Original Research Article

APPLICATION OF ODD PAIRS OF PARTITIONS OF AN EVEN NUMBER OF A NEW FORMULATION IN VALIDATING THE TWIN PRIME CONJECTURE

Abstract

The Twin Prime Conjecture posits the existence of infinitely many pairs of prime numbers $(p, p + 2)$, where both p and $p + 2$ are prime. Despite centuries of investigation, a definitive proof remains elusive. Prime numbers, defined by their indivisibility except by one and themselves, display an apparently erratic distribution. Researchers have utilized a combination of theoretical insights, computational analysis, and innovative mathematical techniques in their quest to solve this conjecture. However, the unpredictable nature of prime occurrences has kept this problem open in Number Theory. This study introduces a novel approach involving the partitioning of even numbers into pairs of odd numbers. We demonstrate that within the set of all such pairs, there exists a proper subset that includes all prime numbers. Notably, this proper subset consistently contains at least two prime numbers differing by 2, providing a potential pathway to validating and proving the Twin Prime Conjecture.

Keywords: Twin Prime Conjecture, Twin primes, Even numbers, Odd numbers, Prime numbers

1. Introduction

The Twin Prime Conjecture, which asserts the existence of infinitely many twin primes, stands as one of the oldest and most captivating unsolved problems in mathematics. Initially posed by Landau in 1912, this conjecture is often mentioned alongside the Riemann Hypothesis and the Goldbach Conjecture as part of Hilbert's eighth problem, introduced in 1900 [1]. The distribution of prime numbers presents profound and complex challenges, with the Twin Prime Conjecture, Goldbach Conjecture, Prime Number Theorem, and Riemann Hypothesis being among the most renowned problems in this domain [2].

Twin primes are pairs of prime numbers that differ by exactly two. The quest to prove the infinitude of twin primes has engaged mathematicians for decades. In 1919, Viggo Brun made significant progress by demonstrating that the sum of the reciprocals of the twin primes converges, though he did not prove the conjecture itself [3]. More recently, in 2014, Yitang Zhang gained substantial attention by proving that there are infinitely many pairs of primes separated by at most 70,000,000 [4]. Following Zhang's breakthrough, the collaborative project "Polymath8" further reduced this bound to 246, and even to 12, assuming the validity of the Elliott–Halberstam Conjecture [5].

Recent advancements have continued to push the boundaries of our understanding of twin primes. Researchers have been focusing on developing new sieve methods and leveraging computational power to explore the distribution of primes with smaller gaps [11]. For example, the Maynard-Tao method introduced by James Maynard and Terence Tao has provided new insights into bounded gaps between primes, allowing for further reduction of the gaps and enhancing our understanding of prime clustering [12].

Formally, the Twin Prime Conjecture posits that there exist infinitely many pairs of primes $(p, p+2)$ such that both p and $p+2$ are prime [6]. Despite significant efforts, the conjecture remains neither proven nor disproven, maintaining its status as one of the most famous unsolved problems in number theory [7]. This study proposes the utilization of a method of partitioning an even number of a new formulation into all pairs of odd numbers [8] as a new approach towards the solution to the Twin Prime Conjecture.

2. Preliminary

The algorithm for partitioning an even number into pairs of odd numbers leverages a new formulation of an even number as $(P_1+P_2)+(P_2-P_1)^n$ to demonstrate that any even number can always be divided in such a manner [9]. The algorithm involves selecting odd numbers in the interval [1, $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n]$] and even number within the same intervals. By using these values, a set containing all odd pairs of partitions of $(P_1+P_2)+(P_2-P_1)^n$ is generated [8]. From this set of pairs of odd numbers, important patterns emerge. For instance, it has been shown that, this set contains at least one pair of primes whose sum equals $(P_1+P_2)+(P_2-P_1)^n$ [13], validating the Strong Goldbach Conjecture. Additionally, that for any even positive integer $2k$, there exist infinitely many pairs of consecutive prime numbers whose difference is $2k$ [14], results that are consistent with the Polignac's Conjecture.

In this paper, we further explore another important pattern associated with the Twin Prime Conjecture that concerns a fixed even gap of 2 between prime numbers. We demonstrate that the set of all pairs of partitions generated encompasses all possible pairs of odd numbers whose sum is the even number $(P_1+P_2)+(P_2-P_1)^n$, demonstrating the existence of at least one pair of primes that differ by 2 in the subset of primes of the set containing all pairs of odd numbers.

