	TS	AOL	ACL	Avg
EC1	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{35}{4}\ \frac{43}{4}\ \frac{51}{4}$
EC2	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	7 9 11	9 11 13	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$
EC3	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	8 10 12	9 11 13	9 11 13	$\frac{67}{8}\ \frac{83}{8}\ \frac{99}{8}$
EC4	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	8 10 12	10 12 14	9 11 13
EC5	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{37}{4}\ \frac{45}{4}\ \frac{53}{4}$
EC6	8 10 12	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	6 8 10	7 9 11	$\frac{57}{8}\ \frac{73}{8}\ \frac{89}{8}$

Table 3: Sub-decision matrix for SWHSs considering the sub-indicators of economic

	EC1 $\frac{35}{4}\ \frac{43}{4}\ \frac{51}{4}$	EC2 $\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	EC3 $\frac{67}{8}\ \frac{83}{8}\ \frac{99}{8}$	EC4 9 11 13	EC5 $\frac{37}{4}\ \frac{45}{4}\ \frac{53}{4}$	EC6 $\frac{57}{8}\ \frac{73}{8}\ \frac{89}{8}$
ICS	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$2\ \frac{5}{2}\ 3$
TS	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$2\ \frac{5}{2}\ 3$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$
AOL	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$2\ \frac{5}{2}\ 3$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{3}{2}\ 2\ \frac{5}{2}$
ACL	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{5}{2}\ 3\ \frac{7}{2}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$

Table 4: Weighted sub-decision matrix for the SWHSs considering the sub-indicators of economic

	EC1	EC2	EC3	EC4	EC5	EC6	EC(Avg)
ICS	$\frac{595}{32}\ \frac{903}{32}\ \frac{1275}{32}$	$\frac{289}{16}\ \frac{441}{16}\ \frac{625}{16}$	$\frac{1005}{64}\ \frac{1577}{64}\ \frac{227}{64}$	$\frac{171}{8}\ \frac{253}{8}\ \frac{351}{8}$	$\frac{703}{32}\ \frac{1035}{32}\ \frac{143}{32}$	$\frac{57}{4}\ \frac{365}{16}\ \frac{267}{8}$	$\frac{7037}{384}\ \frac{3567}{128}\ \frac{15133}{384}$
TS	$\frac{525}{32}\ \frac{817}{32}\ \frac{117}{32}$	$\frac{119}{8}\ \frac{189}{8}\ \frac{275}{8}$	$\frac{67}{4}\ \frac{415}{16}\ \frac{297}{8}$	$\frac{153}{8}\ \frac{231}{8}\ \frac{325}{8}$	$\frac{703}{32}\ \frac{1035}{32}\ \frac{143}{32}$	$\frac{855}{64}\ \frac{1387}{64}\ \frac{204}{64}$	$\frac{6559}{384}\ \frac{10111}{384}\ \frac{1443}{384}$

AOL	665/32 989/32 1377/32	153/8 231/8 325/8	603/32 913/32 1287/32	18 55/2 39	703/32 1035/32 1431/32	171/16 73/4 445/16	1167/64 1775/64 2511/64
ACL	665/32 989/32 1377/32	323/16 483/16 675/16	603/32 913/32 1287/32	45/2 33 91/2	629/32 945/32 1325/32	399/32 657/32 979/32	1831/96 921/32 3887/96

Table 5: Decision matrix for the SWHSs considering all the sustainability indicators

	EC 51 63 75	EN 387/8 483/8 579/8	SO 365/8 461/8 557/4	MA 577/8 721/8 865/8	MN 303/8 383/8 463/8	RE 513/8 641/8 769/8	LC 77/2 97/2 117/2
ICS	7037/384 3567/128 15/38	6867/320 10351/320 1489/320	1825/128 8743/384 1794/384	985/64 13925/576 2013/576	1083/80 175/8 2577/80	8233/512 12829/512 1844/512	2221/160 3573/160 104/32
TS	6559/384 10111/384 14	2863/160 4533/160 6587/160	707/48 2239/96 407/12	2323/144 226/9 5197/144	2377/160 3767/160 5477/160	4149/256 6455/256 9273/256	1171/80 1863/80 543/16
AOL	1167/64 1775/64 25/64	1253/64 9749/320 14001/320	2821/192 4471/192 6505/192	4693/288 7295/288 3491/96	4503/320 7219/320 211/64	4167/256 6481/256 9307/256	2497/160 3921/160 113/32
ACL	1831/96 921/32 388/96	6213/320 9681/320 13917/320	2881/192 4555/192 6613/192	155/9 1271/48 2717/72	303/20 383/16 1389/40	4177/256 6491/256 9317/256	76/5 120/5 174/5

Table 6: Normalized decision matrix for the SWHSs and determination of Fuzzy best and worst value

