Original Research Article

IDEAL ARRIVAL AND SERVICE CONTROL MODEL POLICY BASED ON FUZZY QUEUING SYSTEM OF UNCERTAINTY MEASURE

Abstract:

In this work, we investigate how to optimize costs for a single-server queuing system operating in a fuzzy, unpredictable environment. Constructing the overall optimal cost and cost function of the queuing system under uncertainty in the fuzzy paradigm is the aim of the inquiry. The fuzzy analysis is carried out to offer a more practical answer to the issues at hand, as opposed to the model's usual crisp responses. The model's crisp and fuzzy systems have different theoretical advances that have been determined, and the estimated costs are easily verifiable and comparable. Lastly, sensitivity analysis has also been carried out utilising numerical analysis to evaluate the theoretical conclusions of the model that is being studied.

Keywords: Fuzzy Optimization, Queuing model, Cost Function, Sensitivity Analysis, Uncertainty measure.

1. Introduction

Decisions like whether the water is safe to drink, how terrible the accident was, or even if a grading system is in place, are frequently presented to us when we are unable to reach a solid conclusion. Fuzzy information gives us more specific knowledge, which aids in decision-making in many circumstances when there is ongoing ambiguity. Numerous authors have discussed fuzzy logic and fuzzy set theory, which are used in fuzzyness. The difficulties and solutions associated with making judgments in fuzzy settings have been thoroughly studied by Bellman and Zadeh [2], and Kaufmann [17] has given a summary of the theory of fuzzy subsets for a number of application fields. Additionally, Zimmermann [38] covered fuzzy set theory and its applications in a variety of real-world practical contexts. Zadeh [39] created and applied fuzzy set theory to the theory of possibility, arguing that it forms the foundation of possibility theory. Markovian queues have a significant role in queuing theory research. Mishra and Shukla [23] detailed the computational approach to the cost analysis of the machine interference model for the theory of queues, whereas Mishra and Yadav [22] created and discussed the computational approach to cost and profit analysis of timed queuing networks in queuing theory. While Sharma [32] went into great length about the ideal flow control of a multi-server time sharing queuing network with priority in queuing theory, Priya and Sudhesh [28] addressed the topic of transient analysis of a discrete-time infinite server queue with system disaster.

The problem of employing an artificial neural network programme to simulate an M/M/1 queuing system was tackled by Sundari and Palaniammal [35], whereas Singh et al.

[34] studied a finite queuing model with reneging for a single server using queuing theory. The literature has widely established the importance of Morkovian queues,

and fuzzy queuing models seem to be far more useful in this context than normal crisp theory, providing more realistic solutions in a wide range of real-world applications.The arrival and service rates at the service station are fully probabilistic, but the mean arrival rate, the mean service rate, or both seem more possibilistic than probabilistic. Similarly, the numerical expressions are possibilistic.Conversely, fuzzy queuing models are more applicable and more grounded in reality than crisp queuing models in a wider range of domains. While Buckley [3] went into detail about the basic queuing theory based on possibility theory, Prado and Fuente [26] addressed the issues and applications of queuing theory in Markovian decision processes using fuzzy set theory. The analysis of fuzzy queues and their applications in a variety of fields were covered by Li and Lee [21].

In terms of queuing models, they fall into two categories: normative and descriptive. Normative type models are the most appropriate models for the given situation, while descriptive type queuing models are actual models that are observed in real-world scenarios.We optimise the arrival parameters, service, number of servers, queue discipline, controls, etc. for the specified queuing models in normative type models. As a result, normative models serve as an inspiration for queuing, whereas descriptive models represent queuing in actual situations. Queuing decision models or design and control models are other names for the descriptive kind of queuing models. Models in this category have parameters that are calculated in a way that should maximise model performance. These queuing models were developed by Negi and Lee [24] and examined in a fuzzy setting. Kao et al. [16] focused on the parametric programming of such queuing models and analysed them in a fuzzy environment, while Jo et al. [14] examined the performance evaluation of networks based on fuzzy queuing systems. In queuing theory, control models are crucial, and among them, the service control is dependent on a number of indicators. In queuing theory, control models play a crucial role. Among these models, service control is dependent on a number of different metrics, including service rate, server count, queue discipline, or a mix of these. In queuing theory, arrival control is also crucial and can be achieved by allocating arriving patrons to certain servers or by dispersing them among them. Arrivals can be managed by toll devices or workable restrictions, such as setting up parameters for physical space and working hours, among other things. In queuing theory, Buckley et al. Chen [6] described the bulk arrival queuing model with fuzzy parameters and varying batch sizes and given the solution through fuzzy set theory. Ke and Lin [18] used a nonlinear programming technique to work on the fuzzy analysis of queuing systems with an unreliable server. [4] worked on defining these parameters in the context of queuing theory and offered a fuzzy expert system solution.