3. Methodology and Summary of Results

The Twin prime conjecture ascertains that there exist infinitely many pairs of prime numbers that differ by 2 [15]. The algorithm for generating all pairs of odd numbers for a given even number $(P_1+P_2)+(P_2-P_1)^n$ proposed by Sankei et al.[8], implies that, a given even number say 10 can be partitioned using any even number and all odd numbers in the interval [1, $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n]$]. For $(P_1+P_2)+(P_2-P_1)^n=10$, the even number belonging to the even half-closed interval is [2,4] and the set of odd numbers in the half open-closed interval is [1,3,5] as i) $10 - (2 + 1) = 7$, ii) $10 - (2 + 3) = 5$ and iii) $10 - (2 + 5) = 3$. The distinct pairs from these partitions satisfying the Strong Goldbach Conjecture are (3,7) and (5,5) since $3 + 7 = 5 + 5 = 10$. Although this algorithm for partitioning an even number into all pairs of odd numbers have been extensively used to show that the Strong Goldbach Conjecture holds true[10], it can be extended to find its application in the Twin Prime Conjecture in such a way that from the set say A containing all pairs of odd numbers generated from partitioning $(P_1 + P_2) + (P_2 - P_1)^n$,we can derive the proper subset B of A, = $\{ y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}, y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)} - 2, y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)} - 4, \ldots, y_5, y_3, y_1, z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}, z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)} - 2, \ldots, z_5, z_3\ z_1 \}$of all prime numbers from the least to the largest. Further, from the set B, the possibility exist of another proper subset C containing at least two prime numbers that differ by 2. We illustrate this concept with the following two examples:

Example 1

For instance in partitioning 10 as in above, the set $A = \{ (3,7),(5,5)\}$and the set B = { 3,5,7}. Notice that from the set B, we obtain two pairs of prime numbers (3,5) and (5,7) that defer by 2.

Example 2

Step 1: Let the $Even\ number\ \ (P_1 + P_2) + (P_2 - P_1)^n = 46$, for $n = 1$

Remark 1

Multiples of even numbers say d in the range

$d \in 1 < E \leq \frac{1}{2}(46) = \{2,4,6,8,10,12,14,16,18,20,,22\}$. For example 2, we let $d = 22$.

Step 2 : $d = 22$

Step 3 : Take odd numbers in the range $1 \le O \le \frac{1}{2}(46) \rightarrow (1,3,5,7,9,11,13,15,17,19,21,23)$

Then we partition 46 as follows:

I. $46 - (22+1) = 23$

II. $46 - (22+3) = 21$

III. $46 - (22+5) = 19$

IV. $46 - (22+7) = 17$

V. $46 - (22+9) = 15$

VI. $46 - (22+11) = 13$

VII. $46 - (22+13) = 11$

VIII. $46 - (22+15) = 9$

IX. $46 - (22+17) = 7$

X. $46 - (22+19) = 5$

XI. $46 - (22+21) = 3$

XII. $46 - (22+23) = 1$

The partitions of 46 are therefore: $\big((22+1), 23\big), \big((22+3), 21\big), \big((22+5), 19\big), \big((22+7), 17\big), \big((22+9), 15\big), \big((22+11), 13\big), \big((22+13), 11\big), \big((22+15), 9\big), \big((22+17), 7\big), \big((22+19), 5\big), \big((22+21), 3\big), ((22+23), 1)$ $\Rightarrow$ the set A = {

$(23{,}23), (25{,}21), (27{,}19), (29{,}17), (31{,}15), (33{,}13), (35{,}11), (37{,}9), (39{,}7), (41{,}5), (43{,}3), (45{,}1)$ }, which are all pairs of odd numbers. From the set A, we derive a set B containing the list of all odd numbers in ascending order so that the set $B = \{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43,45\}$. From the set B, we further obtain the proper subset C of prime numbers as the set , $C = \{3,5,7,11,13,17,19,23,29,31,37,41,43\}$. The proper subset C contains prime numbers that can be written as pairs (3,5), (5,7), (11,13), (17,19), (29,31) and (41,43) that differ by 2.

4. Computational approach

The algorithm for partitioning the even number $(P_1 + P_2) + (P_2 - P_1)^n$ into pairs of odd numbers has been tested computationally and used to extend the Strong Goldbach Conjecture for all even numbers not larger than 9×10^{18} through a program that was coded in an Integrated Development Environment called Apache Netbeans IDE 15. Java Runtime Environment (Java 8 Update 371 (64-bit)) was pre-installed to provide an environment for the program to execute. Additionally, Java Development Kit (Java(TM) SE Development Kit 19 (64-bit) was also installed to provide resources needed for the program to execute [10]. So that ultimately, the program executes in different sequential stages to partition an even number into all pairs of odd numbers. The same computational approach is used to show the validity of the Twin Prime Conjecture for large values. Since a computational approach does not constitute a proof as it provides only empirical evidence supporting the conjecture, this study uses example 3 below as one case of verifying the Twin Prime conjecture for large values.

