Modeling Heat and Mass Transfer in a Wet Terracotta Tube Channel for Evaporative Cooling Application: Influence of geometrical parameters

ABSTRACT

This study investigates the influence of design parameters on the performance of a terracotta tube-type evaporative cooling system. A mathematical model was developed based on double film theory and energy and mass conservation equations of the humid air and the wet tube wall in a one-dimensional geometry by applying correlations for heat and mass transfer coefficients and air psychometric properties.A system of non-linear differential equations was established and analytically integrated to obtain the relationship between the operating and the geometrical parameters of the system. Various geometrical parameters were simulated to assess their effects on outlet air temperature, cooling capacity, and wet-bulb effectiveness. The results indicate that increasing the tube equivalent diameter results in higher outlet temperatures and cooling capacity but decreases wet-bulb effectiveness. Conversely, an increased flatness ratio significantly enhances cooling performance and wet-bulb effectiveness due to a larger surface area for heat exchange. Additionally, longer tubes correlate with lower outlet temperatures and higher cooling capacity, indicating improved cooling performance. These findings emphasize the importance of selecting appropriate tube dimensions to optimize cooling efficiency in evaporative cooling systems. By balancing hydraulic diameter, flatness ratio, and tube length, engineers can design compact and effective cooling solutions suitable for various applications, ultimately contributing to energy-efficient and sustainable cooling technologies.

Keywords: Mathematical modeling, porous terracotta tube, direct evaporative cooling, tubular heat and mass exchanger.

1. INTRODUCTION

As global temperatures continue to rise and urbanization accelerates, the demand for energy-efficient cooling solutions has never been more critical. Traditional air conditioning systems, while effective, contribute significantly to energy consumption and greenhouse gas emissions, prompting researchers and engineers to explore alternative cooling technologies. Among these, evaporative cooling stands out as a sustainable and eco-friendly solution, leveraging the natural process of water evaporation to provide effective temperature regulation in indoor environments.

Terracotta, a porous ceramic material, has garnered attention for its potential in evaporative cooling applications due to its unique properties. The material's thermal mass and porosity can enhance heat and mass transfer processes, making it an attractive choice for evaporative cooling systems. Due to its high porosity, low density, large specific surface area, and high thermal conductivity[1], porous ceramic enhances EC performance in several ways. Firstly, its porosity and capillary action improve surface hydrophilicity, enhancing wettability. Secondly, its large specific surface area increases the contact between the working air and the water film. Thirdly, the porous structure acts as a water reservoir, allowing for intermittent rather than continuous water spraying, which reduces the energy consumption of the circulating pump[2]. Among the various shapes of porous ceramic media, hollow clay tubes arranged to form a bundle also gain popularity as an evaporative cooling medium. Early research in porous ceramic tubular evaporative cooling focuses on cross-flow semi-indirect configurations[3], [4]. Except for Semi-IEC, the integration of porous ceramic tubes with heat pipes (HP) for indirect evaporative cooling systems has also been extensibly investigated. The use of porous ceramic tubes allows for effective heat and mass transfer, while heat pipes can provide efficient heat recovery for sensible cooling application. For instance, Amer and Boukhanouf[5] conducted an experimental investigation to evaluate the effect of various operation conditions on a novel heat pipe and ceramic tube based evaporative cooler. Their cooler was able to drop the inlet air temperature by 12℃, and wet bulb effectiveness of 86% was achievable. More recently, Rajski et al. [6] developed mathematical model to investigate the performance of gravity-assisted heat pipe-based indirect evaporative cooler (GAHP-based IEC). They state that the proposed cooler can be used as a complementary device to the conventional HVAC systems. By modeling the heat and mass transfer dynamics within wet terracotta tube channels, researchers can elucidate the influence of various geometrical parameters, such as tube diameter, length, and flatness ratio, on the overall cooling performance.

Several studies have explored the impact of tube geometry on evaporative cooling efficiency. Adam et al.[7] performed analysis of indirect evaporative cooler performance under various heat and mass exchanger dimensions and flow parameters and found that the optimal dimensions that give good efficiency in climates with moderate humidity, the length of the duct should be between 0.6 to 1.0 m, the width of the channel between 0.3 to 0.5 m, and the channel gap between 0.004 to 0.008 m. Similarly, Sun et al.[8] reported that increasing the equivalent diameter of porous ceramic pipes in an indirect evaporative cooler resulted in lower outlet temperatures and higher wet-bulb effectiveness.

