Original Research Article

Heat Transfer Characteristics of a Quiescent Fluid Over a Moving Flat Surface

ABSTRACT

Optimizing thermal control methods and enhancing the efficiency of various engineering processes, such as the cooling of hot moving surfaces, is imperative. Therefore, it is crucial to investigate heat transfer and temperature distributions across moving surfaces in stationary fluids. In this study, the combined effects of viscous dissipation, temperature-dependent viscosity, and a moving surface in a quiescent fluid on the temperature and velocity profiles are investigated.Atwo-dimensional steady laminar boundary layer flow of an incompressible Newtonian fluid, driven by the movement of the surface where viscosity is an inverse function of temperature, is analyzed. The effects of flow parameters, including Reynolds number, Eckert number, Prandtl number, variable viscosity, and surface velocity on temperature and velocityprofiles, are determined. The governing boundary layer partial differential equations are transformed into non-dimensional form and solved using the central finite difference numerical method, implemented in MATLAB software.The numerical results demonstrate that an increase in the Reynolds number and the variable viscosity parameter leads to an increase in the velocity profiles.Additionally,an increase in Reynolds number, Prandtl number, variable viscosity, and surface velocity leads to a decrease in temperature profiles. Finally, an increase in the Eckert number results in higher temperature profiles. Therefore, with suitable flow parameters, the temperature of the fluid can be regulated. These findings are useful in cooling hot sheets or metallic plates drawn over a quiescent fluid to obtain high-quality final products.

Keywords: Quiescent fluid, Temperature-dependent viscosity, Viscous dissipation, Reynolds number, Prandtl number, Eckert number

1. INTRODUCTION

The study of heat transfer and temperature distribution over moving surfaces in quiescent fluids is of significant interest in both industrial and scientific contexts[1]. These scenarios are commonly encountered in applications such as aerodynamic heating, condensation of liquid films, wire drawing, and cooling of sheets or metallic plates [2-7]. Understanding the temperature and velocity profiles that develop in such systems is essential for optimizing thermal management strategies and improving the efficiency and safety of various engineering processes.

In quiescent fluids, the absence of bulk fluid motion means that heat transfer occurs primarily through conduction and natural convection. The interaction between the moving surface and the stationary fluid creates complex thermal gradients, which are influenced by factors such as the surface speed, fluid properties, and ambient conditions [8,9]. Accurately understanding heat transfer and temperature distribution in such systems requires a comprehensive approach that takes into account the boundary layer governing equations of fluid dynamics.

“Various mathematical models for boundary layer flow over a moving surface have been formulated, considering various aspects”[10-14].Jain et al. [15] “explored boundary layer fluid flow over an absorptive surface in the presence of slip and mixed convection. In their study, they showed that thermo-physical parameters influence fluid dynamics in the boundary layer region”. Yacob et al. [16] “conducted research on boundary layer fluid flow in a quiescent fluid over a moving surface. Their results indicated that the heat transfer rate at the surface decreases with an increasing convective parameter”. Razzaq et al. [8] “considered the non-similar aspects of forced convection from a moving heated surface subject to external fluid flow. The governing partial differential equations were solved numerically, and the results indicated that an increase in the Prandtl number increased the convective heat transfer coefficient”. Jan et al. [17] “formulated a model of viscous fluid flow over a moving heated cylinder. Their results indicated that the thermal boundary layers increased with curvature, while temperature profiles increased with the Prandtl number”. Bhattacharyya et al. [18] “conducted a study on the thermal boundary layer for the flow of incompressible Newtonian fluid over an exponentially stretching sheet with an exponentially moving free stream. Their results showed that the temperature in the boundary layer decreases with increasing values of the Prandtl number”.

“Most studies on boundary layer fluid flow are based on the assumption of constant physical properties of the fluid. However, these properties vary, especially fluid viscosity. To accurately predict flow and heat transfer rates, it is necessary to account for the variation in viscosity” [19-21]. “Fluid viscosity is affected by two main factors: pressure and temperature. This study considers the effects of temperature only. Temperature-dependent viscosity affects the design and operation of equipment such as pumps, pipelines, and heat exchangers” [22,23]. “Accurate knowledge of viscosity variations in these equipment helps optimize flow conditions and energy efficiency. In lubrication systems, the viscosity of lubricants changes with temperature, influencing their effectiveness. Ensuring the correct viscosity at operating temperatures is crucial for reducing wear and maintaining machinery” [24,25]. Therefore, continuous research is needed to better understand the microscopic mechanisms governing temperature-dependent viscosity and to develop more accurate predictive models for a wider range of fluids.

“Several studies have been conducted on boundary layer flow, taking into consideration temperature-dependent viscosity. Shah et al. [26] conducted a study on second-order fluid with variable viscosity. Their results indicated that the velocity distribution was enhanced due to a large temperature-dependent viscous parameter. The study on the flow of fluids with variable viscosity was considered” in [27]. Their findings revealed that the temperature-dependent parameter results in an increase in temperature profiles. Zaman et al. [28] studied boundary layer flow over a thin cylinder, taking into account temperature-dependent viscosity. Their results indicated that temperature profiles decrease with increasing Prandtl number. Khan et al. [29] explored the computational aspects of temperature-dependent viscosity for flow over a non-linear stretching sheet. Their results indicated that the temperature profiles decrease with increasing temperature-dependent viscosity parameters.