Subsequently, an emerging paradigm in queuing models emerged, wherein the models are optimised using unreliable data inputs. By using a fuzzy expert system, the model is developed using this input data uncertainty. Fuzzy coefficients and parameters of the queuing models are optimised using fuzzy optimisation techniques in fuzzy expert systems. The single value simulation on fuzzy variables in queuing theory was worked on by Chanas and Nowakowski [5], while the economic study of the M/M/1/N queuing system cost model in a hazy environment was covered by Fazlollahtabar and Gholizadeh [10].While Palpandi and Geetharamani [25] worked on the evaluation of performance measures of bulk arrival queue with fuzzy parameters using a robust ranking technique, Prameela and Kumar [27] described the FM / FEk /1 queuing model with Erlang service under various types of fuzzy numbers, and Sanga et al.Using a parametric nonlinear programming technique, [31] worked on the

FM/FM/1 double orbit retrial queue with customers' joining strategies. When cost coefficients, arrival, and service parameters are exact and known, a variety of techniques are used to derive the solutions of design and control models for performance measures. In contrast, the standard queuing decision models do not offer trustworthy estimates of the parameters of the models under consideration due to imprecision and ambiguity situations that are outside of human control if these parameters of these models are imprecise and vary over time, such as waiting cost per unit. In such a scenario, an intervention is required to determine the impact that could enable the system to function. These scenarios can be handled more skillfully by fuzzy queuing decision models, which may be investigated and explored. Fathi Vajargah and Ghasemalipour [9] worked on the simulation research of a random fuzzy queuing system with Barak and Fallahnezhad [1] discussing the cost analysis of fuzzy queuing systems.In such a scenario, an intervention is required to determine the impact that could enable the system to function. These scenarios can be handled more skillfully by fuzzy queuing decision models, which may be investigated and explored. While Fathi Vajargah and Ghasemalipour [9] worked on the simulation research of a random fuzzy queuing system with several servers, Barak and Fallahnezhad [1] talked about the cost analysis of fuzzy queuing systems. While Kannadasan and Sathiyamoorth [15] worked on the analysis of M/M/1 queue with working vacation in fuzzy environment, Enrique and Enrique [8] discussed the simulation of fuzzy queuing systems with a variable number of servers, arrival, and service rates, and Gou et al.[11] explained the alternate queuing technique for multiple criteria decision making as well as the hesitant fuzzy linguistic entropy and cross-entropy metrics. Ke and Lin [18] conducted a study on the fuzzy analysis of queuing systems with an unreliable server: A nonlinear programming approach, whereas Chen et al. [7] examined the analysis of strategic consumer behaviour in fuzzy queueing systems. While Qin et al. [29] focused on linguistic interval-valued intuitionistic fuzzy archimedean power muirhead mean operators for multiattribute group decision-making, Keith and Ahner [20] conducted a survey of decision making and optimisation under uncertainty. The Markovian arrival and service queuing model under fuzziness was optimised by Singh et al. [33]. Mishra et al. [36] have analysed the Markovian queueing model using neural networks and the signed distance approach.The current work employs a single server and both arrival and control queuing models in an unpredictable, fuzzy environment. The fuzzy environment of uncertainty produces better results for the queuing model with one or more uncertain parameters that need to be optimised for designing and controlling the queuing model under consideration because the fuzzy parameter estimates are more practical and realistic than the crisp model estimates. While fuzzy model FM/FM/1 reflects and discusses real scenarios of waiting lines and their outcomes, crisp model M/M/1 characterises ideal situations of waiting time analysis and its conclusions. It is also a well-known fact that there is never an optimal condition when there is a lineup or queue.As a key performance metric under the control design of arrival and service of FM/FM/1model and its optimisation, we aim to construct the total cost function of arrival and control queuing model with single server in fuzzy environment of uncertainty in the current inquiry. Here, we suggest fuzzifying the total cost function using a trapezoidal system of fuzzy numbers, and then defuzzifying the model FM/FM/1 using an effective signed distance method (SDM). The process of optimising the total cost function yields a system of nonlinear equations involving model parameters. These equations are solved using R software to determine the model's optimal performance measure. Here, the optimal total optimal cost of the queuing model FM/FM/1 under consideration serves as the optimal performance metric. Finally, the model's numerical demonstration has been used to do the sensitivity analysis. In comparison to earlier models, the FM/FM/1 model under examination is perhaps more economical and efficient.

Notations: Following notations are used in the paper.
TC= Optimal Total Cost (OTC) , k = Service cost per unit (SC), c = Waiting cost per unit (WC), λ = Arrival rate of customer (ARC), μ = Service rate. (SR), n = Number of customer (NC), $\tilde{k}$ = Fuzzified Service cost per unit (FSC), $\tilde{c}$ = Fuzzified Waiting cost per unit(FWC), $\tilde{\lambda}$ = Fuzzified Arrival Rate (FAR), $\tilde{n}$ = Fuzzified Number of customer (FNC)

2.Arrival Control Model

In general, the arrival control queuing model is a stochastic input-output system where the input process is controlled by whether entering consumers are accepted or rejected. These models incorporate as special examples a number of popular queuing systems. These models take into account the form of both individually and socially optimal acceptance rules when there are incentives and waiting costs associated with accepted clients (Johansen and Stidham [13]).

