Comparison of the decomposition of static interquark potential in SU(3) lattice quantum chromodynamics

Abstract

We study the decomposition of interquark potential and quantitatively compare the utility of $V(r) \approx V_{abel}(r) + V_{offdiag}(r)$ and $V(r) \approx V_{mon}(r) + V_{mod}(r)$ by calculating respective average relative deviation. We also study the slope of $V_{abel}(r)$ and compare it with the slope of original SU(3) interquark potential over large distances so that we can study their exact confinement behavior. Remarkably, we found that for 24^4 lattice at $\beta = 6.0$, the sum of potentials $V_{mon}(r) + V_{mod}(r)$ matches the original interquark potential $V(r)$ more than $V_{abel}(r) + V_{offdiag}(r)$. Comparing our results we found that perfect abelian dominance is not seem to satisfy even for sufficiently large physical spatial volume (approximately larger than $(2fm)^3$).

Keywords

Quark Confinement, Dual superconductor picture, Monopole, Abelian projection, Maximally Abelian Gauge, Interquark potential, Abelian dominance, String tension, Lattice QCD.

1. Introduction

At low energies the strong interaction becomes extremely strong, and make it impossible to use perturbation theory to calculate behavior of quarks and gluons inside the hadron. Quark confinement is one of the prominent feature of QCD at low energy so nonperturbative methods are required to study quark confinement [1] . Nambu,'t Hooft and Mandelstam presented a possible mathematical foundation of dual superconductor picture which predicts confinement in QCD is dual version of electric charge condensation in ordinary superconductor [2]. To adopt this method a way of Abelian gauge fixing proposed by t'Hooft [3] which extracts infrared relevant Abelian degrees of freedom from QCD [4-7]. To solve QCD in strong coupling regime , lattice QCD is the only known method. When the quarks are pull apart the origin of the linear potential may be traced as flux tube and the interquark potential defined by a sum of coulomb and linear confinement term [8,9,10] given as

$$V(r)=\frac{-A}{r}+\sigma r+C \tag{1}$$

with the string tension σ , the color coulomb coefficient ‘A’ and the irrelevent constant ‘C’.The string tension describes the energy per unit length of the color flux tube and mathematically given by the slope of potential at large distances.

In this paper we will find the mathematical results of the decomposition of static potential more explicitly by calculating average relative deviation as well as comment on the perfect abelian dominance of quark confinement.

2. The Static potential Decomposition

The lattice gauge field $U(s,\mu)=e^{iagA\mu(s)}\in SU(3)$, with lattice spacing ‘a’ , gauge coupling constant ‘g’and the gluon field A_μ leads to the Cartan decomposition of the SU(3) group [11]

$$U^{MA}(s,\mu)=M(s,\mu)u(s,\mu)\in SU(3) \tag{2}$$

The abelian link variable $u(s,\mu)\in U(1)^2$ behaves as QED gauge field in lattice gauge theory ,while the off diagonal factor $M(s,\mu)\in SU(3)/U(1)^2$ behaves as charged matter field. Wilson loop used to calculate the interquark potential in lattice QCD. The $q\bar{q}$ potential from the wilson loop is calculated as [1,11]

$$V(r)=-\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C[U(s,\mu)]\rangle \tag{3}$$

where C denotes the $R\times T$ rectangular loop and $\langle ...\rangle$ means the statistical average over the gauge configuration.The Abelian part of the interquark potential is given by

$$V_{abel}(r)=-\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C[u(s,\mu)]\rangle \tag{4}$$

The off diagonal factor gives the off diagonal potential

$$V_{Offdiag}(r)=-\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C[M(s,\mu)]\rangle \tag{5}$$

The another way of abelian projection defined as coset decomposition of the non abelian lattice gauge field $U(s,\mu)$ into the abelian field u(s,μ) and the coset field $C(s,\mu)$ given as [12,13]

$$U(s,\mu)=C(s,\mu)u(s,\mu) \tag{6}$$

The abelian gauge field can in turn be decomposed into the monopole and photon part [14]

$$u(s,\mu)=u_{mon}(s,\mu)u_{ph}(s,\mu) \tag{7}$$

The modified non abelian gauge field is defined

$$\tilde{U}(s,\mu)=C(s,\mu)u_{ph}(s,\mu) \tag{8}$$

$u_{ph}(s,\mu)$ is the abelian projection of $\tilde{U}(s,\mu)$ and involves no monopoles.

The original non abelian potential from the non Abelian Wilson loop is calculated as given in equation (3)[12,13].