“In boundary layer flow studies, one crucial aspect that cannot be overlooked is viscous dissipation, which refers to the conversion of kinetic energy into thermal energy within the fluid due to viscous forces” [30-32]. “Viscous dissipation plays a significant role in altering the temperature field within the boundary layer, thereby affecting the overall heat transfer and fluid flow characteristics” [33-36]. Ignoring viscous dissipation can lead to inaccurate predictions, particularly in processes involving high-speed flows or fluids with significant temperature gradients. Ibrahim [37] “considered a viscous fluid flow caused by a rotating disk, using the finite difference numerical method to solve the governing partial differential equations. The numerical results indicated that temperature profiles within the boundary layer region increased with viscous dissipation and Prandtl number". A study on boundary layer flow over a moving surface in the presence of viscous dissipation was considered in [38], where the surface was assumed to move in the same or opposite direction to the free stream. Their results showed that the temperature profiles increased with increasing values of the viscous dissipation parameter. Ajaykumar et al. [39] "studied the effects of viscous dissipation on natural convection for an incompressible fluid over a moving flat surface. Their results indicated that the thermal boundary layer was greatly influenced by viscous dissipation". Masthanaiah et al. [40] "conducted a study on the impact of viscous dissipation on cold liquid through a horizontal channel, revealing that temperature increases within the boundary layer region with increasing Eckert number".Sinha et al. [41] studied the effects of viscous dissipation in a Newtonian fluid flow along an inclined permeable plate. Their results showed that fluid velocity and temperature increase with increasing Eckert number.

The main purpose of this study is to investigate the combined impact of temperature-dependent viscosity, viscous dissipation, and a moving surface in a quiescent fluid on temperature profiles. Based on an extensive literature review, no other study has addressed this specific research gap. The viscosity is considered an inverse function of fluid temperature. The partial differential equations governing the flow have been transformed into a non-dimensional form and solved using the finite difference method. The effects of varying flow parameters on the temperature profiles within the boundary layer region have been investigated.

2. MATHEMATICAL FORMULATION

Here, the two-dimensional laminar flow of an incompressible Newtonian fluid along a moving flat sheet is considered. The sheet moves along the x-axis, causing fluid flow, whereas the y-axis is perpendicular to it, as shown in Figure 1. The velocity components parallel to the x and y axes are given by u and v,respectively.The flow is considered steady, with temperature-dependent viscosity and viscous dissipation effects influencing the thermal properties within the boundary layer region.The moving sheet surface is considered to be maintained at a constant temperature $T_w$and moves with velocity U_w. Additionally, the temperature of the quiescent fluid is considered to be T_∞such that T_w is higher than T_∞.

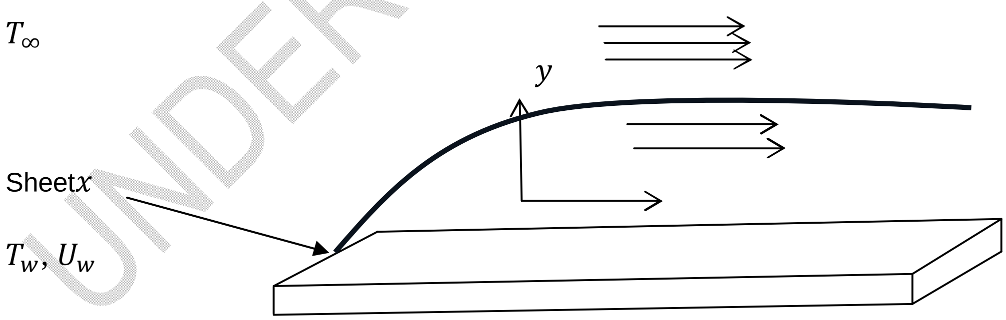

FIGURE 1: Flow configuration

The basic assumptions for the formulation of this study are as follows:

- The flow is laminar and steady.
- The fluid is Newtonian.
- The fluid is incompressible, meaning its density is constant.
- Viscosity is temperature-dependent.
- The effects of viscous dissipation are considered.

Taking into consideration the description and assumptions highlighted, the boundary layer equations governing the flow are given as follows:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

Energy:

$$\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{1}{C_P}K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu}{C_P}\left(\frac{\partial u}{\partial y}\right)^2 \quad (4)$$

Where K is the thermal conductivity of the fluid and C_P is the specific heat capacity.

The boundary layer thickness is assumed to be very thin compared to the length of the surface. Therefore, the velocity component normal to the surface is much smaller than the velocity parallel to the surface, meaning that $v \ll u$, and thus $\frac{\partial v}{\partial y} \approx 0$, $\frac{\partial v}{\partial x} \approx 0$. The gradient components normal to the surface are larger than those along the surface, so $\frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2}$ and $\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}$. Therefore, $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 T}{\partial x^2}$ can be neglected.