Chen [6] proposed a solution for a bulk arrival queuing model with fuzzy parameters and variable batch sizes, while Johansen and Stidham [13] investigated the control of arrivals to a stochastic input-output system. Ke and others. Walker and Bright [37] went into great length on modelling arrival-to-departure sequence disorder in flowcontrolled manufacturing systems, and [19] proposed a method for managing arrivals for a markovian queuing system with a second optional service. The solution for queue-based modelling of the aircraft arrival process at a single airport was presented by Itoh and Mitici [12], while Samanta [30] investigated the D-BMAP/G/1 queuing system's waiting-time analysis. Below is a discussion of the arrival control system's mathematical formulations for both the fuzzy and crisp paradigms:

1.1Arrival Control Model: Crisp Mathematical Formulation

The model's total cost function is specified as

$$TC = k\mu + cE(n)$$

This implies that $TC = k\mu + \frac{c\lambda\lambda}{\mu}$

In terms of service least cost, we have

$$\frac{d}{d\mu}[TC] = \frac{d}{d\mu}\{TC = k\mu + \frac{c\lambda}{\mu}\}$$

For stationary values $\frac{d}{d\mu}[TC]=0$, this ultimately yields

$$\mu = \left(\frac{c\lambda}{\mu}\right)^{\frac{1}{2}}$$

Along with having satisfied $\frac{d^2}{d\mu^2}[TC] = \frac{2c\lambda}{\mu^3} > 0$for minimum cost.

1.2 Fuzzy Mathematical Model

Further, we define a trapezoidal fuzzy number $\tilde{A}$ = (a,b,c,d)with membership function

$$\mu_A(X) = \begin{cases} L(X) = \frac{x-a}{a-b}, when\ a \le x \le b \\ 1, when\ b \le x \le c \\ R(X) = \frac{c-x}{c-d}, when\ c \le x \le d \\ 0, otherwise \end{cases}$$

Now, we want to use trapezoidal fuzzy numbers $\tilde{k}, \tilde{c}, \lambda$ to fuzzify cost coefficients and arrival rates k, c. λ. We've gone on as

$\tilde{k}$ = (k_1, k_2, k_3, k_4) , $\tilde{c}$=(c_1, c_2, c_3, c_4), $\tilde{\lambda}$ =($\lambda_1, \lambda_2, \lambda_3, \lambda_4$)

Thus, fuzzified total cost turns out to be as T$\tilde{C}$ = $\tilde{k}\mu + \frac{\tilde{c}\tilde{\lambda}}{\mu}$which implies that

$\tilde{T}$ C = { $k_1\mu + \frac{c_1\lambda_1}{\mu}$, $k_2\mu + \frac{c_2\lambda_2}{\mu}$, $k_3\mu + \frac{c_3\lambda_3}{\mu}$, $k_4\mu + \frac{c_4\lambda_4}{\mu}$ }

Which is finally expressed as $\tilde{T}$ C= (W,X,Y,Z)

W=$k_1\mu + \frac{c_1\lambda_1}{\mu}$, X = $k_2\mu + \frac{c_2\lambda_2}{\mu}$, Y=$k_3\mu + \frac{c_3\lambda_3}{\mu}$, Z= $k_4\mu + \frac{c_4\lambda_4}{\mu}$

Now, we define following as

$C_L(\alpha) = W + (X - W)\alpha = k_1\mu + \frac{c_1\lambda_1}{\mu} + [k_2\mu + \frac{c_2\lambda_2}{\mu} - (k_1\mu + \frac{c_1\lambda_1}{\mu})]\alpha$ *and*

$C_R(\alpha) = Y - (Y - Z)\alpha = k_3\mu + \frac{c_3\lambda_3}{\mu} - [k_3\mu + \frac{c_3\lambda_3}{\mu} - (k_4\mu + \frac{c_4\lambda_4}{\mu})]\alpha$

Next, we define SDM and apply for present model.

$$\tilde{T}C_{ds} = \frac{1}{2}\int_0^1 [C_L(\alpha) + C_R(\alpha)]d\alpha$$

$$\tilde{T}C_{ds} = \frac{1}{4}[(k_1 + k_2 + k_3 + k_4)\mu + \frac{1}{\mu}(C_1\lambda_1 + C_2\lambda_2 + C_3\lambda_3 + C_4\lambda_4)]$$

Given that we now have stationary value, the minimal fuzzified cost can be achieved.

$$\frac{d\tilde{T}C_{ds}}{d\mu} = \frac{1}{4}[(k_1 + k_2 + k_3 + k_4) - \frac{1}{\mu^2}(C_1\lambda_1 + C_2\lambda_2 + C_3\lambda_3 + C_4\lambda_4)]=0,$$