	EC 51 63 75	EN 387/8 483/8 579/8	SO 365/8 461/8 557/4	MA 577/8 721/8 865/8	MN 303/8 383/8 463/8	RE 513/8 641/8 769/8	LC 77/2 97/2 117/2
ICS	36/677 47/102 62/65	31/249 272/539 1	0 38/145 1	0 103/262 571/652	0 194/493 498/565	0 19/43 497/506	0 338/861 36/41
TS	0 277/701 148/169	0 125/343 321/395	10/689 79/283 293/484	7/211 251/577 327/353	15/241 264/559 461/472	1/160 67/149 586/591	13/370 215/492 396/425
AOL	24/487 365/802 353/373	1/17 151/344 437/484	3/224 62/223 455/753	27/667 419/942 627/668	20/793 66/155 93/101	1/103 336/739 519/520	23/287 425/861 1
ACL	4/47 1/2 1	11/207 104/241 698/781	20/869 72/247 340/547	5/61 395/796 1	43/565 161/328 1	1/86 142/311 1	31/506 55/117 69/71
F^*	4/47 1/2 1	31/249 272/539 1	20/869 72/247 340/547	5/61 395/796 1	43/565 161/328 1	1/86 142/311 1	23/287 425/861 1
F^-	0 277/701 148/169	0 125/343 321/395	0 38/145 1	0 103/262 571/652	0 194/493 498/565	0 19/43 497/506	0 338/861 36/41

Table 7: Sustainability separation measures for the SWHSs

	ICS		TS		AOL		ACL	
Indicators	CS	d	CS	d	CS	d	CS	d
EC	0.49	1.01	0.42	1.07	0.48	1.01	0.53	0.97
EN	0.54	0.95	0.39	1.10	0.47	1.03	0.46	1.04
SO	0.43	1.08	0.30	1.18	0.30	1.18	0.31	1.17
MA	0.42	1.07	0.46	1.03	0.47	1.03	0.53	0.97
MN	0.42	1.07	0.50	0.99	0.45	1.04	0.52	0.98
RE	0.47	1.02	0.48	1.01	0.48	1.01	0.49	1.01
LC	0.42	1.07	0.47	1.03	0.52	0.97	0.50	0.99
$\tilde{S}_I^{SWHSs}$	0.49		0.42		0.48		0.53	
SSM	3.20		3.03		3.19		3.33	

Discussion

The cosine similarity measures function by the cosine of the angle between two vectors and determines whether two vectors are pointing in the same direction. Hence, operating within [0,1], it is expected that the best performance will be the indicator closest to one and worst will be closest to zero, for all the solar water heating systems. This is presented in Figure 3.