Recent research has largely focused on round tubular EC, but flat tubular variants, which offer several advantages, have not been as thoroughly investigated. Unlike round tubular ECs, flat tubular ECs provide superior wetting characteristics, leading to better formation of water films and more efficient use of water's latent heat. Additionally, flat tubular ECs lead to more compact systems rather than round tubular configuration. These advantages have made flat tubular ECs a growing area of interest. Hasan and Sirén [9] provides a comprehensive overview of evaporative cooling systems, highlighting the advantages of flat tubular configurations in terms of surface area and heat exchange efficiency. The increased surface area in flat tubes facilitates better contact between the air and the wetted surface, leading to enhanced evaporative cooling performance, particularly in hot and dry climates. Liu et al.[10] developed a direct-expansion ice thermal storage system that incorporates a multi-channel flat-tubular evaporator, analyzing key factors to enhance system performance. Existing studies have explored the impact of geometric parameters on the cooling performance of Terracotta Tubular ECs (TTEC) through experimental testing and numerical simulations. Despite these advancements, existing studies still lack efficiency and effectiveness in evaluating and predicting TTEC performance. Optimization strategies for this kind of cooler remain underexplored, and there is a pressing need for direct, pragmatic approaches for it performance evaluation. To date, no research has developed performance prediction models for TTEC using analytical methods or engaged in comprehensive multi-objective optimization.

This study focuses on the innovative application of wet terracotta tube channels in evaporative cooling systems, where the unique properties of terracotta such as its thermal mass and porosity can enhance heat and mass transfer processes. By modeling the heat and mass transfer dynamics within these tube channels, we aim to elucidate the influence of various geometrical parameters, including tube diameter, length, and flatness ratio, on the overall cooling performance.Understanding the intricate relationship between these geometrical factors and system efficiency is essential for optimizing the design of terracotta tube-based evaporative coolers. As we delve into the modeling of these systems, we not only seek to advance the scientific understanding of heat and mass transfer in porous materials but also to contribute to the development of more efficient and sustainable cooling technologies. This research holds the potential to inform the design of next-generation evaporative cooling systems that can significantly reduce energy consumption while providing effective climate control in a variety of applications, particularly in hot and arid regions.

2. MATERIAL AND METHODS

2.1. Geometric parameters of the tubular heat and mass exchanger

The heat and mass exchanger geometry is shown in Figure 1.

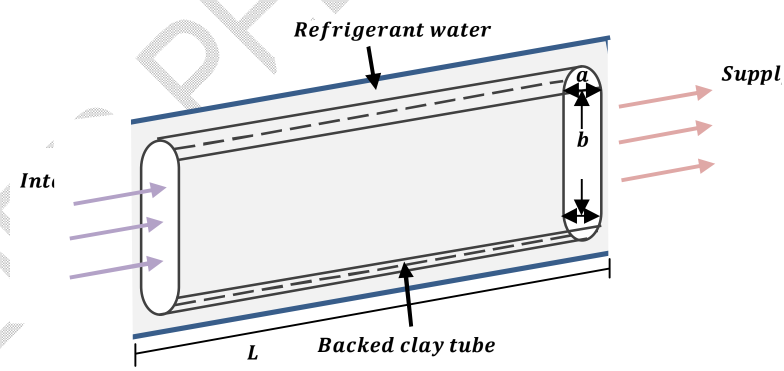

Figure 1: Baked clay tube heat and mass exchanger geometry

To characterize the geometry of this type of heat and mass exchanger, three parameters are essential: tube short axis (a), tube long axis (a+b), and tube length (L). The other parameters, namely, perimeter (P), cross-section area (A_c), exchange surface area (A_w), tube Flatness Ratio (R_F), and characteristic length (represented by hydraulic diameter (D_H) or equivalent diameter (D_e), are derived using specific formulas.