Yielding result as $\mu = (\frac{C_1\lambda_1 + C_2\lambda_2 + C_3\lambda_3 + C_4\lambda_4}{k_1 + k_2 + k_3 + k_4})^{\frac{1}{2}}$ having satisfied the condition as

$$\frac{d^2\tilde{T}C_{ds}}{d\mu^2} = \frac{1}{2}[\frac{C_1\lambda_1 + C_2\lambda_2 + C_3\lambda_3 + C_4\lambda_4}{\mu^3}] > 0$$

1.3 Arrival Control Model: Crisp Computation:

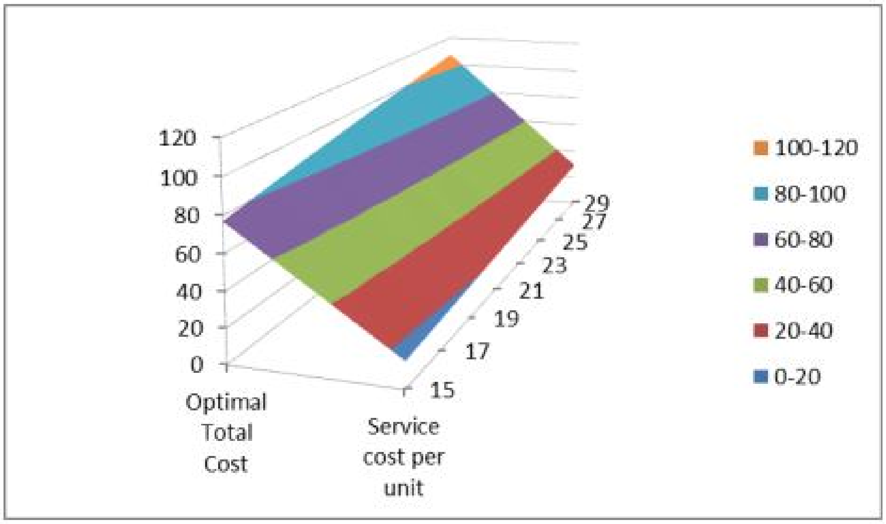
Following table 1 represents computation of cost function under crisp environment.

Table - 1(Computation table for k, TC)

k	c	λ	μ	TC
11	7	5	1.78	39.24
13	7	5	1.64	42.66

15	7	5	1.526	45.82
17	7	5	1.431	48.785
19	7	5	1.357	51.57
21	7	5	1.290	54.22
23	7	5	1.233	56.739
25	7	5	1.18	58.66

Figure-1: Variation of optimal total cost and service cost per unit



1.4Arrival Control Model: Fuzzy Computation:

Following tables from table 2 represents computation of cost function under fuzzy environment

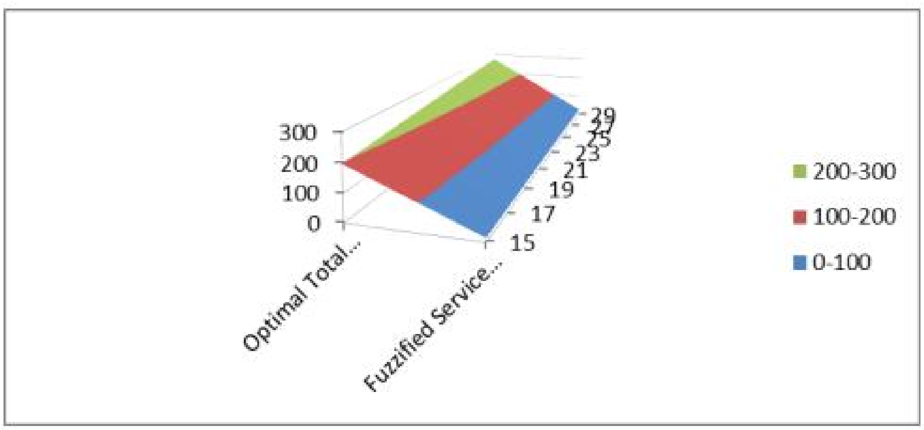
Table: 2 (Computation table for $\tilde{k}$,$\tilde{T}$ C)

$\tilde{k}$				$\tilde{c}$			
$k_{1,}$	$k_{2,}$	$k_{3,}$	k_4	$c_{1,}$	$c_{2,}$	$c_{3,}$	c_4
8	10	12	14	4	6	8	10
10	12	14	16	4	6	8	10
12	14	16	18	4	6	8	10
14	16	18	20	4	6	8	10

16	18	20	22	4	6	8	10
18	20	22	24	4	6	8	10
20	22	24	26	4	6	8	10
22	24	26	28	4	6	8	10

