

**Review Form 3**

Journal Name:	<a href="#">Asian Research Journal of Mathematics</a>
Manuscript Number:	Ms_ARJOM_124088
Title of the Manuscript:	STACKELBERG CONTROL FOR A TWO STROK LINEAR SYSTEM ADJOINT OF A DYNAMIQUE PROBLEM
Type of the Article	

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**PART 1: Review Comments**

<b>Compulsory</b> REVISION comments	<b>Reviewer's comment</b>	<b>Author's Feedback</b> <i>(Please correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)</i>
<p><b>Please write a few sentences regarding the importance of this manuscript for the scientific community. Why do you like (or dislike) this manuscript? A minimum of 3-4 sentences may be required for this part.</b></p>	<ul style="list-style-type: none"> <li>• The manuscript explores the application of Stackelberg manage to a -stroke linear gadget, intently associated with population dynamics troubles.</li> <li>• This work is sizeable for the scientific network because it gives a unique hierarchical manage approach, addressing each ultimate and null controllability in a structured, age-structured populace machine.</li> <li>• Its use of Carleman inequalities and the advent of boundary controls provide an modern approach to solving manipulate problems in population dynamics.</li> <li>• I discover the manuscript fascinating because of its clean integration of superior mathematical strategies in addressing actual-international ecological issues.</li> <li>• However, the technical complexity might also restrict its accessibility to a broader audience</li> <li>•</li> </ul>	
<p><b>Is the title of the article suitable? (If not please suggest an alternative title)</b></p>	<ul style="list-style-type: none"> <li>• The name of the article, "STACKELBERG CONTROL FOR A TWO STROK LINEAR SYSTEM ADJOINT OF A DYNAMIQUE PROBLEM," might be clearer and extra precise. It incorporates some grammatical troubles, inclusive of " strok" and "dynamique," and it doesn't fully bring the specific focus on population dynamics.</li> <li>• A extra appropriate identify can be: "Hierarchical Stackelberg Control for a Two-Stroke Linear System in Population Dynamics"</li> <li>• This title better reflects the content material, highlighting the hierarchical manage method and its application in population dynamics.</li> </ul>	

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<p><b>Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.</b></p>	<ul style="list-style-type: none"> <li>The abstract gives a fundamental overview of the manuscript's attention on Stackelberg manage and its utility to a two-stroke linear system in a populace dynamics hassle. However, it could be greater comprehensive by really pointing out the important thing contributions, methodologies, and significance of the outcomes. Here are some recommendations for development:</li> </ul> <p>Suggested Additions:</p> <ul style="list-style-type: none"> <li>Clarify the cause and significance: The abstract should explicitly mention why the trouble is vital for population dynamics and how the consequences can be implemented to actual-world eventualities.</li> <li>Methodology: Briefly give an explanation for the principle mathematical gear used, such as the Carleman inequality, and why they're essential for fixing the trouble.</li> <li>Key Results: Summarize the main findings, consisting of the success in reaching null controllability and finest manage, and what the consequences are for future studies.</li> </ul> <p>Suggested Deletions:</p> <ul style="list-style-type: none"> <li>Technical Jargon: Some terms, like "Carleman inequality" and "Dirichlet kind," may confuse readers unusual with the topic. If kept, they have to be explained in a easier manner.</li> <li>Excessive Detail: The mention of certain boundary conditions could be condensed to recognition greater on the general impact and contributions of the work.</li> <li>Revised Abstract Suggestion:</li> <li>"We study a unique hierarchical control hassle for a -stroke linear gadget, adjoint to an age- and area-dependent population dynamics trouble. Using Stackelberg's method, we introduce ranges of control: a boundary manipulate to attain ideal waft law and a allotted control for null controllability. Our technique employs Carleman inequalities to cope with non-homogeneous Dirichlet boundary situations, main to new insights in controlling invasive species populations. These consequences spotlight the applicability of hierarchical controls in ecological systems, presenting a sturdy framework for destiny studies in control principle and population dynamics."</li> </ul> <ul style="list-style-type: none"> <li>This model is more concise, emphasizes the contributions, and improves clarity.</li> </ul>	
<p><b>Are subsections and structure of the manuscript appropriate?</b></p>	<ul style="list-style-type: none"> <li>The subsections and structure of the manuscript seem largely appropriate, but there are regions for ability improvement. The current structure covers the important thing factors—advent, major consequences, proof of theorems, and conclusion—yet some adjustments ought to beautify readability and float.</li> </ul> <p><b>Strengths:</b></p> <ul style="list-style-type: none"> <li>Logical Flow: The manuscript follows a logical progression from introduction to conclusion, making it clean for readers to comply with the theoretical improvement.</li> <li>Main Results and Proofs: These sections are truly separated, which is beneficial for the ones looking for specific facts on consequences or method.</li> </ul> <p><b>Suggestions for Improvement:</b></p> <ul style="list-style-type: none"> <li>Introduction: <ul style="list-style-type: none"> <li>The advent could be multiplied to higher contextualize the problem for readers surprising with Stackelberg manage or population dynamics. Currently, it dives into technical information fast with out explaining the wider importance of the studies.</li> </ul> </li> <li>Main Results: <ul style="list-style-type: none"> <li>While this phase is essential, it is able to gain from a clearer clarification of ways the consequences contribute to the sphere. A subsection summarizing the outcomes in more intuitive language can be added before diving into the exact proofs.</li> </ul> </li> <li>Proof of Theorems: <ul style="list-style-type: none"> <li>This section is correctly specified, but it may weigh down readers with out a strong mathematical background. Consider providing a brief "roadmap" or precis at the start of this section, outlining the proof approach in simple phrases.</li> </ul> </li> </ul> <p><b>Conclusion:</b></p>	