The governing equations (1)-(4), considering temperature-dependent viscosity and boundary layer approximations, take the form of equations (5)-(7), as given below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu(\theta)}{\rho}\frac{\partial^2 u}{\partial y^2} \quad (6)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_P}K\frac{\partial^2 T}{\partial y^2} + \frac{\mu(\theta)}{\rho C_P}\left(\frac{\partial u}{\partial y}\right)^2 \quad (7)$$

The boundary conditions for the flow are as follows:

$$\left.\begin{array}{ll} u = U_w,\ v = 0,\ T = T_w & \text{at } y = 0 \\ u = 0,\ v = 0, T = T_\infty & \text{at } y \to \infty \end{array}\right\} \quad (8)$$

This study considers the inverse relationship between fluid viscosity and temperature as given by [42] as follows:

$$\mu(\theta) = \frac{\mu_\infty}{1 + \left(\frac{\mu_\infty - \mu_w}{\mu_w}\right)\theta} \quad (9)$$

Where μ_∞ is the constant reference viscosity in the quiescent fluid, and μ_w is the viscosity at the sheet surface temperature.

The boundary layer equations (5-7) are transformed into a non-dimensional form using the non-dimensional variables given in equation (10).

$$x^* = \frac{x}{L},\quad y^* = \frac{y}{L},\quad u^* = \frac{u}{U_\infty},\quad v^* = \frac{v}{U_\infty},\quad \theta = \frac{T - T_\infty}{T_w - T_\infty}. \quad (10)$$

Where L and U_∞ are the characteristic length and velocity, respectively.

The non-dimensional form of Equation (9) is given as follows:

$$\frac{\mu(\theta)}{\mu_\infty} = \frac{1}{1+\varepsilon\theta} \quad (11)$$

Where ε is the variable viscosity parameter.

The non-dimensional variables given in equation (10) and the non-dimensional form of temperature-dependent viscosity given in equation (11) are employed to transform the governing boundary layer equations into the non-dimensional form as follows:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \quad (12)$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re(1+\varepsilon\theta)} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (13)$$

$$u^* \frac{\partial \theta}{\partial x^*} + v^* \frac{\partial \theta}{\partial y^*} = \frac{1}{Pr.Re} \frac{\partial^2 \theta}{\partial y^{*2}} + \frac{Ec}{Re(1+\varepsilon\theta)} \left(\frac{\partial u^*}{\partial y^*}\right)^2 \quad (14)$$

Where Re, Pr, and Ec are dimensionless parameters given, respectively, as follows:

$$Re = \frac{\rho u L}{\mu}, \; Pr = \frac{\mu C_P}{K}, \; Ec = \frac{U^2}{C_P(T_W - T_\infty)} \quad (15)$$

The boundary conditions in the non-dimensional form are given as:

$$\left.\begin{array}{lll} u^* = \frac{U_W}{U_\infty} = a, & v^* = 0, \theta = 1 & \text{at } y = 0 \\ u^* = 0, & v^* = 0, \theta = 0 & \text{at } y \rightarrow \infty \end{array}\right\} \quad (16)$$

Where a is the non-dimensional velocity of the sheet.

3.METHOD OF SOLUTION

The steady two-dimensional boundary layer flow over a moving sheet in a quiescent incompressible fluid, considering temperature-dependent viscosity and viscous dissipation effects, has been formulated. The non-dimensional coupled non-linear partial differential equations (12-14) with boundary conditions (16) are solved numerically using the central finite difference numerical technique.

The central finite difference form of the non-dimensional equations (12-14) is given as follows:

$$\frac{u(i+1,j) - u(i-1,j)}{2\Delta x} + \frac{v(i,j+1) - v(i,j-1)}{2\Delta y} = 0 \quad (17)$$

$$u(i,j)\left[\frac{u(i+1,j) - u(i-1,j)}{2\Delta x}\right] + v(i,j)\left[\frac{u(i,j+1) - u(i,j-1)}{2\Delta y}\right] = \frac{1}{Re\{1+\varepsilon\theta(i,j)\}}\left[\frac{u(i,j+1) - 2u(i,j) + u(i,j-1)}{(\Delta y)^2}\right] \quad (18)$$

$$u(i,j)\left[\frac{\theta(i+1,j) - \theta(i-1,j)}{2\Delta x}\right] + v(i,j)\left[\frac{\theta(i,j+1) - \theta(i,j-1)}{2\Delta y}\right] = \frac{1}{\mathrm{Pr}.Re}\left[\frac{\theta(i,j-1) - 2\theta(i,j) + \theta(i,j+1)}{(\Delta y)^2}\right] + \frac{Ec}{Re\{1+\varepsilon\theta(i,j)\}}\left[\frac{u(i,j+1) - u(i,j-1)}{2\Delta y}\right]^2 \quad (19)$$