$\tilde{\lambda}$				μ	$\tilde{T}$ C
λ_1	λ_2	λ_3	λ_4		
2	4	6	8	1.90	41.95
2	4	6	8	1.75	45.60
2	4	6	8	1.63	48.98
2	4	6	8	1.53	52.15
2	4	6	8	1.44	55.13
2	4	6	8	1.37	57.96
2	4	6	8	1.31	60.66
2	4	6	8	1.26	63.24

Figure-1: Optimal total cost and fuzzified service



1. Service Control Model: Crisp Mathematical Formulation

This is the model's total cost function.

$$TC = kn\mu + cE(n).$$

This implies that

$$TC = kn\mu + c\frac{\lambda}{\mu}$$

In terms of service least cost, we have

$$\frac{d\,[TC]}{d\mu} = \frac{d}{d\mu}\left(kn\mu + \frac{C\lambda}{\mu}\right)$$

For stationary values $\frac{d\,[TC]}{d\mu} = 0$, this ultimately yields

$$\mu = \left(\frac{C\lambda}{kn}\right)^{\frac{1}{2}}$$

Alongwith having satisfied

$$\frac{d^2[TC]}{d\mu^2} = \frac{2c\lambda}{\mu^3} > 0$$

For minimum cost.

1.5Service Control Model: Fuzzy Mathematical Model:

Now, we wish to fuzzify cost coefficients and arrival rates k, c, λ, and n with the help of trapezoidal fuzzy numbers (defined by function A) as , $\tilde{k}, \tilde{c}$, $\tilde{\lambda}$ and $\tilde{n}$ respectively depicted as

$\tilde{k} = (k_1, k_2, k_3, k_4)$, $\tilde{c} = (c_1, c_2, c_3, c_4)$, $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ $\tilde{n} = (n_1, n_2, n_3, n_4)$

Now, we define fuzzified total cost as

$\widetilde{TC} = \tilde{k}\tilde{n}\mu + \frac{\tilde{c}\tilde{\lambda}}{\mu}$, which implies that

$$\tilde{T}C = \left\{ k_1 n_1 \mu + \frac{c_1\lambda_1}{\mu},\ k_2 n_2 \mu + \frac{c_2\lambda_2}{\mu},\ k_3 n_3 \mu + \frac{c_3\lambda_3}{\mu},\ k_4 n_4 \mu + \frac{c_4\lambda_4}{\mu} \right\}$$

This is further simplified as $\tilde{T}C = (W, X, Y, Z)$ where

Now , we define left and right cuts as

$$C_L(\alpha) = W + (X - W)\alpha = k_1 n_1 \mu + \frac{c_1\lambda_1}{\mu} + \left[k_2 n_2 \mu + \frac{c_2\lambda_2}{\mu} - \left(k_1 n_1 \mu + \frac{c_1\lambda_1}{\mu}\right)\right]\alpha$$

and

$C_R(\alpha) = Y - (Y - Z)\alpha \;=\; k_3 n_3 \mu \;+\; \frac{C_3 \lambda_3}{\mu - \lambda_3}$ - $[k_3 n_3 \mu \;+\; \frac{C_3 \lambda_3}{\mu - \lambda_3}$ - $(k_4 n_4 \mu \;+\; \frac{C_4 \lambda_4}{\mu})]\alpha$

Now , we apply SDM as

$\tilde{T}C_{ds} = \frac{1}{2}\int_0^1 [C_L(\alpha) + C_R(\alpha)]d\alpha$

$\tilde{T}C_{ds} = \frac{1}{4}[(k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4)\mu \;+\; \frac{1}{\mu}(C_1 \lambda_1 + C_2 \lambda_2 + C_3 \lambda_3 + C_4 \lambda_4)]$

Now, in order to attain minimum fuzzified cost , we have now stationary value

$\frac{d\tilde{T}C_{ds}}{d\mu} = \frac{1}{4}[(k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4) - \frac{1}{\mu^2}(C_1 \lambda_1 + C_2 \lambda_2 + C_3 \lambda_3 + C_4 \lambda_4)] = 0,$

Yielding result as

$$\mu = \left(\frac{C_1 \lambda_1 + C_2 \lambda_2 + C_3 \lambda_3 + C_4 \lambda_4}{k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4}\right)^{\frac{1}{2}}$$

Having satisfied the condition as

$\frac{d^2\tilde{T}C_{ds}}{d\mu^2} = \frac{1}{2}\left[\frac{C_1 \lambda_1 + C_2 \lambda_2 + C_3 \lambda_3 + C_4 \lambda_4}{\mu^3}\right] > 0$