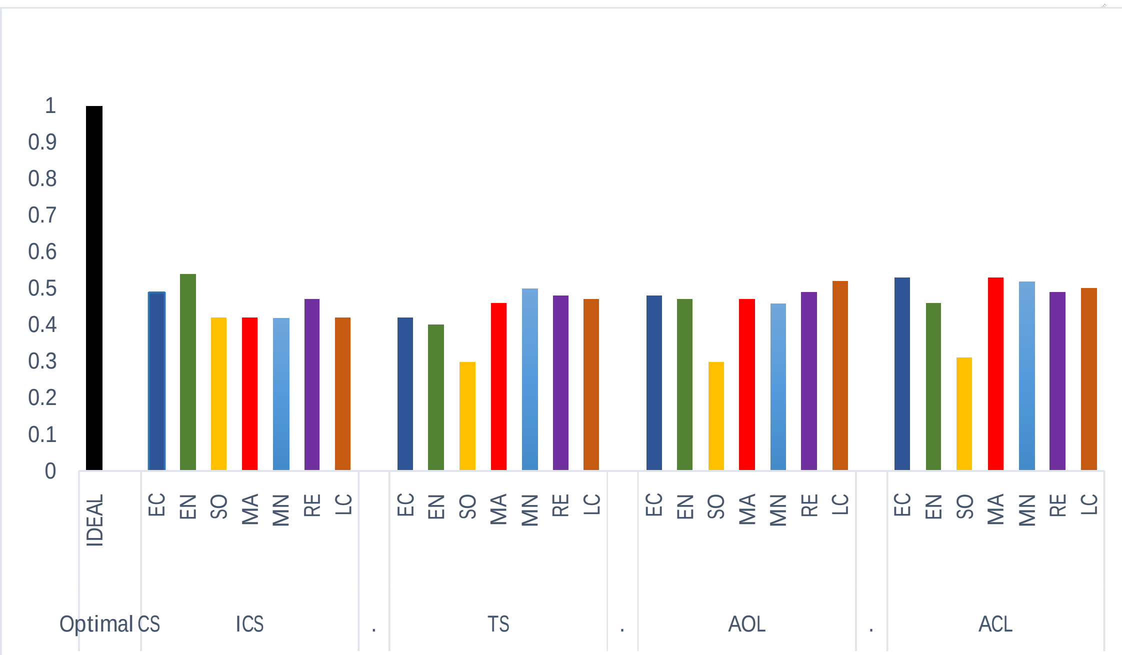

Figure 3: Cosine similarity measures for the sustainability indicators of SWHSs

The highest economic indicator was in ACL at 53% and the lowest inTS at 42%. This shows that it is important for sustainable economic ease and growth, across the domestic and importantly the industrial sector that the rate of heating using SWHSs should be improved for economic sustainability. The highest environmental indicator was in ICS at 54% and the lowest in TS at 39%. The social indicator was highest in ICS with 43% and joint lowest in TS and AOL at 30%. This indicates the level of acceptability and usage of these system in developing and underdeveloped regions, which shows the need for more local adoption and international investment to promote sustainable development by the improvement of this social indicator of sustainability. The manufacturing indicator was highest in ACL with 53% and lowest in ICS at 42%. It is important that in the pre-design and design stage of production of these systems, considerations must be made to prioritize fast and adequate operations primarily thereby improving the manufacturing indicator, while also considering the societal implication on the capital of low to middle class income earners, further improving acceptability. The maintenance indicator was highest in ACL with 52% and lowest in ICS at

42%. This indicates that for improvement in the maintenance indicator, it is advantageous to ensure ease in part interchange and overall maintenance. Furthermore, it is important to consider all classes of users and the living traffic level of the populaces in the pre-design stage of the heating systems. The reliability indicator was highest in ACL with 49% and lowest in ICS at 47%. This indicates that while the systems can satisfy domestic and industrialusers, for improvement in the reliability indicator, the system should be able to satisfy overall daily usage across all sectors, reducing cost while maximizing the potential of the system. The life-cycle indicator was highest in AOL with 52% and lowest in ICS 42%. This indicates that for improvement to the life-cycle of a SWHSs it is necessary to operate sustainably and satisfactorily in the short and long-term, thereby increasing its acceptability.

In order to improve the overall sustainability of the SWHSs, while maintaining balance so that no indicator starts to perform poorly. It's important to analyse the performance of the indicators with respect to their Euclidean distances, as presented in Figure 4.

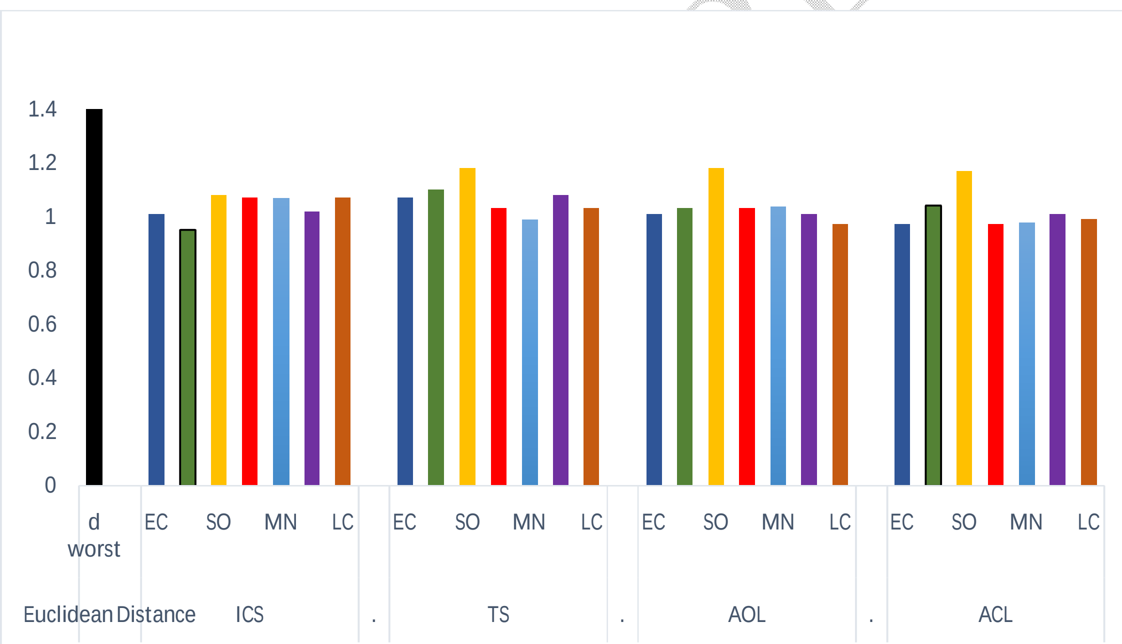

Figure 4: Euclidean distance of the sustainability indicators relative to worst performance

This identifies which indicators require improvement in the adoption of a system. Hence, contained within $[0,2^{1/2}]$, the indicators with the best sustainable performance index are closest 0, while the indicators with the worst sustainable performance index are closest to $2^{1/2}$. For the ICS the indicators with the worst performances are manufacturing, maintenance, and life-cycle indicators. In the TS, the indicators with the worst performances are economic, environmental and social indicators. In the AOL, the indicators with the worst performances are the social, and maintenance indicators. And lastly in the ACL, the indicators with the worst performances are social, and environmental indicators. The optimum sustainability measure of the sustainability indicators is; 53%, 54%, 43%, 53%, 52%, 49%, and 50% for economic, environmental, social, manufacturing, maintenance, reliability, and life-cycle respectively. The SSM values are 3.20, 3.03, 3.19 and 3.33 for ICS, TS, AOL and ACL respectively. The SSM indicates the sustainable performance of the SWHSs considering the sustainability indicators while the performance index obtained from the separation measures under the fuzzy VIKOR model indicates the distance to ideal satisfactory performance. The environmental indicator was the overall optimum sustainability at 54%. It can be stated that, this was as a result of the environmentally friendly nature of the systems.