Similarly, $$V_{mon}(r)=-\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C[u_{mon}(s,\mu)]\rangle \tag{9}$$

also, $$V_{mod}(r)=-\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C[\tilde{U}(s,\mu)]\rangle \tag{10}$$

For the decomposition (2) several simulation results [11,15,16] informed that the original $Q\bar{Q}$ potential and abelian projected potential have almost the same slope at large distances . This property of string tension is called Abelian dominance. On the other hand for the off diagonal part there is almost no slope, so $V_{offdiag}(r)$ is pure Coulombic potential. From the analysis of static interquark potential $V(r)\approx V_{abel}(r)+V_{offdiag}(r)$ also has been verified [15]. Similarly for decomposition (6) in the MA gauge, the lattice QCD simulation curves [12,13,17,18] implied that

$V_{mon}(r)$ is almost linear at large distances and has small curve at small distances. The modified (monopoleless) components of the potential is similar to the original coulomb part both at small and large distances. Potential analysis informed that the non abelian potential V(r) is well approximated by the sum of potential $V(r) \approx V_{mon}(r) + V_{mod}(r)$ [17] .

3. Comparison of Decomposed potentials with the SU(3) static potential using average relative deviation method

To compare the sum of potential $V_{abel}(r) + V_{offdiag}(r)$ and $V_{mon}(r) + V_{mod}(r)$ to original SU(3) static potential V(r), we calculate the average relative deviation percentage as following.

The average relative deviation percentage of the sum of potential $V_{abel}(r) + V_{offdiag}(r)$ is given as

$$(\Delta d)_1 = \frac{1}{N} \sum_i \frac{V(r) - [V_{abel}(r) + V_{offdiag}(r)]}{V(r)} \tag{11}$$

where N is the total number of observation done.

Similarly , the average relative deviation for the sum of $V_{mon}(r) + V_{mod}(r)$ is calculated as [18]

$$(\Delta d)_2 = \frac{1}{N} \sum_i \frac{V(r) - [V_{mon}(r) + V_{mod}(r)]}{V(r)} \tag{12}$$

To study the comparison between the original SU(3) interquark potential and the sum of decomposed potentials obtained through the abelian projection, we observe the lattice results of 24^4 lattice at β=6.0 [17]. The plot of static potential V(R) versus R shown in figure 1. In figure the green square curve depicts the combined effect of the abelian and off-diagonal potentials i.e. $V_{abel}(r) + V_{offdiag}(r)$ similarly the orange diamond curve corresponds to the summation of the decomposed potentials, includes the monopole and modified potential $V_{mon}(r) + V_{mod}(r)$ while the blue circle curve represents the original SU(3) interquark potential V(r).

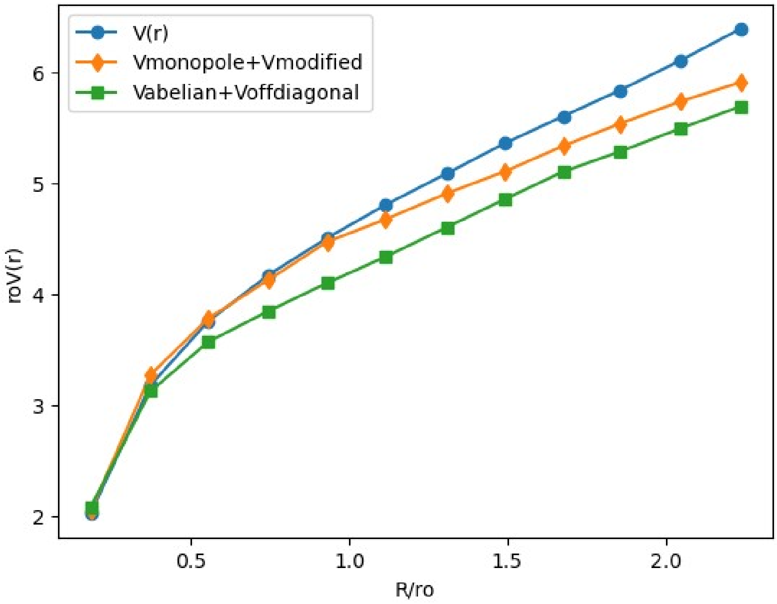

FIG. 1 : The curve of V(r) , $V_{abel}(r)+V_{offdiag}(r)$ *and* $V_{mon}(r)+V_{mod}(r)$ *for lattice* 24^4 *at β = 6.0*

Our calculations reveal an average relative deviation percentage $(\Delta d)_1$ of approximately 7.404% for the sum of the abelian and off-diagonal potentials , indicating a resemblance of about 92.596% with the original interquark potential V(r). Similarly, the average relative deviation for the sum of the monopole and modified potential is approximately 2.572%, implying a resemblance of approximately 97.28% with the non-abelian potential V(r).