- ***Perimeter***

$$P = \pi a + 2b \quad (1)$$

- ***Cross-section area***

$$A_c = \frac{a^2\pi}{4} + ab \tag{2}$$

- ***Wet surface area***

The wet surface area is computed using eq. (3)

$$A_w = L[\pi a + 2b] \tag{3}$$

- ***Hydraulic and equivalent diameters***

Before determining the heat and mass transfer coefficients, it is necessary to establish an appropriate Reynolds number for the flat tube geometry. For air flowing through non-circular tubes, usually, the hydraulic diameter is used for Reynolds number calculations, which is:

$$D_H = \frac{4A_c}{P} \tag{4}$$

However, as mentioned Cheng et al.by[11] for non-circular channels, it is recommended to use equivalent diameters instead of hydraulic diameters in the flow analysis. Using the equivalent diameter allows for keeping the same mass flow rate of air in the equivalent tube as that in the flat tube. Hence, in this study, the equivalent diameter is treated as the characteristic diameter for the flat tube. Its expression is given by:

$$D_e = \sqrt{\frac{4A_c}{\pi}} \tag{5}$$

- ***Tube Flatness Ratio (FR)***

The tube flatness ratio is defined as the ratio of the tube's long axis and the tube's short axis:

$$R_F = \frac{a + b}{a} \tag{6}$$

When the equivalent diameter and the flatness ratio are known, a and b can be determined by solving the equation below:

$$A_c = \frac{\pi {D_e}^2}{4} = \frac{a^2\pi}{4} + ab \tag{7}$$

For $R_F = 1 \rightarrow b = 0 \; and \; a = D_e \rightarrow this \; correspond \; to \; the \; circular \; tube$

For $R_F = 2 \rightarrow b = a \; and \; a = D_e\sqrt{(\pi/4)/(1 + \pi/4))}$

For $R_F = 3 \rightarrow b = 2a \; and \; a = D_e\sqrt{(\pi/4)/(2 + \pi/4))}$

For $R_F = 4 \rightarrow b = 3a \ and \ a = D_e\sqrt{(\pi/4)/(3+\pi/4))}$

The geometric dimensions of the tubes are listed in **Error! Reference source not found.**.

Table 1: Geometric dimensions of the tube

Tube	D_e (mm)	a (mm)	b (mm)	P_w (mm)	A_c (mm^2)	D_h (mm)	R_F
Round	15	15	0	47.12	176.71	15	1
AR2	15	9.95	9.95	51.15	176.71	13.82	2
AR3	15	7.96	15.93	56.88	176.71	12.43	3
AR4	15	6.83	20.50	62.46	176.71	11.32	4

Error! Reference source not found. shows that the equivalent diameter is larger than the hydraulic diameter with increasing differences at higher flatness ratios.

2.2. Description of the physical model

The physical description of the tubular heat and mass exchanger involves a control volume where heat and mass conservation laws are applied to analyze the cooling process. The system features porous tube that allows water to seep through via capillary action, moving toward the inner surface, and evaporates in contact with hot dry air flowing inside the tube, creating a cooling effect at the tube-air interface.

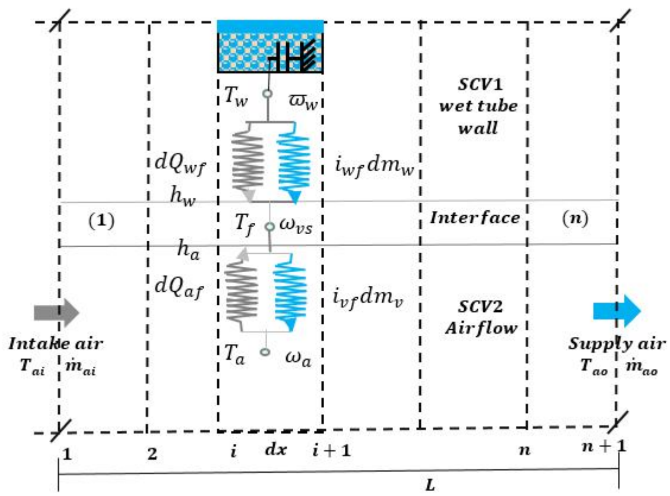

Figure 2: Prototype of the terracotta tube evaporative cooler

2.2.1. Simplifying assumptions

In the following analysis, it is assumed that:

- Heat and mass are transferred perpendicularly through the channel walls;
- Within the air streams, the convective heat transfer is considered the dominant mechanism for heat transfer;
- The interior surface of the tube's wall is completely and continuously wet;
- All outside boundaries are adiabatic;

- Tube-air interface temperature is assumed identical to the wet bulb temperature of the intake air;
- Air is treated as an incompressible gas and the velocity and properties of the supply air are considered to be uniform in a differential control volume;
- No condensation happens in the air channel;
- The heat and mass transfer coefficients are constant in a differential control volume.