Conclusion

The optimal performance of established sustainability indicators was derived from the outcome of the sustainability assessment of the Solar Water Heating Systems (SWHSs) using Multi-Criteria Decision Model (MCDM) through the evaluation of the relative significance of the indicators considering their weights in the assessment. Four SWHSs namely; integrated collector storage (ICS), thermosyphon (TS), active open-loop (AOL) and active closed-loop (ACL). Have been assessed in this article. Also, the article was able to identify the indicators necessary for improvement following the sustainability assessment of SWHSs. The framework for the sustainability assessment of the SWHSs included the traditional sustainability indicators such as economic (EC), environmental (EN), and social (SO), and peculiar sustainability indicators namely manufacturing (MA), maintenance (MN), reliability (RE), and life-cycle (LC).The sustainability indicators definition and weighting for the assessment were adapted using linguistic term of Triangular Fuzzy Numbers (TFNs) membership function, to address the multidimensional nature of the indicators and sub-indicators and manage the uncertainty in the weighting process with its computational strength. It is evident and as expected that in the result across all SWHSs the environmental indicator was the highest and closest to the optimal ideal cosine similarity measure. Hence, the general average performance of the highest percentile of all the sustainability indicators is largely due to regional selection of these systems. And while some SWHSs performed relatively well with the traditional indicators, the results from the peculiar indicators revealed other weaknesses.It is essential to ensure balance for the systems across all sustainability indicators. It is best to improve all indicators to increase the sustainable similarity measure, as increasing a single indicator can adversely affect other indicators.

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1.

2.

3.

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APPENDIX

Table I: Determination of weights for sub-indicators of environment indicator.

ENVIRONMENT	ICS	TS	AOL	ACL	Avg
EN1	10 12 14	7 9 11	9 11 13	9 11 13	$\frac{35}{4}\ \frac{43}{4}\ \frac{51}{4}$
EN2	8 10 12	6 8 10	8 10 12	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{59}{4}\ \frac{75}{4}\ \frac{91}{4}$
EN3	9 11 13	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	8 10 12	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$
EN4	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	9 11 13	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{67}{8}\ \frac{83}{8}\ \frac{99}{8}$
EN5	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	8 10 12	8 10 12	8 10 12
					$\frac{387}{8}\ \frac{483}{8}\ \frac{579}{8}$

Table II Determination of weights for sub-indicators of social indicator.

SOCIAL	ICS	TS	AOL	ACL	Avg
SO1	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	9 11 13	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{71}{8}\ \frac{87}{8}\ \frac{103}{8}$
SO2	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	7 9 11	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{57}{8}\ \frac{73}{8}\ \frac{89}{8}$
SO3	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	5 7 9	6 8 10	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{25}{4}\ \frac{33}{4}\ \frac{41}{4}$
SO4	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	9 11 13	8 10 12	8 10 12
SO5	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	7 9 11	$\frac{55}{8}\ \frac{71}{8}\ \frac{87}{8}$
SO6	8 10 12	9 11 13	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$
					$\frac{365}{8}\ \frac{461}{8}\ \frac{557}{4}$

Table III: Determination of weights for sub-indicators of manufacturing.

MANUFACUTURING	ICS	TS	AOL	ACL	Avg
MA1	6 8 10	6 8 10	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	$\frac{25}{4}\ \frac{33}{4}\ \frac{41}{4}$
MA2	8 10 12	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	9 11 13	10 12 14	$\frac{71}{8}\ \frac{87}{8}\ \frac{103}{8}$
MA3	8 10 12	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	8 10 12	9 11 13	$\frac{67}{8}\ \frac{83}{8}\ \frac{99}{8}$
MA4	8 10 12	9 11 13	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{35}{4}\ \frac{43}{4}\ \frac{51}{4}$

	(col. 1)	(col. 2)	(col. 3)	(col. 4)	(Avg)
MA5	$\frac{11}{2}\ \frac{15}{2}\ \frac{19}{2}$	6 8 10	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	8 10 12	$\frac{27}{4}\ \frac{35}{4}\ \frac{43}{4}$
MA6	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	8 10 12	9 11 13	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{35}{4}\ \frac{43}{4}\ \frac{51}{4}$
MA7	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	8 10 12	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	8 10 12	$\frac{33}{4}\ \frac{41}{4}\ \frac{49}{4}$
MA8	6 8 10	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	8 10 12	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$
MA9	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	9 11 13	8 10 12	8 10 12	$\frac{69}{8}\ \frac{85}{8}\ \frac{101}{8}$
					$\frac{577}{8}\ \frac{721}{8}\ \frac{865}{8}$

Table IV: Determination of weights for sub-indicators of maintenance indicator.