Example 3

For $2n = \mathbf{1,000,000}$, some partitions of this even number includes:

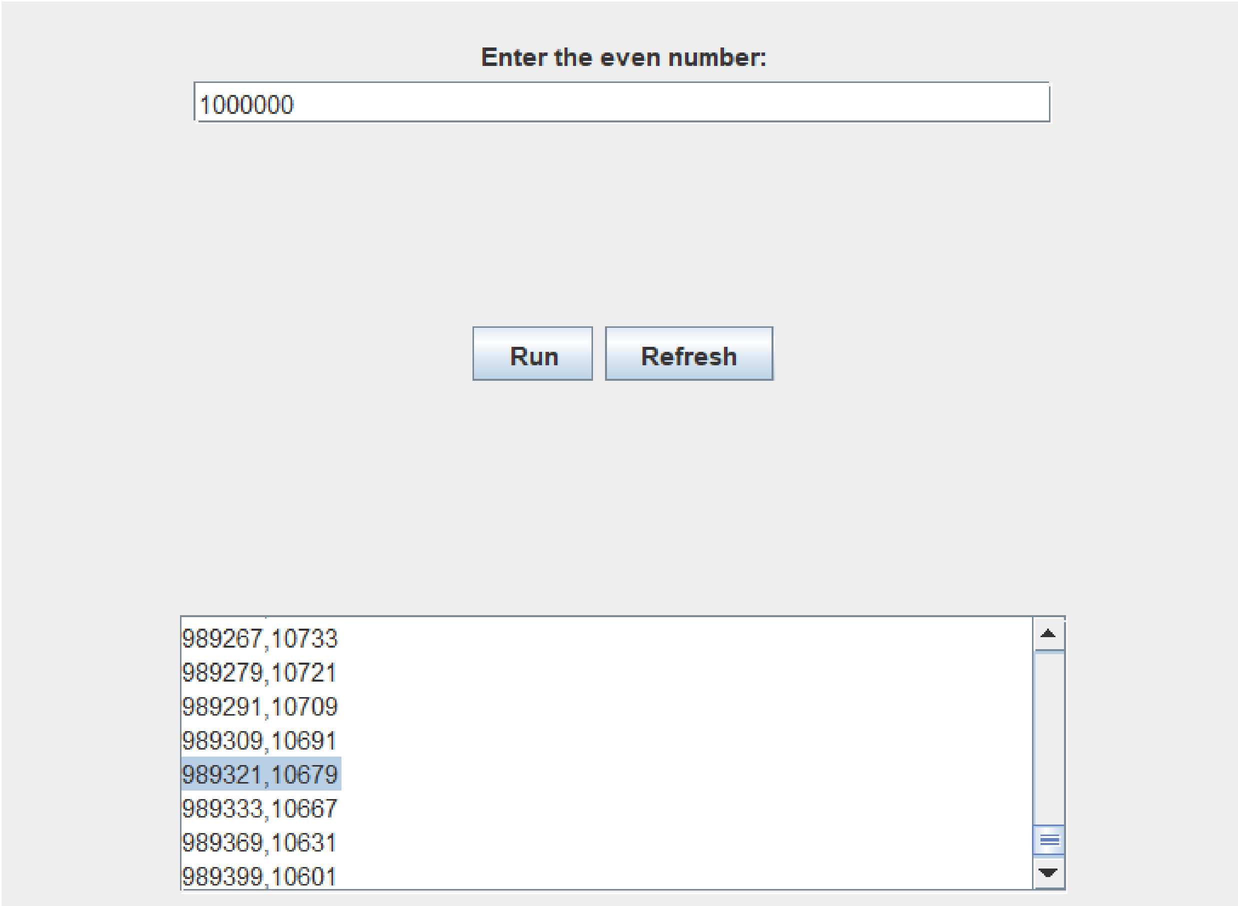

Fig. 1. **Computational approach**

Since the list of all partitions is quite long, the authors decided to select the following pairs of odd numbers from the entire set of pairs of odd numbers: that is, the set *A*= { …, (**989321**, 10679), (**989323**, 10677), (989333, 10667), (989335, 10665), (989369, 10631), (989371, 10629), (989399, 10601), (989401, 10599), (989411, 10589), (989413, 10587), (989441, 10559) and (989443, 10557), (989447, 10553), (989449, 10551), …}. From this set, we obtain a proper subset with the two prime numbers **989321** and **989323** that differ by 2. Notice that the pairs of odd numbers is not complete and therefore the possibility is that, there could be more pairs of prime numbers that differ by 2.