By employing these assumptions, the mathematical model of the evaporative cooler can now be derived from energy and mass conservation equations.

2.2.2. Governing equations

The physical model is divided into numerous control volumes with an infinitesimal wet surface area of $\left|A_w.\frac{dx}{L}\right|$. A set of differential equations is established for one control volume using the energy and mass conservation laws. **Error! Reference source not found.** shows an elementary control volume of length dx with three zones: the wet clay tube zone (SCV1), the airflow zone, and the tube-air interface zone (SCV2). The wet clay tubes exchange sensible heat dQ_{wf} and latent heat $i_{wf}dm_w$ with the interface through the tubes' walls. It is also indicated that the interface draws part of its evaporation energy dQ_{af} from the process air. The water vapor produced is then absorbed by the process air as a form of latent heat $i_{vf}d\dot{m}_v$.

Let dm_e be the evaporation rate, ϖ the specific humidity of clay tubes, and ω_a the specific humidity of process air:

$$\varpi_t = \frac{m_w}{m_t} \quad (8)$$

$$\omega_a = \frac{\dot{m}_v}{\dot{m}_a} \quad (9)$$

Considering the heat and mass transfer process at the water-air interface of the evaporative cooling system, the general energy and mass conservation equations are given below:

The mass balance across each control volume is given by:

For the dry air

$$d\dot{m}_a = 0 \quad (10)$$

For the dry tube

$$dm_t = 0 \quad (11)$$

For the liquid water

$$dm_w = m_t d\varpi_t = -dm_e \quad (12)$$

For the water vapor

$$d\dot{m}_v = \dot{m}_a d\omega_a = dm_e \quad (13)$$

The mass balance of the elementary control volume comprising all the fluids is written as:

$$dm_t + dm_w + d\dot{m}_a + d\dot{m}_v = 0 \quad (14)$$

Carrying eqs(10),(11),(12), and (13) into eq. (14), we find:

$$m_t d\varpi_t = -\dot{m}_a d\omega_a \quad (15)$$

If $i_t, i_w, i_a and\ i_v$, are the enthalpies of dry tubes, water, dry air and water vapor respectively, by neglecting heat exchange with the environment and variations in kinetic and potential energies, the energy balance for each sub-control volume is:

$$CV1:\ d(m_t i_t) + d(m_w i_w) = -dQ_{wf} - i_{wf} dm_e \quad (16)$$

$$CV2:\ d(\dot{m}_a i_a) + d(\dot{m}_v i_v) = -dQ_{af} + i_{vf} dm_e \quad (17)$$

Where i_{wf} and i_{vf} are the specific enthalpies of the water and the water vapor evaluated at the wet surface-air interface temperature T_f.

$$i_{wf} = c_{pw} T_f \quad (18)$$

$$i_{vf} = i_0 + c_{pv} T_f \quad (19)$$

Where i_0 is the latent heat of vaporization of water at 0 ^{o}C.
The energy balance of the elementary control volume comprising all the fluids is written as:

$$d(m_t i_t) + d(m_w i_w) + d(\dot{m}_a i_a) + d(\dot{m}_v i_v) = 0 \quad (20)$$

Carrying eqs(16)&(17) into eq.(21), we find:

$$dQ_{wf} + dQ_{af} = (i_{vf} - i_{wf}) dm_e \quad (21)$$

We assume that the water vapor produced at the wet surface-air interface is saturated at water film temperature (T_f), which means that equilibrium between the liquid and vapor phases is established at the interface, so the previous expression takes the form:

$$dQ_{wf} + dQ_{af} = i_{fg} dm_e \quad (22)$$

Where i_{fg} is the latent heat of the vaporization of water at the water film temperature.
Eq.(22) indicates that the net sensible heat flux at the water-air interface represents the energy required for the evaporation process.