The comparison between the original SU(3) interquark potential and the decomposed potentials obtained through the two different types of abelian projection methods concludes that for the given lattice , $V_{mon}(r)+V_{mod}(r)$ results are approximately 4.832 % more satisfactory than $V_{abel}(r)+V_{offdiag}(r)$ is crucial for understanding confinement phenomena in gauge theories.

4. The perfect Abelian dominance

In the context of quantum chromodynamics the string tension is a parameter used to describe the behavior of the interquark potential at large distances which is represented by the slope of the potential at large distances. The string tension for the given graph is calculated as

$$\sigma=\frac{\Delta(r_0 V(r))}{\Delta(R/r_0)} \tag{13}$$

where $\Delta(r_0 V(r))$ is change in interquark potential and $\Delta(R/r_0)$ is change in distance between two observation points at large distances. The investigation demonstrates a close alignment between the abelian projected string tension and the full string tension, a phenomenon referred to as abelian dominance. To delve into abelian dominance further, we examine the 24^4 lattice at $\beta = 6.0$. The string tension $(\sigma_{abelian})$ and (σ), calculated from the slopes of $V_{abel}(r)$ and $V(r)$ respectively, as depicted in figure 2. The observation of string tension for the given lattice implies that the $\frac{\sigma_{abelian}}{\sigma} \approx 0.808$.

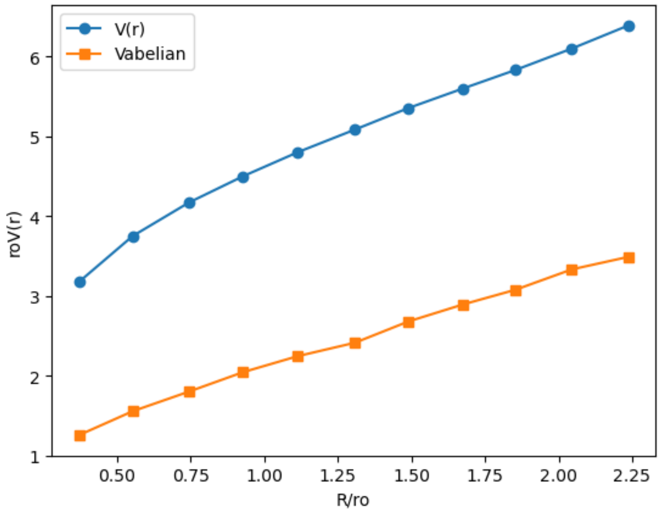

FIG. 2 : Slope of V(r) and $V_{abel}(r)$ for 24^4 lattice at β = 6.0.

To compare our findings with lattice data, we summarize several pioneering studies of Abelian dominance in the table below.

β	$L^3 L_t$	La(fm)	$\frac{\sigma_{abelian}}{\sigma}$	Ref
6.4	32^4	1.86	1.015	[15]
6.0	32^4	3.27	1.009	[15]
5.8	$16^3 32$	2.37	1.00	[15]
6.0	$16^3 32$	1.64	0.94	[15]
6.0	$12^3 32$	1.25	0.94	[15]
6.2	$16^3 32$	1.20	0.95	[15]
5.9	$10^3 16$	1.23	0.93	[19]
6.0	16^4	1.68	0.90	[19]
6.0	$16^3 32$	1.6	0.83	[20]

Table 1- Results of abelian dominance from several lattice studies [15].

Sakumichi et. Al [15] concluded from the lattice studies that physical spatial volume approximately larger than $(2fm)^3$ is necessary for perfect abelian dominance of the string tension. However, for the lattice we utilized, specifically 24^4 lattice at $\beta=6.0$, the lattice spacing defined by $La\approx 2.232\, fm$ with $a/r_0=0.186(4)$,where $r_0=0.5\, fm$ [17]. Because our value is greater than 2fm, we should have achieved perfect Abelian dominance, but our result is quite different as $\frac{\sigma_{abelian}}{\sigma}\approx 0.808$ and does not agree with perfect Abelian dominance to be realized for sufficiently large physical spatial volume.

5. Summary and Conclusions

Our results suggest that out of both two approaches of decomposition of interquark potential , the summation $V_{mon}(r)+V_{mod}(r)$ gives approximately 4.832% better results than $V_{abel}(r)+V_{offdiag}(r)$ in the case of 24^4 lattice at $\beta = 6.0$. We also concluded that despite having sufficiently large spatial size, it could not satisfy the perfect Abelian dominance.

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