MAINTENANCE	ICS	TS	AOL	ACL	Avg
MN1	6 8 10	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	7 9 11	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	7 9 11
MN2	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	7 9 11
MN3	7 9 11	8 10 12	8 10 12	8 10 12	$\frac{31}{4}\ \frac{39}{4}\ \frac{47}{4}$
MN4	8 10 12	9 11 13	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	8 10 12	$\frac{65}{8}\ \frac{81}{8}\ \frac{97}{8}$
MN5	8 10 12	8 10 12	8 10 12	8 10 12	8 10 12
					$\frac{303}{8}\ \frac{383}{8}\ \frac{463}{8}$

Table V: Determination of weights for sub-indicators of reliability indicator.

RELIABILITY	ICS	TS	AOL	ACL	Avg
RE1	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$
RE2	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	$\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	8 10 12	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{71}{8}\ \frac{87}{8}\ \frac{103}{8}$
RE3	7 9 11	7 9 11	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{29}{4}\ \frac{37}{4}\ \frac{45}{4}$
RE4	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{31}{4}\ \frac{39}{4}\ \frac{47}{4}$
RE5	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	8 10 12	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{65}{8}\ \frac{81}{8}\ \frac{97}{8}$
RE6	8 10 12	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	9 11 13	$\frac{33}{4}\ \frac{41}{4}\ \frac{49}{4}$
RE7	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	7 9 11	7 9 11	7 9 11	$\frac{57}{8}\ \frac{73}{8}\ \frac{89}{8}$
RE8	7 9 11	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	7 9 11	$\frac{29}{4}\ \frac{37}{4}\ \frac{45}{4}$

$\frac{513}{8}\ \frac{641}{8}\ \frac{769}{8}$

Table VI: Determination of weights for sub-indicators of life-cycle indicator.

LIFE CYCLE	ICS	TS	AOL	ACL	Avg
LC1	$8\ 10\ 12$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$8\ 10\ 12$	$8\ 10\ 12$
LC2	$7\ 9\ 11$	$8\ 10\ 12$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{31}{4}\ \frac{39}{4}\ \frac{47}{4}$
LC3	$\frac{13}{2}\ \frac{17}{2}\ \frac{21}{2}$	$7\ 9\ 11$	$8\ 10\ 12$	$\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$
LC4	$7\ 9\ 11$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$8\ 10\ 12$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$
LC5	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$8\ 10\ 12$	$8\ 10\ 12$	$\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	$\frac{31}{4}\ \frac{39}{4}\ \frac{47}{4}$
					$\frac{77}{2}\ \frac{97}{2}\ \frac{117}{2}$

Table VII: Sub-decision matrix for SWHSs considering the sub-Indicators of environment.

	EN1 $\frac{35}{4}\ \frac{43}{4}\ \frac{51}{4}$	EN2 $\frac{59}{4}\ \frac{75}{4}\ \frac{91}{4}$	EN3 $\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$	EN4 $\frac{67}{8}\ \frac{83}{8}\ \frac{99}{8}$	EN5 $8\ 10\ 12$
ICS	$\frac{5}{2}\ 3\ \frac{7}{2}$	$2\ \frac{5}{2}\ 3$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$
TS	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{3}{2}\ 2\ \frac{5}{2}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$
AOL	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$2\ \frac{5}{2}\ 3$	$2\ \frac{5}{2}\ 3$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$
ACL	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$2\ \frac{5}{2}\ 3$

Table VIII: Sub-decision matrix for SWHSs considering the sub-Indicators of social.

	SO1 $\frac{71}{8}\ \frac{87}{8}\ \frac{103}{8}$	SO2 $\frac{57}{8}\ \frac{73}{8}\ \frac{89}{8}$	SO3 $\frac{25}{4}\ \frac{33}{4}\ \frac{41}{4}$	SO4 $8\ 10\ 12$	SO5 $\frac{55}{8}\ \frac{71}{8}\ \frac{87}{8}$	SO6 $\frac{17}{2}\ \frac{21}{2}\ \frac{25}{2}$
ICS	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$2\ \frac{5}{2}\ 3$
TS	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{5}{4}\ \frac{7}{4}\ \frac{9}{4}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$
AOL	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$\frac{3}{2}\ 2\ \frac{5}{2}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$

ACL	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$

Table IX: Sub-decision matrix for SWHSs considering the sub-Indicators of manufacturing.