Making $v(i,j+1)$, $u(i+1,j)$, and $\theta(i+1,j)$ the subject of the formulas in equations (17-19), respectively, yields:

$$v(i,j+1) = v(i,j-1) + \frac{\Delta y}{\Delta x}\left[u(i-1,j) - u(i+1,j)\right] \quad (20)$$

$$u(i+1,j) = u(i-1,j) + \frac{2\Delta x\left\{\frac{1}{Re\{1+\varepsilon\theta(i,j)\}}\left[\frac{u(i,j-1)-2u(i,j)+u(i,j+1)}{(\Delta y)^2}\right] - v(i,j)\left[\frac{u(i,j+1,)-u(i,j-1)}{2\Delta y}\right]\right\}}{u(i,j)} \quad (21)$$

$$\theta(i+1,j) = \theta(i-1,j) + \frac{2\Delta x\left\{\frac{1}{Pr.Re}\left[\frac{\theta(i,j-1)-2\theta(i,j)+\theta(i,j+1)}{(\Delta y)^2}\right] + \frac{Ec}{Re\{1+\varepsilon\theta(i,j)\}}\left[\frac{u(i,j+1)-u(i,j-1)}{2\Delta y}\right]^2 - v(i,j)\left[\frac{\theta(i,j+1)-\theta(i,j-1)}{2\Delta y}\right]\right\}}{u(i,j)} \quad (22)$$

MATLAB software was used to solve the central finite difference equations (20-22) to obtain the results discussed in section 3.

4. RESULTS AND DISCUSSION

The analysis of flow over a moving sheet in a quiescent fluid, considering the effects of temperature-dependent viscosity and viscous dissipation, yields the dimensionless parameters Reynolds number (Re), Eckert number (Ec), Prandtl number (Pr), variable viscosity (ε), and surface velocity (a). The base values of these non-dimensional parameters were selected in accordance with a published work [43] as Re = 5.0, Ec = 0.4, Pr = 0.71, ε = 0.2, and a = 0.6. The effects of these non-dimensional parameters on the temperature distribution within the boundary layer region have been investigated. The computations were performed using $\Delta x = \Delta y = 0.01$.The convergence of the solutions was tested by running the program using Δx = 0.01, 0.015, 0.02, 0.025, 0.03 and Δy =0.01, 0.015, 0.02, 0.025, 0.03 It was observed that there were no significant changes in results, ensuring that the central finite difference method used in the study converges. The results are presented graphically, as shown in Figures 2-8.

Figure 2 illustrates the effects of varying Reynolds number (Re) on temperature profiles. It is observed that an increase in Reynolds number results in a decrease in temperature profiles. Reynolds number is the ratio of inertial to viscous forces. An increase in Reynolds number increases inertial forces, which dominate viscous forces. This, in turn, decreases friction between the surface and fluid, resulting in a reduction of dissipated heat within the thermal boundary layer and hence a decrease in temperature profiles. Conversely, when Reynolds number is decreased, viscous forces dominate the inertial forces. These large viscous forces result in increased friction between the surface and the fluid, translating into increased dissipation of heat within the boundary layer and hence an increase in temperature profiles.

Figure 3 depicts the impact of the Eckert number (Ec) on the temperature profiles. It is observed that an increase in the Eckert number leads to an increase in the magnitude of temperature profiles. An increase in the Eckert number implies that the fluid absorbs more heat energy released from internal viscous forces, which in turn increases the temperature profiles.

Figure 4 illustrates the effects of the Prandtl number (Pr) on the thermal boundary layers. It is observed that an increase in the Prandtl number results in a decrease in the magnitude of the thermal boundary layers. The Prandtl number is the ratio of momentum diffusivity to thermal diffusivity. A fluid with a high Prandtl number has relatively low thermal conductivity, which results in the reduction of the thermal boundary layer thickness.

Figure 5 illustrates the impact of the variable viscosity parameter (ε) on the thermal boundary layer. It is observed that an increase in the variable viscosity parameter results in a decrease in the thermal boundary layer. An increase in the variable viscosity parameter results in a decrease in viscous forces. This decrease in viscous forces leads to reduced friction between the sheet surface and the fluid, consequently lowering the dissipation of heat within the thermal boundary layer. Conversely, a decrease in the variable viscosity parameter increases the dimensionless viscosity and thus increases viscous forces, which dominate the inertial forces. These large viscous forces, in turn, increase friction between the fluid and the surface, translating into increased dissipation of heat within the thermal boundary layer.

Figure 6 depicts the effects of sheet surface velocity (a) on the temperature profiles. It is observed that an increase in sheet surface velocity decreases the magnitude of the temperature profiles. This decrease is attributed to the increase in fluid velocity and, consequently, a decrease in viscous forces. The decrease in viscous forces results in a reduction of viscous dissipation within the thermal boundary layer, thus leading to a decrease in the temperature profiles.