$$i_{fg} = i_0 + (c_{pv} - c_{pw}) T_f \quad (23)$$

The value of i_{fg} is can be calculated with a relative error of less than 1% by the following formula valid between 0 and 180°C[12]:

$$i_{fg} = 2501.6 - 2.18 T_f\ (kJ.kg^{-1}) \quad (24)$$

Where T_f is the water film temperature.
The equations established previously are written in terms of flows of mass and heat, and now these same flow equations will be expressed in terms of transfer potentials.
Vapor density gradients are responsible for mass transfer. The mass flow between the interface and the airstream is defined by the relationship:

$$dm_e = h_m (\rho_{vf} - \rho_v) \frac{A_w}{L} dx \quad (25)$$

Where ρ_v is the partial density of the water vapor in air and ρ_{vf} is the partial density of the saturated water vapor at T_f and is, therefore, a function of T_f: $\rho_{vf} = f(T_f)$ and in which, h_m is the mass transfer coefficient. $\rho_v(T)$ is linked to the specific humidity by eq.(26).

$$\rho_v = \rho_a \omega_a \quad (26)$$

Where ρ_a is the density of the dry air. Since the partial density of water vapor is proportional to the moisture content of the air, the difference between the moisture content of the saturated air at the interface and that of the passing air is the driving force for water evaporation in the wet channel. Then, eq. (25) can be rewritten as follows:

$$dm_e = \rho_a h_m (\omega_{vs} - \omega_a) \frac{A_w}{L} dx \quad (27)$$

Where $\omega_a$is the moisture content of the air (kg/kg_{as}) and ω_{vs} is the moisture content of saturated air close to the wet surface. ω_{vs} is evaluated at water film temperature using eq. (28)[13].

$$\omega_{vs}(T_f) = 0.622\frac{P_{vs}(T_f)}{101325 - P_{vs}(T_f)} \tag{28}$$

In which $P_{vs}(T_f)$ is the saturated vapor pressure calculated at wet surface temperatureT_f and whose correlation expression is given by[14].
The mass change for each sub-control volume can be rewritten as follows:

$$\frac{m_t d\varpi_t}{dt} = -\rho_a h_m(\omega_{vs} - \omega_a)\frac{A_w}{L}dx \tag{29}$$

$$\dot{m}_a d\omega_a = \rho_a h_m(\omega_{vs} - \omega_a)\frac{A_w}{L}dx \tag{30}$$

In a similar way to mass transfer, temperature gradients are the potentials that cause heat transfer. The enthalpy change for each sub-control volume can be rewritten as follows:

$$d(m_t i_t) + d(m_w i_w) = m_t di_t + i_t dm_t + i_w dm_w + m_w di_w \tag{31}$$

$$d(\dot{m}_a i_a) + d(\dot{m}_v i_v) = i_a d\dot{m}_a + \dot{m}_a di_a + i_v d\dot{m}_v + \dot{m}_v di_v \tag{32}$$

$$d(m_t i_t) + d(m_w i_w) = m_t(c_{pt} + \varpi_t c_{pw})dT_w + c_{pw}T_w dm_w \tag{33}$$

$$d(\dot{m}_a i_a) + d(\dot{m}_v i_v) = \dot{m}_a(c_{pa} + \omega_a c_{pv})dT_a + (i_0 + c_{pv}T_a)d\dot{m}_v \tag{34}$$

The heat fluxes transferred between subsystem CV2 and the interface, and between subsystem CV1 and the interface, are respectively defined by the relationships:

$$dQ_{wf} = k_w(T_w - T_f)\frac{A_w}{L}dx \tag{35}$$

$$dQ_{af} = h_a(T_a - T_f)\frac{A_w}{L}dx \tag{36}$$

Where h_a is the convective heat transfer coefficient between air and the interface. For the flowing air condition, h_a is a function of air flow rate and channel characteristics. A is the total internal surface area of a tube, and L is the tube length. k_w is the total heat transfer coefficient between the whole system's thermal mass and the interface and is defined as follows:

$$k_w = \frac{1}{\frac{1}{h_{wt}} + \frac{\delta}{\lambda_{wt}}} \tag{37}$$

Where h_{wt} represents the convective heat transfer coefficient between the refrigerant water and the tube's external wall. δ and λ_{wt} represent the thickness and thermal conductivity of wet channel walls respectively. The porous ceramic wall is saturated with water and its thermal conductivity should take into account both the dry ceramic and water thermal conductivities. This is computed as follows:

$$\lambda_{wt} = \frac{\lambda_w[\lambda_w + \lambda_t - (1-\sigma)(\lambda_w - \lambda_t)]}{\lambda_w + \lambda_t + (1-\sigma)(\lambda_w - \lambda_t)} \tag{38}$$