	MA1 $\frac{25}{4}\ \frac{33}{4}\ \frac{41}{4}$	MA2 $\frac{71}{8}\ \frac{87}{8}\ \frac{103}{8}$	MA3 $\frac{67}{8}\ \frac{83}{8}\ \frac{99}{8}$	MA4 $\frac{35}{4}\ \frac{43}{4}\ \frac{51}{4}$	MA5 $\frac{27}{4}\ \frac{35}{4}\ \frac{43}{4}$	MA6 $\frac{35}{4}\ \frac{43}{4}\ \frac{51}{4}$	MA7 $\frac{33}{4}\ \frac{41}{4}\ \frac{49}{4}$	MA8 $\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	MA9 $\frac{69}{8}\ \frac{85}{8}\ \frac{101}{8}$
ICS	$\frac{3}{2}\ 2\ \frac{5}{2}$	$2\ \frac{5}{2}\ 3$	$2\ \frac{5}{2}\ 3$	$2\ \frac{5}{2}\ 3$	$\frac{11}{8}\ \frac{15}{8}\ \frac{19}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{3}{2}\ 2\ \frac{5}{2}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$
TS	$\frac{3}{2}\ 2\ \frac{5}{2}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{3}{2}\ 2\ \frac{5}{2}$	$2\ \frac{5}{2}\ 3$	$2\ \frac{5}{2}\ 3$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$
AOL	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$2\ \frac{5}{2}\ 3$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$
ACL	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$\frac{5}{2}\ 3\ \frac{7}{2}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$2\ \frac{5}{2}\ 3$	$2\ \frac{5}{2}\ 3$	$2\ \frac{5}{2}\ 3$

Table X: Sub-decision matrix for SWHSs considering the sub-Indicators of maintenance.

	MN1 7 9 11	MN2 7 9 11	MN3 $\frac{31}{4}\ \frac{39}{4}\ \frac{47}{4}$	MN4 $\frac{65}{8}\ \frac{81}{8}\ \frac{97}{8}$	MN5 8 10 12
ICS	$\frac{3}{2}\ 2\ \frac{5}{2}$	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$2\ \frac{5}{2}\ 3$	$2\ \frac{5}{2}\ 3$
TS	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$2\ \frac{5}{2}\ 3$
AOL	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$
ACL	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$	$2\ \frac{5}{2}\ 3$	$2\ \frac{5}{2}\ 3$

Table XI: Sub-decision matrix for SWHSs considering the sub-Indicators of reliability.

	RE1 $\frac{19}{2}\ \frac{23}{2}\ \frac{27}{2}$	RE2 $\frac{71}{8}\ \frac{87}{8}\ \frac{103}{8}$	RE3 $\frac{29}{4}\ \frac{37}{4}\ \frac{45}{4}$	RE4 $\frac{31}{4}\ \frac{39}{4}\ \frac{47}{4}$	RE5 $\frac{65}{8}\ \frac{81}{8}\ \frac{97}{8}$	RE6 $\frac{33}{4}\ \frac{41}{4}\ \frac{49}{4}$	RE7 $\frac{57}{8}\ \frac{73}{8}\ \frac{89}{8}$	RE8 $\frac{29}{4}\ \frac{37}{4}\ \frac{45}{4}$
ICS	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$
TS	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$
AOL	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$
ACL	$\frac{19}{8}\ \frac{23}{8}\ \frac{27}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{9}{4}\ \frac{11}{4}\ \frac{13}{4}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$

Table XII: Sub-decision matrix for SWHSs considering the sub-Indicators of life-cycle.

	LC1 $8\ 10\ 12$	LC2 $\frac{31}{4}\ \frac{39}{4}\ \frac{47}{4}$	LC3 $\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	LC4 $\frac{15}{2}\ \frac{19}{2}\ \frac{23}{2}$	LC5 $\frac{31}{4}\ \frac{39}{4}\ \frac{47}{4}$
ICS	$2\ \frac{5}{2}\ 3$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{13}{8}\ \frac{17}{8}\ \frac{21}{8}$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$
TS	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{7}{4}\ \frac{9}{4}\ \frac{11}{4}$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$
AOL	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$2\ \frac{5}{2}\ 3$
ACL	$2\ \frac{5}{2}\ 3$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$	$\frac{17}{8}\ \frac{21}{8}\ \frac{25}{8}$	$2\ \frac{5}{2}\ 3$	$\frac{15}{8}\ \frac{19}{8}\ \frac{23}{8}$

Table XIII: Weighted sub-decision matrix for the SWHSs considering the sub-indicators of environment.