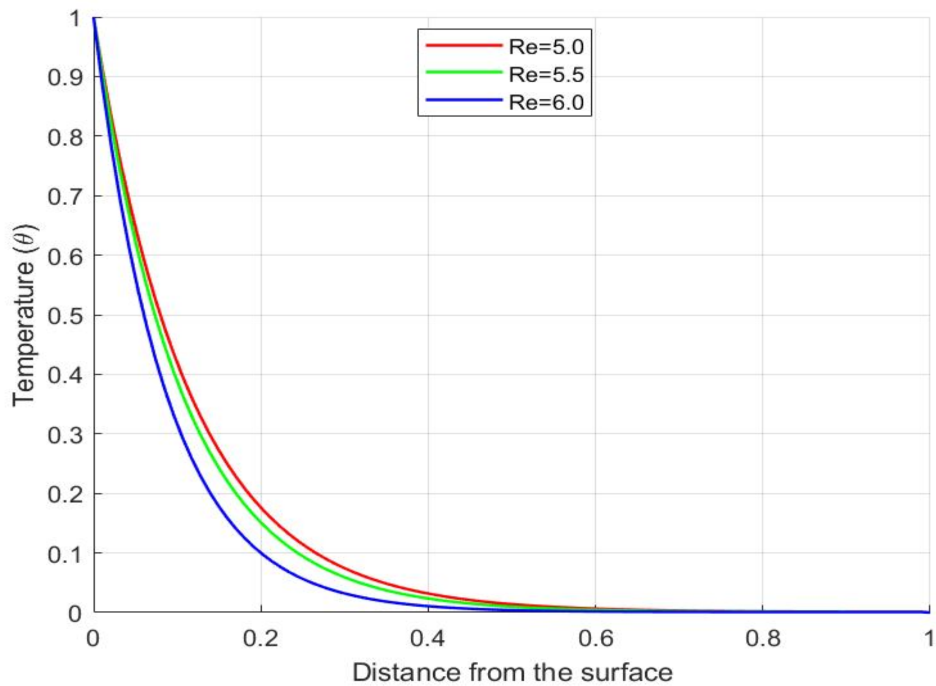

FIGURE 2: Effects of varying Reynolds number on temperature profiles

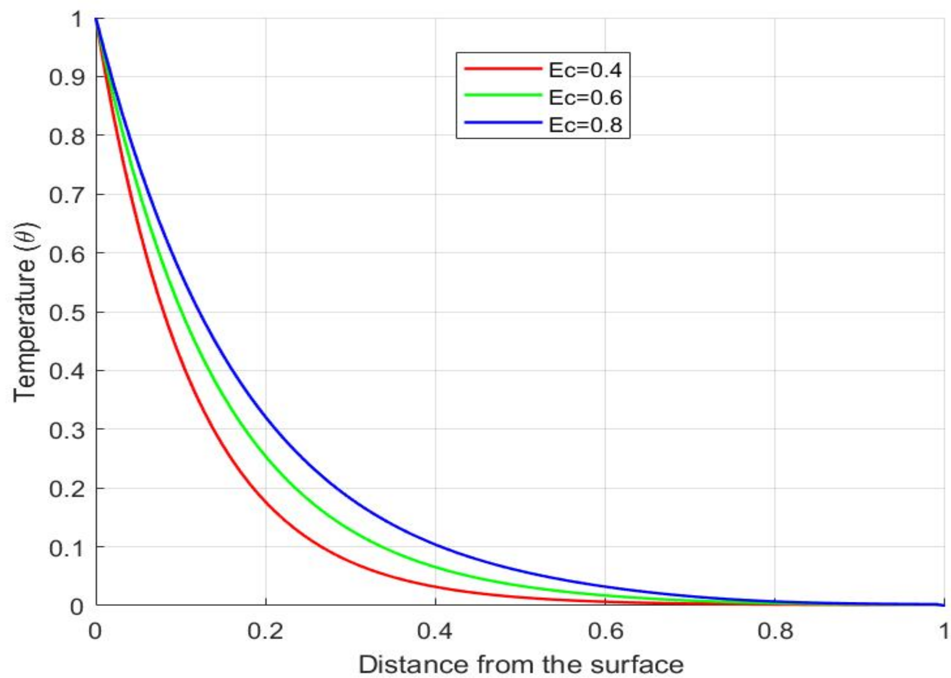

FIGURE 3: Effects of varying Eckert number on temperature profiles

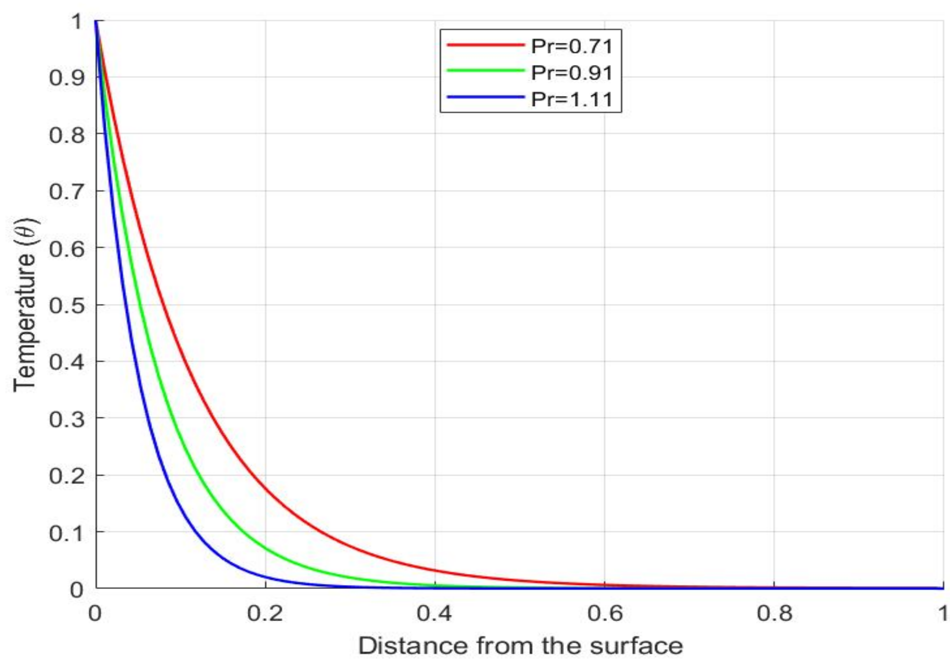

FIGURE 4: Effects of varying Prandtl number on temperature profiles

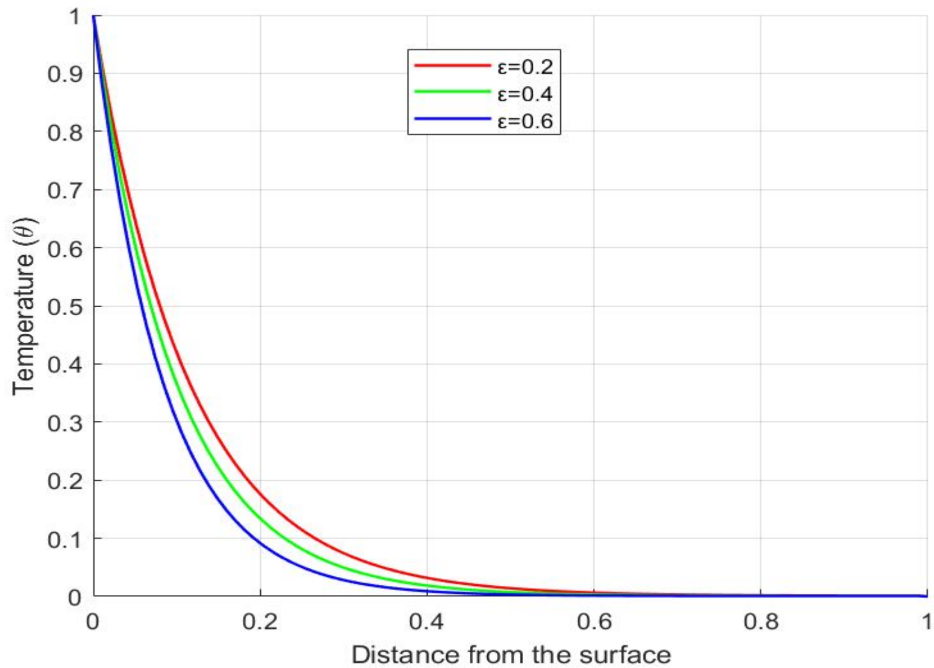

FIGURE 5: Effects of varying viscosity parameter on temperature profiles

Figure 7 depicts the effects of Reynolds number (Re) on the velocity profiles. An increase in Reynolds number results in an increase in velocity profiles. The Reynolds number represents the ratio of inertial forces to viscous forces. An increase in Reynolds number leads to a larger inertial force. The effects of inertial forces become more pronounced due to the decrease in resistance to shear stress between the layers of the fluid, which reduces viscous forces, causing the fluid to accelerate. Conversely, a decrease in Reynolds number leads to larger viscous forces, which oppose the motion of the fluid, resulting in a decrease in velocity profiles

Figure 8 illustrates the effects of the variable viscosity parameter (ε) on the velocity profiles. It is observed that an increase in the variable viscosity parameter increases the velocity profiles. In this study, viscosity is considered an inverse function of temperature, implying that as the temperature of the fluid increases, its viscosity decreases. The variable viscosity parameter typically represents this relationship; therefore, an increase in the variable viscosity parameter implies that viscosity decreases more rapidly with increasing temperature. As the viscosity decreases, the fluid experiences less internal friction or resistance to flow. This reduction in resistance allows the fluid to move more easily, leading to an increase in its velocity.