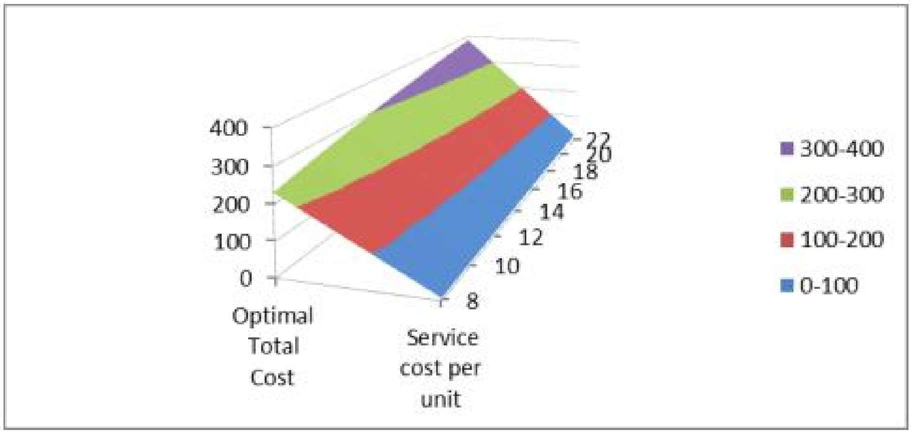
1.6Service Control Model: Crisp Computation

Following tables 3 represents computation of different cost functions under crisp environment.

Table - 3 (Computation table for k, TC)

k	c	λ	n	μ	TC
6	8	6	10	0.89	107.33
8	8	6	10	0.77	123.93
10	8	6	10	0.69	138.56
12	8	6	10	0.63	151.79
14	8	6	10	0.58	163.95
16	8	6	10	0.54	175.28
18	8	6	10	0.50	186
20	8	6	10	0.48	196

Figure-3: Variation of Optimal Total Cost and Service cost per unit



1.7Service Control Model: Fuzzy Computation

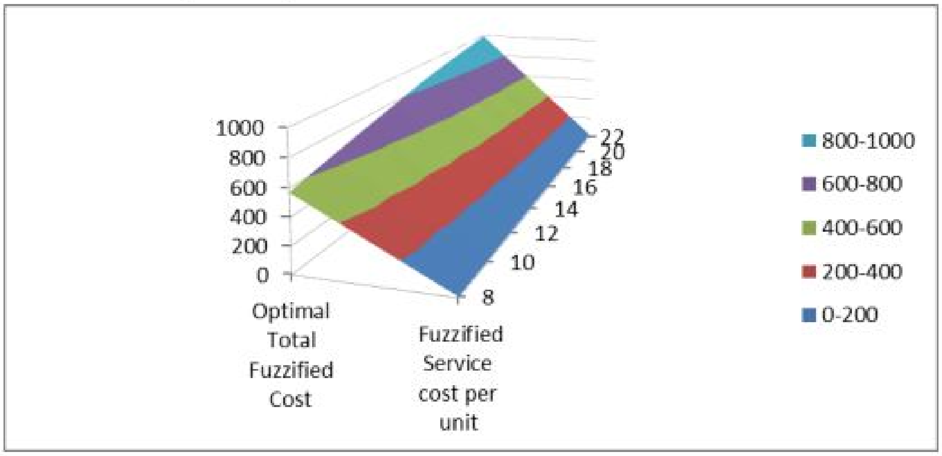
Following tables 4 represents computation of different cost functions under fuzzy environment

Table: 4 (Computation table for $\tilde{k}$,$\tilde{T}$C)

$\tilde{\lambda}$				$\tilde{c}$			
λ_1	λ_2	λ_3	λ_4	c_1	c_2	c_3	c_4
5	7	9	11	7	9	11	13
5	7	9	11	7	9	11	13
5	7	9	11	7	9	11	13
5	7	9	11	7	9	11	13
5	7	9	11	7	9	11	13
5	7	9	11	7	9	11	13
5	7	9	11	7	9	11	13
5	7	9	11	7	9	11	13

$\tilde{k}$				$\tilde{n}$				μ	$\tilde{T}$C
k_1	k_2	k_3	k_4	n_1	n_2	n_3	n_4		
3	5	7	9	5	7	9	11	1.26	134.24
5	7	9	11	5	7	9	11	1.109	153.17
7	9	11	13	5	7	9	11	1.00	170
9	11	13	15	5	7	9	11	0.91	185.31
11	13	15	17	5	7	9	11	0.87	192.84
13	15	17	19	5	7	9	11	0.79	212.66
15	17	19	21	5	7	9	11	0.75	225.08
17	19	21	23	5	7	9	11	0.71	236.86

Figure-4: Optimal total fuzzified cost and fuzzified service cost



2. Arrival and Service Control Model: Crisp Mathematical Model

The definition of the model's total cost function is

$$TC = kn\mu + \frac{c\lambda}{\mu - \lambda}$$

In terms of service least cost, we have

$$\frac{d\,[TC]}{d\mu} = \frac{d}{d\mu}\left(kn\mu + \frac{C\lambda}{\mu - \lambda}\right)$$

For stationary values $\frac{d\,[TC]}{d\mu} = 0$,

This ultimately yields $\mu = \lambda + \left(\frac{c\lambda}{kn}\right)^{\frac{1}{2}}$(for max service), along with satisfied

$$\frac{d^2[TC]}{d\mu^2} = \frac{2c\lambda}{(\mu - \lambda)^3} > 0$$

For minimum cost.

