INTEGER PROGRAMMING OPTIMIZATION-BASED SIMULATION IN ALLOCATING MEDICAL DOCTORS IN RURAL PUBLIC HEALTH FACILITIES IN KENYA

Abstract: The efficient distribution of physicians within rural public hospitals is essential for maximizing the use of healthcare resources and enhancing patient access to high-quality treatment. This work uses a simulation approach based on integer programming optimization to handle the doctor allocation problem in rural public hospitals in Kenya. An objective function, constraints, and decision variables are included into an integer programming paradigm. The goal is to create an allocation strategy that takes into account limitations of physician availability, facility capacity, and patient demand, all while minimizing patient waiting times. Subsequently, the model incorporates simulation approaches to replicate patient flow and assess the effectiveness of various allocation strategies. The present study adds to the corpus of knowledge in healthcare operations research by offering significant insights into the optimization of doctor allocation in public hospitals. The study's conclusions have applications for those in charge of healthcare policy.

Comment [H1]: This does not comprehensively summarize the the research work. Try going through the following headings, making at least one sentence on each of the topics introduction, problem statement, research method, result, conclusions and recommendation

Keywords: integer programming, decision variables, health system planning, distribution of physicians, rural public hospitals.

I. Introduction

A mathematical optimization or feasibility program where some or all of the variables are limited to be integers is known as an integer programming problem. The phrase is often used to describe integer linear programming (ILP), in which the constraintsand the goal function are both linear.The significance of allocating resources in healthcare systems efficiently has come to light more and more in recent years. A crucial part of this procedure is assigning physicians to public health facilities since it has an immediate influence on patients' access to prompt, high-quality medical care. However, there is a big challenge especially in Kenya's rural areas, lack of doctors, an unequal distribution of doctors, and inefficient allocation procedures, which result in lengthy wait times. Previous research on the distribution of doctors relies on heuristic or rule-based strategies, frequently ignoring the advantages of optimization-based techniques.

Health care planning can be seen as the science of predicting the number of resources needed to deliver health care service at specified levels of cost and quality.Mathematical modeling and simulation in this study is used for planning and automation of processes based on the design and control of health care processes that includes allocation of physicians and for budgeting.If health systems are to serve people best [13], [3], then they must operate efficiently and equitably, and appropriate valuation methods are needed to determine how to do this. With the advances in computing power over the past few decades, advanced mathematical optimization algorithms can now be run on personal computers and can be used to provide comprehensive, evidence-based recommendations for policymakers on how to prioritize health spending considering policy objectives, interactions of interventions, real-world system constraints and budget envelopes.

An automated solutions for Integer Linear Programming (ILP) issues was created, [18]. The computational procedure that involved creating precise answers to integer linear programming problems by utilizing an object-oriented was applied as the methodology. The results showed that the developed Visual basic programming software was robust to parameter variations and very accurate.

Akkermans*et al.* [18] offered an integer linear programming two-phase method to handle the shift and break design problem in which the process of assigning breaks to each instance of a shift, one by one, until no improvement was observed. This methodology outperforms the state-of-the-art strategy for the shift and break design problem on a set of benchmark instances that include both randomly generated and real-life examples.

A capacity expansion model with exponential demand and continuous time intervals and facility sizes was done,[10]. The capacity expansion as the product of limited and discrete choices as routine planning sessions was modelled. It was shown that a capacity increase or decrease occurs in some fixed bed amount.A general health care delivery system as a network of queuing stations and incorporating the queuing network into an optimization frameworkwas modelled, [4], to determine the optimal capacity levels subject to a specified level of system performance.An algorithm combining branch-and-bound with outer approximation cutting plane method to solve the nonlinear optimization problem with discrete variables was used, but the algorithm could require complete enumeration of all integer solutions, leading to very large solution times.While capacity planning has challenged health care decision makers and researchers for decades [7], [8] and[12], there is a renewed sense of urgency to address this problem. Furthermore, to the perennial struggle between the continually increasing costs of highly specialized and scarce inputs such as skilled and flexible staff, advanced clinical and medical technology and equipment, physical space and supplies, and declining government and private reimbursements [6] and[11], the demand for inpatient care has been growing substantially. An increased inpatient admissions coupled with fewer hospitals and fewer hospital attendants would support the argument in favor of capacity increases [1], [2] and[5].