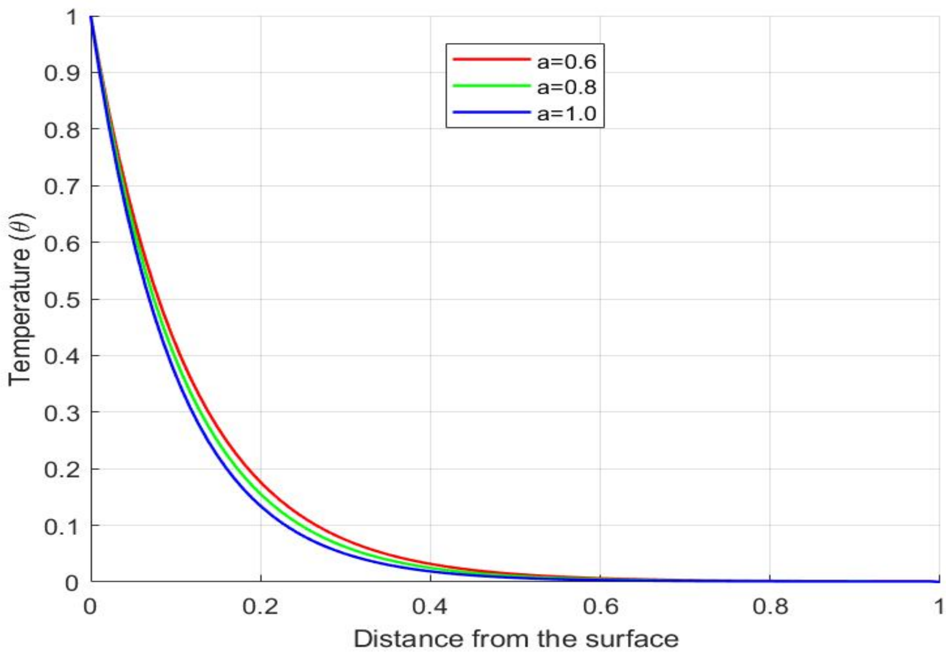

FIGURE 6: Effects of varying velocity of the surface on temperature profiles

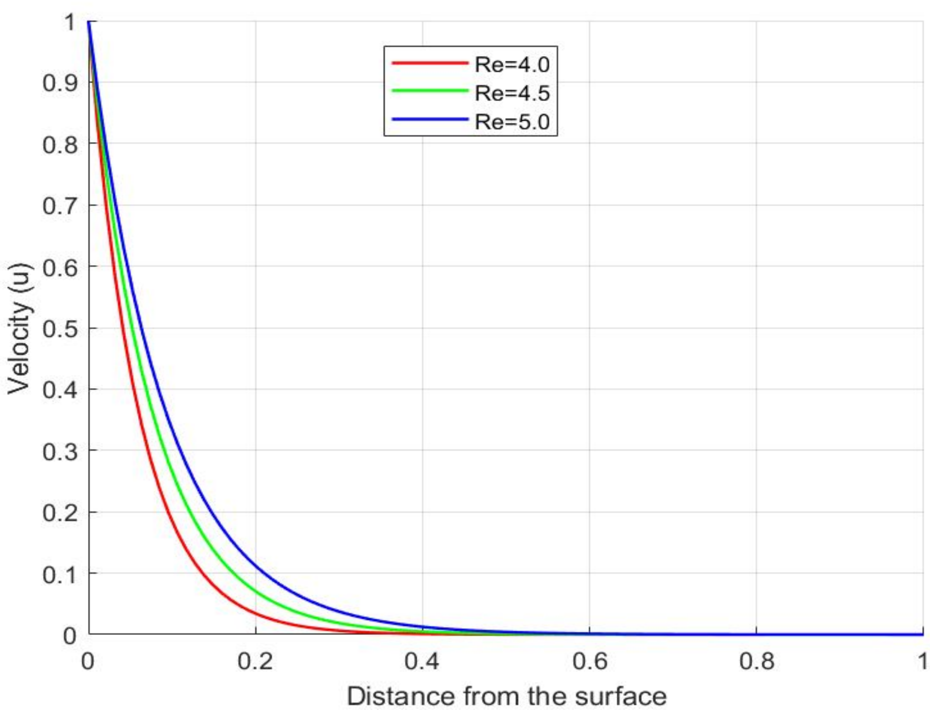

FIGURE 7: Effects of Varying Reynolds Number on Velocity Profiles

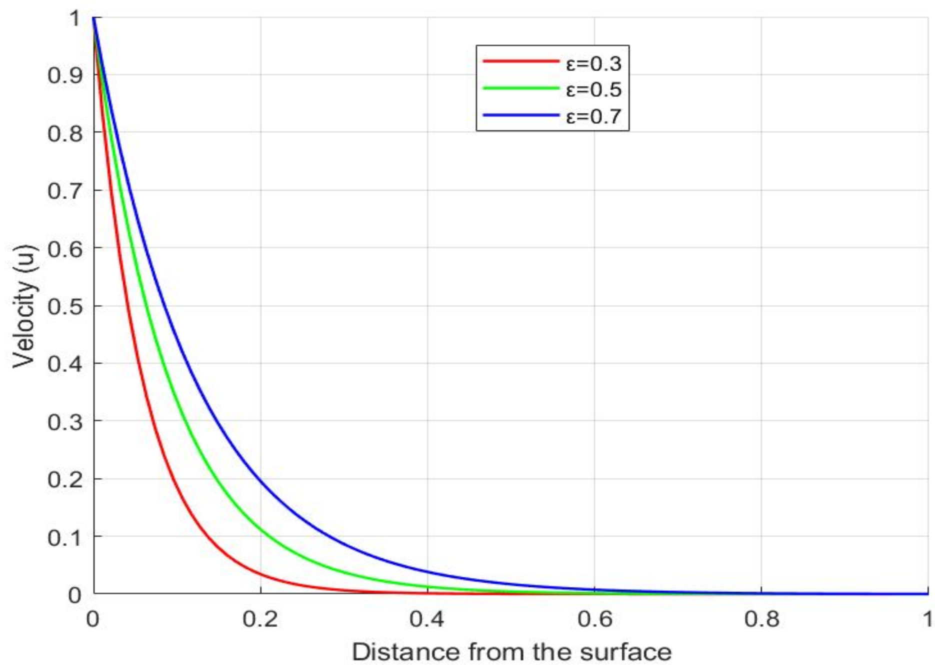

FIGURE 8: Effects of the Variable Viscosity Parameter on Velocity Profiles

The combined effects of temperature-dependent viscosity, viscous dissipation, and a moving surface in a quiescent fluid find application in hot rolling during metal processing and in the lubrication of sliding or rotating bearings [13].

In hot rolling, metals such as steel or aluminum are passed through rollers to reduce thickness and improve mechanical properties. The rolling surface moves over the metal sheet, creating fluid flow in the boundary layer. The metal's surface temperature is extremely high, which affects the viscosity of the lubricant or cooling fluid used to reduce friction and dissipate heat. As the viscosity of the fluid depends on temperature, and viscous dissipation generates additional heat, managing these effects is essential to ensuring smooth operation, avoiding excessive heat buildup, and preventing material defects.

In mechanical systems with sliding or rotating bearings, such as turbines, compressors, or large motors, the surfaces are often lubricated by oil. The moving surfaces create boundary layer flow in the lubricant. Viscous dissipation, caused by friction between the moving parts and the lubricant, generates heat, which changes the lubricant's temperature and affects its viscosity. Temperature-dependent viscosity plays a crucial role in maintaining the correct lubrication film thickness to minimize friction and wear.

In these applications, the combined effects of temperature-dependent viscosity, viscous dissipation, and a moving surface in a quiescent fluid significantly influence the efficiency, performance, and quality of the process or system. Proper management of these effects is crucial for optimizing heat transfer, minimizing friction, and ensuring smooth operation, whether in high-temperature manufacturing, lubrication systems, or thermal management.

5. CONCLUSION