	EN1	EN2	EN3	EN4	EN5	EN(Avg)
ICS	$\frac{175}{8}\ \frac{129}{4}\ \frac{357}{8}$	$\frac{59}{2}\ \frac{357}{8}\ \frac{273}{4}$	$\frac{153}{8}\ \frac{231}{8}\ \frac{325}{8}$	$\frac{1139}{64}\ \frac{1743}{64}\ \frac{2475}{64}$	$19\ \frac{115}{4}\ \frac{81}{2}$	$\frac{6867}{320}\ \frac{10351}{320}\ \frac{14891}{320}$
TS	$\frac{245}{16}\ \frac{387}{16}\ \frac{561}{16}$	$\frac{177}{8}\ \frac{75}{2}\ \frac{455}{8}$	$\frac{323}{16}\ \frac{483}{16}\ \frac{675}{16}$	$\frac{603}{32}\ \frac{913}{32}\ \frac{1287}{32}$	$13\ \frac{85}{4}\ \frac{63}{2}$	$\frac{2863}{160}\ \frac{4533}{160}\ \frac{6587}{160}$
AOL	$\frac{315}{16}\ \frac{473}{16}\ \frac{663}{16}$	$\frac{59}{2}\ \frac{357}{8}\ \frac{273}{4}$	$17\ \frac{105}{4}\ \frac{75}{2}$	$\frac{1005}{64}\ \frac{1577}{64}\ \frac{2277}{64}$	$16\ 25\ 36$	$\frac{1253}{64}\ \frac{9749}{320}\ \frac{14001}{320}$
ACL	$\frac{315}{16}\ \frac{473}{16}\ \frac{663}{16}$	$\frac{885}{32}\ \frac{1425}{32}\ \frac{2093}{32}$	$\frac{255}{16}\ \frac{399}{16}\ \frac{575}{16}$	$\frac{1139}{64}\ \frac{1743}{64}\ \frac{2475}{64}$	$16\ 25\ 36$	$\frac{6213}{320}\ \frac{9681}{320}\ \frac{13917}{320}$

Table XIV: Weighted sub-decision matrix for the SWHSs considering the sub-indicators of social.

	SO1	SO2	SO3	SO4	SO5	SO6	SO(Avg)
ICS	$\frac{1207}{64}\ \frac{1827}{64}\ \frac{2575}{64}$	$\frac{855}{64}\ \frac{1387}{64}\ \frac{2047}{64}$	$\frac{325}{32}\ \frac{561}{32}\ \frac{861}{32}$	$15\ \frac{95}{4}\ \frac{69}{2}$	$\frac{715}{64}\ \frac{1207}{64}\ \frac{1827}{64}$	$17\ \frac{105}{4}\ \frac{75}{2}$	$\frac{1825}{128}\ \frac{8743}{384}\ \frac{17945}{384}$
TS	$\frac{1349}{64}\ \frac{2001}{64}\ \frac{2781}{64}$	$\frac{399}{32}\ \frac{657}{32}\ \frac{979}{32}$	$\frac{125}{16}\ \frac{231}{16}\ \frac{369}{16}$	$15\ \frac{95}{4}\ \frac{69}{2}$	$\frac{825}{64}\ \frac{1349}{64}\ \frac{2001}{64}$	$\frac{153}{8}\ \frac{231}{8}\ \frac{325}{8}$	$\frac{707}{48}\ \frac{2239}{96}\ \frac{407}{12}$
AOL	$\frac{639}{32}\ \frac{957}{32}\ \frac{1339}{32}$	$\frac{741}{64}\ \frac{1241}{64}\ \frac{1869}{64}$	$\frac{75}{8}\ \frac{33}{2}\ \frac{205}{8}$	$18\ \frac{55}{2}\ 39$	$\frac{715}{64}\ \frac{1207}{64}\ \frac{1827}{64}$	$\frac{289}{16}\ \frac{441}{16}\ \frac{625}{16}$	$\frac{2821}{192}\ \frac{4471}{192}\ \frac{6505}{192}$
ACL	$\frac{1207}{64}\ \frac{1827}{64}\ \frac{2575}{64}$	$\frac{855}{64}\ \frac{1387}{64}\ \frac{2047}{64}$	$\frac{375}{32}\ \frac{627}{32}\ \frac{943}{32}$	$16\ 25\ 36$	$\frac{385}{32}\ \frac{639}{32}\ \frac{957}{32}$	$\frac{289}{16}\ \frac{441}{16}\ \frac{625}{16}$	$\frac{2881}{192}\ \frac{4555}{192}\ \frac{6613}{192}$

Table XV: Weighted sub-decision matrix for the SWHSs considering the sub-indicators of manufacturing.

MA1	MA2	MA3	MA4	MA5	MA6	MA7	MA8	MA9	MA(Avg)