OUR RESULTS:

2.1 Arrival and Service Control Model: Fuzzy Mathematical Model

Claim 2.1 Assuming that $\widetilde{TC}$ represents the fuzzified total cost function demonstrate that $\frac{1}{2}\left[\frac{c_3\lambda_3}{(\mu - \lambda_3)^3}\right] > 0$ and hence deduce $\mu = \left(\frac{c_3\lambda_3}{(k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4)}\right)^{\frac{1}{2}} + \lambda_3$.

Proof: We construct fuzzified total cost function as

$$\tilde{TC} = \tilde{k}\tilde{n}\mu + \frac{\tilde{c}\tilde{\lambda}}{\mu - \tilde{\lambda}}$$

With the use of trapezoidal fuzzy numbers (specified by function A), $\tilde{k}, \tilde{c}$, $\tilde{\lambda}$ and $\tilde{n}$ we now want to fuzzify cost coefficients and arrival rates k, c, λ, and n, respectively.

$k \rightarrow \tilde{k}$, $c \rightarrow \tilde{c}$, $\lambda \rightarrow \tilde{\lambda}$, $n \rightarrow \tilde{n}$

$\tilde{k} = (k_1, k_2, k_3, k_4)$, $\tilde{c} = (c_1, c_2, c_3, c_4)$, $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ $\tilde{n} = (n_1, n_2, n_3, n_4)$

Further , we have

$\tilde{TC} = \tilde{k}\tilde{n}\mu + \frac{\tilde{c}\tilde{\lambda}}{\mu - \tilde{\lambda}}$, which implies that

$$\tilde{T}C = ((k_1, k_2, k_3, k_4)(n_1, n_2, n_3, n_4)\mu + \frac{(c_1, c_2, c_3, c_4)(\lambda_1, \lambda_2, \lambda_3, \lambda_4)}{\mu - (\lambda_1, \lambda_2, \lambda_3, \lambda_4)}$$

This is expressed as

$\tilde{T}C = (W, X, Y, Z)$, where

$W = k_1 n_1 \mu - c_1$, $X = k_2 n_2 \mu - c_2$, $Y = k_3 n_3 \mu - \frac{c_3 \lambda_3}{\mu - \lambda_3}$, $Z = k_4 n_4 \mu - c_4$

We now define cuts to the left and right as

$C_L(\alpha) = W + (X - W)\alpha = (k_1 n_1 \mu - c_1) + [(k_2 n_2 \mu - c_2) - (k_1 n_1 \mu - c_1)]\alpha$ and

$C_R(\alpha) = Y - (Y - Z)\alpha = (k_3 n_3 \mu - \frac{c_3 \lambda_3}{\mu - \lambda_3}) - [(k_3 n_3 \mu - \frac{c_3 \lambda_3}{\mu - \lambda_3}) - (k_4 n_4 \mu - c_4)]\alpha$

Next, we apply SDM as

$$\tilde{T}C_{ds} = \frac{1}{2}\int_0^1 [C_L(\alpha) + C_R(\alpha)]d\alpha$$

$$\tilde{T}C_{ds} = \frac{1}{4}[(k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4)\mu - (c_1 + c_2 + c_4) + -\frac{c_3 \lambda_3}{\mu - \lambda_3}$$

Now, for minimum cost with respect to μ

$$\frac{d\tilde{T}C_{ds}}{d\mu} = \frac{1}{4}[(k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4) - \frac{c_3 \lambda_3}{(\mu - \lambda_3)^2}] = 0$$

With sufficient condition

$$\frac{d^2\tilde{T}C_{ds}}{d\mu^2} = \frac{1}{2}\left[\frac{C_3\,\lambda_3}{(\mu-\lambda_3)^3}\right] > 0$$

Which ultimately gives us

$$(k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4)(\mu - \lambda_3)^2 - c_3\,\lambda_3 = 0$$

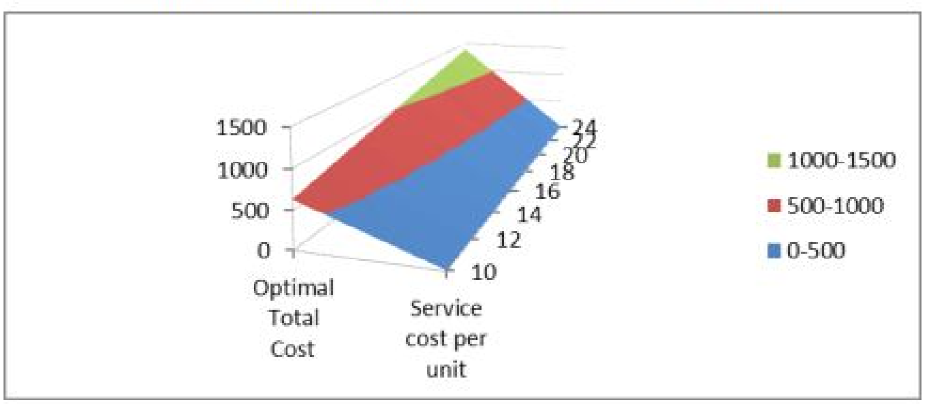
$$\mu = \left(\frac{c_3\,\lambda_3}{(k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4)}\right)^{\frac{1}{2}} + \lambda_3$$