In this paper, the combined effects of temperature-dependent viscosity, viscous dissipation, and a moving surface in a quiescent fluid on temperature and velocity profiles have been investigated. The fluid flow was considered to be caused by the movement of the sheet surface, with viscosity being an inverse function of temperature. The boundary layer equations governing the flow were transformed into non-dimensional form, yielding the following dimensionless parameters: Reynolds number (Re), Eckert number (Ec), Prandtl number (Pr), variable viscosity (ε), and surface velocity (a). The non-dimensional governing equations were solved using the central finite difference numerical method implemented in MATLAB software.

The numerical results demonstrate that an increase in the Reynolds number and the variable viscosity parameter leads to an increase in the velocity profiles.Additionally,an increase in Reynold's number, Prandtl number, variable viscosity, and surface velocity results in a decrease in temperature profiles. Moreover, an increase in the Eckert number was found to increase temperature profiles. Therefore, with suitable flow parameters, the temperature of the fluid can be regulated during the cooling of hot sheets or metallic plates drawing over a quiescent fluid to obtain high-quality final products.

The recommendations for future studies include extending the analysis to unsteady fluid flow, considering different geometries, and examining the effects of magnetic fields and porous media.

Nomenclature

Symbol	Quantity
C_P	Specific heat capacity
Ec	Eckert number $\left(= \frac{U^2}{C_p(T_w - T_\infty)}\right)$
K	Thermal conductivity, $Wm^{-1}K^{-1}$
L	Characteristic length, m
Pr	Prandtl number $\left(= \frac{\mu C_p}{K}\right)$
Re	Reynolds number $\left(= \frac{\rho u l}{\mu}\right)$
T	Dimensional temperature, K
T_w	Temperature at the wall, K
T_∞	Free stream temperature, K
u, v	Dimensional velocity components, ms^{-1}
u^*, v^*	Dimensionless velocity components
U_w	Sheet velocity, ms^{-1}
U_∞	Characteristic velocity, ms^{-1}
x, y	Cartesian coordinate in dimensional form, m
x^*, y^*	Cartesian coordinate in dimensionless form
Δx, Δy	Distance intervals, m
ε	Viscosity variation parameter in dimensionless form
a	Surface velocity in dimensionless form
μ	Coefficient of viscosity, $kgm^{-1}s^{-1}$

μ_∞ Coefficient of viscosity at free stream temperature, $kgm^{-1}s^{-1}$

μ_w Coefficient of viscosity at surface temperature, $kgm^{-1}s^{-1}$

ρ Fluid density, kgm^{-3}

θ Dimensionless temperature

DATA AVAILABILITY STATEMENT

The data used for this study are from the published articles that are cited.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Authorhereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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