An optimal integer linear programming (OILP) algorithm with Lyapunov stability optimization algorithm based on task priorityand the MEC and edge cloud servers separately was done, [16].This was transformed into an integer linear programming problem, and then an integer linear programming algorithm based on CPU utilization optimization was proposed to obtain a resource allocation scheme.

Patient assignment was defined as the process of assigning hospital patients to specific physician services and clinical units based on their diagnosis, [14]. A discreteevent simulation model to evaluate different assignment strategies using a simulation-based optimization approach was applied by evaluating and heuristically optimizing these strategies accounting for expected hospital and physician profit, care quality and patient waiting time. It was shown that the strategies that use heuristically optimized designation of overflow services and units increase expected profit relative to the capacity-based strategy in which overflow patients are assigned to a service and unit with the most available capacity

A mixed-integer linear programming model (MILP) was developed, [15] for resource allocation of data analysis workflows on heterogeneous clusters. The model contained binary decision variables determining assigned to which nodes. Comparative results showed that the approach outperforms related approaches in most cases.

Classical resource allocation methods for query optimization to handle the pricing models in cloud environments was performed [19] using Integer Linear-Programming (ILP). The proposed linear models was implemented in a fast solver for ILP, compared with some existing greedy algorithms. Experimental evaluation showed that the solution offers a good trade-off between the allocation quality and allocation cost.

In order to create a heat exchanger network with numerous utilities from a collection of hot and cold streams and manually chosen utilities, a modified mixed-integer linear programming model was constructed in Microsoft Excel® and reduced using the Solver tool [17]. It was shown that the annualized cost of a heat exchanger network with multiple utilities may be less than that of a network without multiple utilities due to boiler-feed water generation and the low cost of utilities.

This research work seeks to create a simulation framework based on integer programming optimization for assigning physicians to public health institutions in rural Kenya. The goal is to reduce patient wait times while processing patients' demand, facility capacity, and doctor availability that improves patient outcomes and healthcare access by merging simulation and optimization to improve decision-making and resource use.

Although simulation methods have been employed to evaluate the effects of allocation strategies, they have hardly ever been combined with optimization models for integer programming. As a result, there is a gap on the combined application of these methods to create reliable and effective physician allocation plans in public health institutions in Kenya's rural health facilities.

II. Problem Statement

The effective allocation of medical doctors to public health facilities in rural health facilities in Kenya is a difficulty that results in inefficient resource utilization, prolonged patient wait times, and compromised access to healthcare. The lack of integration between optimization and simulation techniques in current allocation methodologies hinders the development of strong and efficient doctor allocation strategies. Thus, an integrated framework integrating simulation and integer programming optimization approaches is required to optimize the distribution of doctors while taking patient demand, facility capacity, and doctor availability into account. Through tackling this issue, the study seeks to increase patient access to care, boost the effectiveness of the healthcare system, and reduce wait times.

III. Mathematical Formulation

Essentially, capacity restrictions, doctor availability, patient demand, and the goal of reducing waiting time or patient waiting costs are all captured in the mathematical formulation of the doctor allocation problem. This optimization problem is solved to produce allocation strategies that maximize resource use, enhance patient access to healthcare, and shorten wait times.Amathematical representation of integer programming optimization-based is subjected to the following assumptions:

1. The system is in steady state in each of the intervals used.
2. Hospitals physician must be integer valued.
3. The arrival rates for different patient streams can be combined and a representative value for the average length of stay per patient.
4. The number of physicians in a facility can be limited.

Let the length planning horizon be indexed by T indexed by $t = 1, 2, 3, \dots, T$.

In most health facilities, the average length of stay has been relatively stable over time [2], therefore the patient's aggregate arrival ratefor a given facility will not be known until the demand presents itself. Applying the above assumptions, three types of decision variables are formed.The objective function minimizes the total cost of patient waiting, changing physician capacity, and operating the existing physician capacity.The aggregate physician capacity planning is represented as a nonlinear inter programming problem as;

$$min \sum_{t=1}^{T} f(x_t, \lambda_t, \mu_t) + \sum_{t=1}^{T} g(x_{t-1}, x_t) + \sum_{t=1}^{T} c(x_t) \dots \dots \dots \dots (1)$$