2.2 Arrival and Service Control Model: Crisp Computation

Table - 5 (Computation table for k, TC)

k	c	λ	n	μ	TC
8	10	6	4	7.36	279.63
10	10	6	4	7.22	337.98
12	10	6	4	7.11	395.33
14	10	6	4	7.03	451.93
16	10	6	4	6.96	507.94
18	10	6	4	6.91	563.45
20	10	6	4	6.86	618.56
22	10	6	4	6.82	673.33

Figure-5: Variation of Optimal Total Cost and Service cost per unit



2.3 Arrival and Service Control Model: Fuzzy Computation

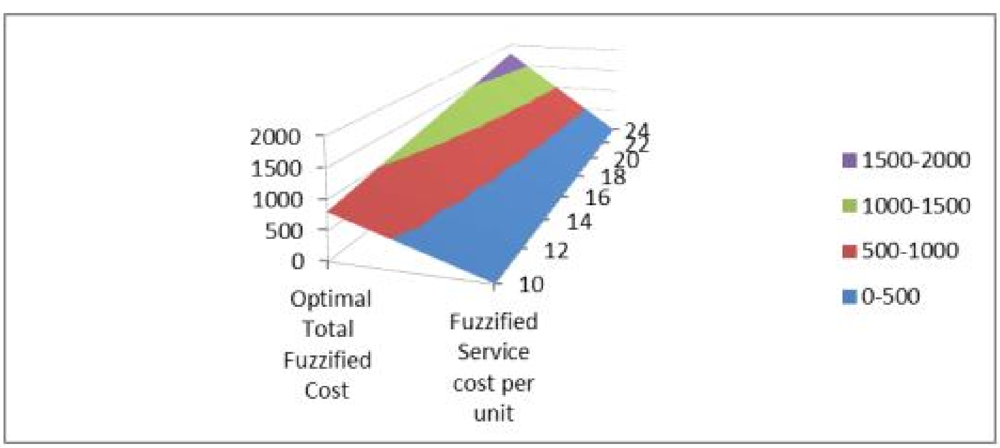
Following tables 6 represents computation of different cost functions under fuzzy environment.

Table: 6 (Computation table for $\tilde{k}$,$\tilde{T}$ C)

$\tilde{\lambda}$				$\tilde{c}$			
λ_1	λ_2	λ_3	λ_4	c_1	c_2	c_3	c_4
3	5	7	9	7	9	11	13
3	5	7	9	7	9	11	13
3	5	7	9	7	9	11	13
3	5	7	9	7	9	11	13
3	5	7	9	7	9	11	13
3	5	7	9	7	9	11	13
3	5	7	9	7	9	11	13
3	5	7	9	7	9	11	13

$\tilde{k}$				$\tilde{n}$				μ	$\tilde{T}$C
k_1	k_2	k_3	k_4	n_1	n_2	n_3	n_4		
5	7	9	11	1	3	5	7	7.72	305.12
7	9	11	13	1	3	5	7	7.64	366.62
9	11	13	15	1	3	5	7	7.6	430.55
11	13	15	17	1	3	5	7	7.55	488.3
13	15	17	19	1	3	5	7	7.51	548.68
15	17	19	21	1	3	5	7	7.5	608.75
17	19	21	23	1	3	5	7	7.46	668.69
19	21	23	25	1	3	5	7	7.44	728.42

Figure-6: Variation of Optimal Total Fuzzified Cost and Fuzzified Service cost/unit



3. Conclusion

For the current study, we constructed a queuing model in a fuzzy environment since it provides a more workable solution when human control over measurements is not feasible. The total optimal cost of the studied queuing model with a single server in a fuzzy environment has been found to offer a better solution than the queuing model in a crisp environment.The results may be seen to be readily comparable for both the fuzzy and the crisp surroundings. Numerous applications including uncertainty may result from the further extending of the studied queuing model to the fuzzy environment. The current work may provide strong results that are crucial for building queuing models for fuzzy environments, and it may be highly instructive for researchers in the field. These queuing models are crucial for creating a more effective waiting line system that will satisfy our workforce demands and address issues with pricing, settlement, arrival management, service quality, decreasing client wait times, and increasing the number of customers served. In addition, these models could be helpful in offering operational management techniques for scheduling and inventory control to improve customer service in businesses where there are naturally occurring lines. Six sigma practitioners should also be aware of the current Markovian queuing model strategies in order to improve customer service in various organisations. For additional research, fuzzy neuro and intuitionistic fuzzy techniques would be more practical for analysing such queuing models.

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