$\lambda_t$is the thermal conductivity of the dry ceramic container and σ is the ceramic container's porosity. λ_w is the thermal conductivity of water.
Finally, by substituting the heat and mass flow expressions in eqs(16), (17), and (21) by their corresponding heat and mass transfer expressions, we obtain the following relationship:

$$\frac{m_t(c_{pt} + \varpi_t c_{pw})dT_w}{dt} + k_w(T_w - T_f)\frac{A_w}{L}dx - c_{pw}(T_w - T_f)dm_e = 0 \tag{39}$$

$$\dot{m}_a(c_{pa} + \omega_a c_{pv})dT_a + h_a(T_a - T_f)\frac{A_w}{L}dx + c_{pv}(T_a - T_f)dm_e = 0 \tag{40}$$

$$k_w(T_w - T_f) + h_a(T_a - T_f) = \rho_a h_m i_{fg}(T_f)(\omega_{vs} - \omega_a) \tag{41}$$

The above equations can also be combined into a system of five differential equations as follows:

$$\begin{cases} \dfrac{d\varpi_t}{dt} = -\dfrac{\rho_a h_m A_w dx}{m_t L}(\omega_{vs} - \omega_a) \\ \dfrac{d\omega_a}{dx} = \dfrac{A_w \rho_a h_m}{L\dot{m}_a}(\omega_{vs} - \omega_a) \\ \dfrac{dT_w}{dt} = \dfrac{[-k_w + c_{pw}\rho_a h_m(\omega_{vs} - \omega_a)]A_w dx}{m_t(c_{pt} + \varpi_t c_{pw})L}(T_w - T_f) \\ \dfrac{dT_a}{dx} = -\dfrac{[h_a + c_{pv}\rho_a h_m(\omega_{vs} - \omega_a)]A_w}{\dot{m}_a(c_{pa} + \omega_a c_{pv})L}(T_a - T_f) \\ k_w(T_w - T_f) + h_a(T_a - T_f) = \rho_a h_m i_{fg}(T_f)(\omega_{vs} - \omega_a) \end{cases} \tag{42}$$

2.3. Analytical model development

This system describes the heat and mass transfer process in the cooler at unsteady-state conditions. Studying such a system would involve solving these equations simultaneously to examine the evolution of the system and the impact of the initial and operating parameters. Unfortunately, it is a system of non-linear equations that cannot be solved numerically by the finite difference method. Nevertheless, at steady-state conditions, this system is reduced to three main equations that can be combined and integrated to obtain an analytical solution. In the absence of heat extraction from the system thermal mass (water-tube assembly), and without heat addition from any external source, the system becomes adiabatic, opened and the process evolves in a steady state for heat and mass transfer. At steady state conditions, the whole thermal mass of the system is in thermal equilibrium with the tube-air interface. In this case, the recirculating water and the interface are at the same temperature, and this temperature is that of the wet bulb temperature of the inlet air. Hence, at a steady state, the energy required to sustain the evaporation is exclusively provided by the convective heat transfer from the air flowing inside the tube channels. Therefore eq. (39) disappears and eq. (41) is simplified to:

$$h_a(T_a - T_{wb}) = \rho_a h_m i_{fg}(T_f)(\omega_{vs} - \omega_a) \tag{43}$$

Therefore, at a steady state, the cooler is described by the following set of equations:

$$\begin{cases} \dfrac{d\varpi_t}{dt} = -\dfrac{\rho_a h_m A_w dx}{m_t L}(\omega_{vs} - \omega_a) \\ \dfrac{d\omega_a}{dx} = \dfrac{A_w \rho_a h_m}{L\dot{m}_a}(\omega_{vs} - \omega_a) \\ \dfrac{dT_a}{dx} = -\dfrac{[h_a + c_{pv}\rho_a h_m(\omega_{vs} - \omega_a)]A_w}{\dot{m}_a(c_{pa} + \omega_a c_{pv})L}(T_a - T_{wb}) \\ h_a(T_a - T_{wb}) = \rho_a h_m i_{fg}(\omega_{vs} - \omega_a) \end{cases} \tag{44}$$

2.3.1. Determination of the exchange surface area

Eqs.(43)&(44) are combined and integrated to determine the wet surface area as a function of the inlet and outlet air temperatures, air physical properties, air mass flow rate, and heat and mass transfer coefficients. From eq.(43):