Subject to

$w(x_t, \lambda_t, \mu_t) \leq \alpha_t$(Patients maximum allowable waiting limit)

$x_0 = p_0$(Initials physician's availability)

$x_{t-1} + x_t^+ - x_t^- = x_t$(Balance equation)

$g(x_{t-1}, x_t) \leq R_t$ (Budget limit)

$x_t \geq 0, x_t^+ \geq 0, x_t^- \geq 0$(Integer valued); $\forall t$ … … … … … … … … … (2)

Sometimes it is cumbersome to handle large values of integer problem since there are no restrictions on how many physicians are added to or removed from service, since this is done in batches corresponding to the size of a health center. This means that constraints associated with change in capacity can be replaced a set of discrete alternative constraints which are chosen in the solution for each period. Equations (1) and (2) becomes nonlinear integer programming problem. For simplicity we set a base value β in multiples which physician capacity can be increased or decreased at n number of distinct levels of capacity increase or decrease. For instance, if p in a period of $t+1$ can be of $(p-n\beta)^+$, $(p-(n-1)\beta)^+$, …, $(p-\beta)^+$, p, $p+\beta$, …, $p+(n-1)\beta$, $p+n\beta$, where $(x)^+ = max\{0, x\}$. Here if we assume that the newly acquired doctors are available, the capacity increases by $i\beta$ at the beginning of period t for $i = 1, 2, 3, \ldots, n$, then $z_{it}^+ = 1$and zero otherwise.Similarly the capacity decreases by $i\beta$ at the beginning of period $t$for $i = 1, 2, 3, \ldots, n$, then $z_{it}^- = 1$ zero otherwise. Therefore the nonlinear zerone inter programming problem is;

$$min \sum_{t=1}^{T} f(x_t, \lambda_t, \mu_t) + \sum_{t=1}^{T} g(x_{t-1}, x_t) + \sum_{t=1}^{T} c(x_t) \ldots \ldots \ldots \ldots (3)$$

(Minimizes the total cost of patient delay changing and operating physician capacity)

Subject to

$w(x_t, \lambda_t, \mu_t) \leq \alpha_t$(Patients maximum allowable waiting limit)

$x_0 = p_0$(Initials physician's availability)

$$x_{t-1} + \sum_{i=1}^{n} i\beta z_{it}^+ - \sum_{i=1}^{n} i\beta z_{it}^- = x_t$$

(Flow balance equation)

$$\sum_{i=1}^{n} z_{it}^{+} + \sum_{i=1}^{n} z_{it}^{-} \leq 1$$

(Allowsonly one choice for changing the capacity in each period)

$g(x_{t-1}, x_t) \leq R_t$ (Budget limit)

$x_t \geq 0,$ (Non-negativity on physicians and capacity selection)

$$z_{it}^{+}, z_{it}^{-} \in \{0, x\} \forall t \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4)$$

Finally the integer programming representation as a network of the above system is developed. Let (t, p) represent a system where there are p physicians in period t and $C(p)$ be a set of reachable capacity levels in the next period if the capacity in the current period is p, that is $C(p)$=$\{(p - n\beta)^{+}, (p - (n-1)\beta)^{+}, \ldots, (p - \beta)^{+}, p, p + \beta, \ldots, p + (n-1)\beta, p + n\beta\}$. Moreover, a set S_t represents all capacity levels reachable in period t from all capacity levels in period $t - 1$. A source node denoted by d_s connected only to node $(0, x_0)$ with zero arc length which is the beginning state. The point $(0, x_0)$ is connectedto all nodes $(1, x^{'}) \forall x^{'} \in C(p_0)$. If $w(x', \lambda_1, \mu_1) \leq \alpha_1$ which is not a violated constraint and to $g(x_0, x') \leq R_t$ the same. The lengths of the arcs are given by $f(x', \lambda_1, \mu_1) + g(x_0, x') + c(x')$. A case where any constraint is violated, then the length of corresponding arc length is set to M. Similar case is done in the set of equations (3) and (4) where node (t, x) for $x \in S_t$ be connected to nodes $(t+1, x^{'}) \forall x^{'} \in C(x)$ with length$f(x', \lambda_{1+1}, \mu_{1+1}) + g(x_t, x') + c(x')$. If $w(x', \lambda_{1+1}, \mu_{1+1}) \leq \alpha_{1+1}$ and $g(x_t, x') \leq R_{t+1}$ and M otherwise. In order to complete the network, each node is assigned $(T, x) \forall x^{'} \in S_T$ is connected to the outer sink node d_t of an arc of zero length.