ICS	(75/8, 33/2, 205/8)	(71/4, 435/16, 309/8)	(67/4, 415/16, 297/8)	(35/2, 215/8, 153/4)	(297/32, 525/32, 817/32)	(595/32, 903/32, 1275/32)	(561/32, 861/32, 1225/32)	(45/4, 19, 115/4)	(1311/64, 1955/64, 2727/64)	(985/64, 13925/576, 20137/576)
TS	(75/8, 33/2, 205/8)	(1207/64, 1827/64, 2575/64)	(1139/64, 1743/64, 2475/64)	(315/16, 473/16, 663/16)	(81/8, 35/2, 215/8)	(35/2, 215/8, 153/4)	(33/2, 205/8, 147/4)	(255/16, 399/16, 575/16)	(621/32, 935/32, 1313/32)	(2323/144, 226/9, 5197/144)
AOL	(325/32, 561/32, 861/32)	(639/32, 957/32, 1339/32)	(67/4, 415/16, 297/8)	(595/32, 903/32, 1275/32)	(405/32, 665/32, 989/32)	(315/16, 473/16, 663/16)	(561/32, 861/32, 1225/32)	(225/16, 361/16, 529/16)	(69/4, 425/16, 303/8)	(4693/288, 7295/288, 3491/96)
ACL	(325/32, 561/32, 861/32)	(355/16, 261/8, 721/16)	(603/32, 913/32, 1287/32)	(665/32, 989/32, 1377/32)	(27/2, 175/8, 129/4)	(665/32, 989/32, 1377/32)	(33/2, 205/8, 147/4)	(15, 95/4, 69/2)	(69/4, 425/16, 303/8)	(155/9, 1271/48, 2717/72)

Table XVI: Weighted sub-decision matrix for the SWHSs considering the sub-indicators of maintenance.

	MN1	MN2	MN3	MN4	MN5	MN(Avg)
ICS	(21/2, 18, 55/2)	(91/8, 153/8, 231/8)	(217/16, 351/16, 517/16)	(65/4, 405/16, 291/8)	16 25 36	(1083/80, 175/8, 2577/80)
TS	(91/8, 153/8, 231/8)	(105/8, 171/8, 253/8)	(31/2, 195/8, 141/4)	(585/32, 891/32, 1261/32)	16 25 36	(2377/160, 3767/160, 5477/160)
AOL	(49/4, 81/4, 121/4)	(91/8, 153/8, 231/8)	(31/2, 195/8, 141/4)	(975/64, 1539/64, 2231/64)	16 25 36	(4503/320, 7219/320, 2115/64)
ACL	(119/8, 189/8, 275/8)	(105/8, 171/8, 253/8)	(31/2, 195/8, 141/4)	(65/4, 405/16, 291/8)	16 25 36	(303/20, 383/16, 1389/40)

Table XVII: Weighted sub-decision matrix for the SWHSs considering the sub-indicators of reliability.

	RE1	RE2	RE3	RE4	RE5	RE6	RE7	RE8	RE(Avg)
ICS	(361/16, 529/16, 729/16)	(1349/64, 2001/64, 2781/64)	(203/16, 333/16, 495/16)	(465/32, 741/32, 1081/32)	(975/64, 1539/64, 2231/64)	(33/2, 205/8, 147/4)	(855/64, 1387/64, 2047/64)	(203/16, 333/16, 495/16)	(8233/512, 12829/512, 18449/512)
TS	(361/16, 529/16, 729/16)	(1349/64, 2001/64, 2781/64)	(203/16, 333/16, 495/16)	(465/32, 741/32, 1081/32)	(1105/64, 1701/64, 2425/64)	(495/32, 779/32, 1127/32)	(399/32, 657/32, 979/32)	(435/32, 703/32, 1035/32)	(4149/256, 6455/256, 9273/256)
AOL	(361/16, 529/16, 729/16)	(71/4, 435/16, 309/8)	(435/32, 703/32, 1035/32)	(527/32, 819/32, 1175/32)	(65/4, 405/16, 291/8)	(561/32, 861/32, 1225/32)	(399/32, 657/32, 979/32)	(435/32, 703/32, 1035/32)	(4167/256, 6481/256, 9307/256)
ACL	(361/16, 529/16, 729/16)	(1207/64, 1827/64, 2575/64)	(435/32, 703/32, 1035/32)	(465/32, 741/32, 1081/32)	(1105/64, 1701/64, 2425/64)	(297/16, 451/16, 637/16)	(399/32, 657/32, 979/32)	(203/16, 333/16, 495/16)	(4177/256, 6491/256, 9317/256)

Table XIX: Weighted sub-decision matrix for the SWHSs considering the sub-indicators of life-cycle.

	LC1	LC2	LC3	LC4	LC5	LC(Avg)
ICS	16 25 36	(217/16, 351/16, 517/16)	(195/16, 323/16, 483/16)	(105/8, 171/8, 253/8)	(465/32, 741/32, 1081/32)	(2221/160, 3573/160, 1049/32)
TS	(15, 95/4, 69/2)	(31/2, 195/8, 141/4)	(105/8, 171/8, 253/8)	(225/16, 361/16, 529/16)	(31/2, 195/8, 141/4)	(1171/80, 1863/80, 543/16)
AOL	(17, 105/4, 75/2)	(527/32, 819/32, 1175/32)	(15, 95/4, 69/2)	(225/16, 361/16, 529/16)	(31/2, 195/8, 141/4)	(2497/160, 3921/160, 1133/32)
ACL	16 25 36	(465/32, 741/32, 1081/32)	(255/16, 399/16, 575/16)	(15, 95/4, 69/2)	(465/32, 741/32, 1081/32)	(76/5, 120/5, 174/5)