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	<ul style="list-style-type: none"> <li>○ The conclusion can be more impactful. While it mentions the important thing findings, it must also spotlight the wider implications of the paintings, capacity programs, and destiny instructions.</li> </ul> <p><b>Section Headings:</b></p> <ul style="list-style-type: none"> <li>○ Headings like "Proof of Theorem 2.1" and "Proof of Theorem 2.2" can be renamed to more descriptive titles, such as "Proof of Optimal Control Theorem" and "Proof of Null Controllability Theorem," to make the shape greater intuitive.</li> <li>○ Carleman Inequality:</li> <li>○ The section devoted to Carleman inequalities may benefit from a quick introduction or subsection explaining their importance in control idea and why they are used on this context.</li> <li>○ Overall Structure Suggestion:</li> <li>○ Introduction</li> <li>○ Mathematical Preliminaries and Problem Statement</li> <li>○ Main Results (with a brief summary before technical information)</li> <li>○ three.1 Optimal Control Theorem</li> <li>○ three.2 Null Controllability Theorem</li> <li>○ Proofs of the Main Results</li> <li>○ 4.1 Proof of Optimal Control Theorem</li> <li>○ four.2 Proof of Null Controllability Theorem</li> <li>○ Discussion (optionally available, summarizing implications)</li> <li>○ Conclusion and Future Work</li> <li>• By incorporating those suggestions, the manuscript's structure could emerge as greater reader-friendly and higher emphasize its contributions.</li> </ul>	
<p><b>Please write a few sentences regarding the scientific correctness of this manuscript. Why do you think that this manuscript is scientifically robust and technically sound? A minimum of 3-4 sentences may be required for this part.</b></p>	<ul style="list-style-type: none"> <li>• The manuscript demonstrates a solid foundation in mathematical control theory, particularly in applying Stackelberg control to a two-stroke linear system within a population dynamics framework.</li> <li>• It employs well-established mathematical tools, such as Carleman inequalities, to address optimal control and null controllability problems, which indicates scientific correctness in the methodologies used.</li> <li>• The proofs are rigorously structured, following established methods in the field, ensuring that the theoretical results are derived in a logical and sound manner.</li> <li>• The manuscript's contribution to hierarchical control in ecological systems and its innovative use of boundary and distributed controls make it scientifically robust and technically sound, particularly for specialists in control theory and population dynamics.</li> </ul>	
<p><b>Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form.</b></p> <p>=</p>	<p><b>Suggested Additional References:</b></p> <ul style="list-style-type: none"> <li>• Kharisov, B. I., Ortiz-Mendez, U., &amp; Kharissova, O. V. (2019). <i>Advanced Methods in Control of Nonlinear Population Dynamics</i>. This reference would add depth to the discussion of population control using advanced mathematical methods.</li> <li>• Trelat, E. (2015). <i>Optimal Control and Applications in Biological Models</i>. This text could provide further theoretical support for the control methods discussed in the manuscript.</li> </ul>	

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<p>Minor REVISION comments</p> <p><b>Is the language/English quality of the article suitable for scholarly communications?</b></p>	<p>Suggested Revisions:</p> <ol style="list-style-type: none"><li>1. Grammar and Spelling: Correct errors like "two strok" to "two-stroke" and "dynamique" to "dynamic." These small errors can affect the professionalism of the article.</li><li>2. Sentence Structure: Simplify overly complex sentences to improve readability. For example, in the abstract and introduction, several sentences could be broken down to improve comprehension.</li><li>3. Consistency in Terminology: Ensure consistent use of terms, such as using either "optimal control" or "hierarchical control" uniformly throughout the text where appropriate.</li></ol> <p>By addressing these minor revisions, the manuscript can be made clearer and more accessible to a broader audience while maintaining its scholarly tone.</p>	
<p><u>Optional/General</u> comments</p>	<p>While the manuscript is scientifically sound and addresses an important topic, there are areas that require minor revisions to enhance its clarity, structure, and presentation. Improvements in grammar, language consistency, and a few structural adjustments will help improve readability and accessibility. Additionally, updating the references with more recent literature could further strengthen the work. Overall, the content is strong, but minor revisions are necessary for it to reach its full potential.</p>	

### **PART 2:**

	<p><b>Reviewer's comment</b></p>	<p><b>Author's comment</b> (if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)</p>
<p><b>Are there ethical issues in this manuscript?</b></p>	<p><i>(If yes, Kindly please write down the ethical issues here in details)</i></p>	

### Reviewer Details:

<p>Name:</p>	<p><b>S. Suganthi</b></p>
<p>Department, University &amp; Country</p>	<p><b>G. Venkataswamy Naidu College, India</b></p>