IV. Results and Discussions

Based on a set of parameters, the optimization model seeks to determine the best distribution of physicians among public rural health facilities.

A linear pattern of increasing/decreasing availability or a constant availability rate could be seen in the linear graph that shows physician availability over time. It is assumed that over the course of the study, the availability will either stay mostly same or vary at a steady pace.

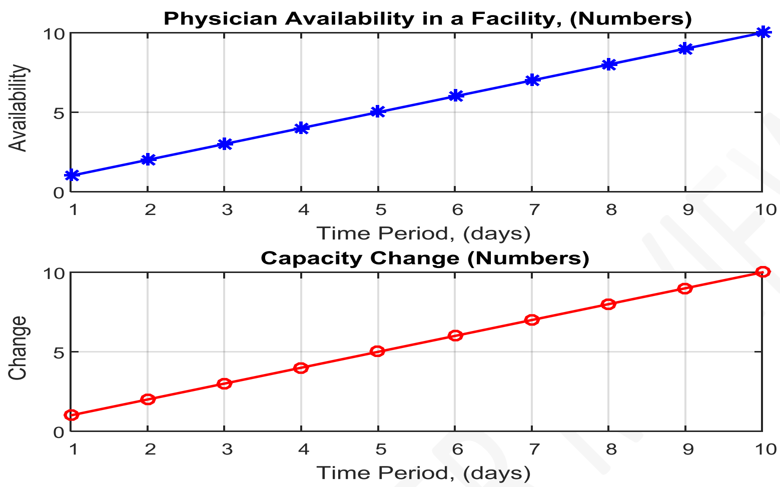

Figure 1: Capacity and availability illustration.

A constant or linearly changing capacity adjustment over the course of the study period is suggested by the linear graph showing capacity change over time. It is assumed that during the course of the planning horizon, the capacity changes will either happen steadily or at a constant rate.An allocation plan detailing the number of physicians assigned to each public health facility is provided by the optimization model. As a result, each hospital would have the ideal number of doctors assigned to it, guaranteeing effective resource use and satisfying staffing requirements.

When distributing medical doctors, the optimization model may take equity and fairness criteria into account. An allocation plan that guarantees an equitable distribution of physicians among facilities while accounting for variables including population size, patient demands, facility capacity, and geographic accessibility would be the anticipated outcome.

The optimization model is useful for assessing various situations or hypothetical analyses. This enables decision-makers to investigate how changes in a range of variables, including population distribution, the availability of physicians, and the opening of new facilities, may affect the best way to distribute medical professionals.

V. Conclusion

In conclusion, the study "Integer Programming Optimization-Based Simulation in Allocating Medical Doctors in Public Health Facilities in a Region" is warranted due to its potential to improve patient access and quality of care, optimize the allocation of healthcare resources, resolve workload imbalances, assist in decision-making and resource planning, and influence public health policy and practice. The study intends to increase the efficacy and efficiency of the healthcare systems in the area by addressing these important factors.

It's crucial to remember that the problem formulation, objective function, restrictions, and data used in the optimization model will all affect the precise expected outcomes. The outcomes would be customized to meet the unique needs and objectives of the public health facility allocation issue in targeted areas.

VI. Nomenclature

λ_t	Aggregate patient arrival rate.
t	Time rate.
$\frac{1}{\mu}$	Patient's length of stay.
μ	Physician's service rate per day.
μ_t	Physician's service rate over a period t.
α_t	Maximum allowable expected delay for patient before seen by physician period t.
p_0	Initial capacity of physician in a facility.
p	Capacity of physician in a facility.
R_t	Amount required to hire additional physician over a period t
x_t	Number of physician in a period t.
x_t^+	Amount of increase in physician capacity at the beginning of period t

x_t^- Amount of decrease in physician capacity at the beginning of period t

$f(x_t, \lambda_t, \mu_t)$ Expected patient waiting cost.

$g(x_{t-1}, x_t)$ Cost of changing physician capacity.

$w(x_t, \lambda_t, \mu_t)$ Expected delay of patient.

$c(x_t)$ Cost of operation

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