4.1 Proposed Future Pathway towards the general proof of the Twin Prime Conjecture

The algorithm for partitioning the even number $(P_1+P_2)+(P_2-P_1)^n$, works such that it is able to generate all pairs of odd numbers associated to the given even number [8]. Since prime numbers greater than 2 are subsets of odd numbers, it has been shown that, from these pairs of odd numbers, one could always obtain a proper subset containing only prime numbers.

By partitioning 10, 46 and 1000000 into all pairs of odd numbers, we generate from these sets, subsets of prime numbers that exhibit interesting patterns where we get at least a pair of prime that differ by 2 in these subsets. Although these results do not constitute a general proof of the Twin Prime Conjecture, this new approach has proved fundamental as one of the approaches that should be explored in an attempt to provide a general proof to the conjecture proposed as follows:

Let $(P_1+P_2)+(P_2-P_1)^n$ be any even number and $A=\{\, y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)},\ y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}-2,\ y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}-4,\ldots,\ y_5, y_3, y_1, z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}, z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}-2,\ldots, z_5, z_3\ z_1\,\}$ be set of all odd numbers less than $(P_1+P_2)+(P_2-P_1)^n$ generated from partitioning the even number. It has been shown that there exist a proper subset say B= $\{\, p_1, p_2, p_3, \ldots\}$ of prime numbers of the set A ,containing at least a pair of prime numbers that differ by 2. By finding all the possible combinations of the differences between any two prime numbers in the set B, the possibility exists that 2 belongs to the set of the differences between these prime numbers. If these results are proven true, it will be a solution to the Twin Prime Conjecture.

Conclusion

The results for partitioning an even number of a new formulation into all pairs of odd numbers has been shown to have significant application to the Twin Prime Conjecture. We obtain a set containing all pairs of odd numbers that has been shown to contain a proper subset of all prime numbers. It has further been shown that the proper subset contains at least one pair of prime number that differ by 2. The study recommends a further exploration of this proper subset of prime numbers to show that there exist at least a pair of prime numbers that differ by 2 for any arbitrary such set.

Disclaimer (Artificial intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

References

1. Ting, J. Y. (2024). On the Universal Presence of Mathematics for Incompletely Predictable Problems in Rigorous Proofs for Riemann Hypothesis, Modified Polignac's and Twin Prime Conjectures. Journal of M, 16(2), 1.
2. Liu, D., & Liu, C. (2023). Corresponding Prime Number Distribution Equation. Journal of Applied Mathematics and Physics, 11(11), 3354-3365.
3. GENSEL, B., & SELLERS, J. (2021). ON THE TWIN PRIME CONJECTURE.
4. Zhang, Y. (2014). Bounded gaps between primes. Annals of Mathematics, 179(3), 1121–1174.
5. Maynard, J. (2015). Small gaps between primes. Annals of Mathematics, Second Series, 181(1), 383–413.
6. Nazardonyavi, S. (2012). Some history about twin prime conjecture. arXiv preprint arXiv:1205.0774.

7. Hunacek, M. (2023). Introduction to number theory. Chapman and Hall/CRC.
8. Sankei, D., Njagi, L., & Mutembei, J. (2023). Partitioning an even number of the new formulation into all pairs of odd numbers. *Journal of Mathematical Problems, Equations and Statistics, 4 (2), 35-37. https://doi. org/10.22271/math. 2023. v4. i2a, 104.*
9. Sankei, D., Njagi, L., & Mutembei, J. (2023). A New Formulation of a Set of Even Numbers. European Journal of Mathematics and Statistics, 4(4), 78–82. https://doi.org/10.24018/ejmath.2023.4.4.249
10. Daniel, S., Njagi, L., & Mutembei, J. (2023). A NUMERICAL VERIFICATION OF THE STRONG GOLDBACH CONJECTURE UP TO 9× 10^ 18. GPH-International Journal of Mathematics, 6(11), 28-37.
11. Maynard, J. (2015). *Small gaps between primes*. Annals of Mathematics, 181(1), 383-413.
12. Zhang, Y. (2014). *Bounded gaps between primes*. Annals of Mathematics, 179(3), 1121-1174.
13. Sankei, Daniel, Loyford Njagi, and Josephine Mutembei. 2024. “A Detailed Proof of the Strong Goldbach Conjecture Based on Partitions of a New Formulation of a Set of Even Numbers”. Asian Research Journal of Mathematics 20 (4):8-17. https://doi.org/10.9734/arjom/2024/v20i4793.
14. Sankei, Daniel, Loyford Njagi, and Josephine Mutembei. 2024. “A Logical Proof of the Polignac’s Conjecture Based on Partitions of an Even Number of a New Formulation”. Asian Research Journal of Mathematics 20 (3):1-11. https://doi.org/10.9734/arjom/2024/v20i3787.
15. Berke, A. (2006). An Introduction to The Twin Prime Conjecture.