Comparison of the decomposition of static interquark potential in SU(3) lattice quantum chromodynamics

Abstract: We study the decomposition of interquark potential and compare the utility of $V(r) \approx V_{abelian}(r) + V_{offdiagonal}(r)$ and $V(r) \approx V_{monopole}(r) + V_{modified}(r)$ by calculating respective average relative deviation. We also study the slope of $V_{abelian}(r)$ and $V_{monopole}(r)$ and compare it with the slope of original SU(3) interquark potential over large distances so that we can study their exact confinement behavior.

PACS numbers: 11.15.Ha, 12.38.−t, 12.38.Aw, 12.38.Gc

1.Introduction： In the context of quark confinement, at low energies the strong interaction becomes extremely sturdy and make it impossible to use perturbation theory to calculate behavior of quarks and gluons inside the hadron. So nonperturbative methods are required to study quark confinement [1].'t Hooft presented a possible mathematical foundation of dual superconductor picture by way of Abelian gauge fixing on $SU(N)/U(1)^{N-1}$ [2]. Abelian projection [2,3] involves decomposition of non-Abelian field into Abelian and non-Abelian parts. The Abelian part is associated with the U(1) subgroup, usually interpreted as an effective electromagnetic like interaction. On the other hand the non-Abelian part of the original non-Abelian gauge field is responsible for the interactions among gluons themselves that are absent in Abelian theories.

The maximally Abelian gauge (MAG) [4] is a specific gauge choice of Abelian projection that maximizes the Abelian subgroup within the non-Abelian gauge group of QCD and the point in space time where the Abelian gauge field become singular emerges as monopole [2,4-6]. Hence in this simplified Abelian framework the dual superconductor picture [7] becomes more tractable.

One of the quantities of interest in lattice QCD simulations is the quark-antiquark potential. This potential reflects the energy between a quark and an antiquark as a function of their separation. Understanding the quark-antiquark potential is crucial for unraveling the mechanism of quark confinement. Calculations of interquark potential is well defined by a sum of coulomb and linear confinement term [1,8-9] as

$$V(r)=\frac{-A}{r}+\sigma r+C \quad (1)$$

with the string tension σ , the color coulomb coefficient 'A' and the irrelevant constant 'C'.

Lattice QCD simulations provide a way to determine the values of these parameters by fitting the simulated data to the expected form of the potential. In context of lattice QCD simulations string tension is defined as the slope of potential at large distance and describes the energy per unit length of the color flux tube form between $q\bar{q}$ pair when pulled apart.

In this paper we will compare the decomposition of static potential more explicitly by calculating average relative deviation as well as comment on the slope of Abelian and monopole part of the interaction potential.

2. Review of Literature :

2.1 Decomposition of Static potential into Abelian and Off diagonal part:

To simplify the analysis of quark confinement the concept of the Cartan decomposition of the maximally Abelianized lattice gauge field [10] $X(s,\mu)\in SU(3)$ is given as -

$$X^{maximally\ Abelian}(s,\mu)=O(s,\mu)A(s,\mu)\in SU(3) \quad (2)$$

where $O(s,\mu)\in SU(3)/U(1)^2$ represent the off-diagonal factor. The Abelian factor $A(s,\mu)\in U(1)^2$ behaves as electromagnetic gauge field in lattice gauge theory .

Wilson loop [1] used to calculate the interquark potential in lattice QCD. The $q\bar{q}$ potential from the Wilson loop is calculated as -

$$V(r)=-\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C[X(s,\mu)]\rangle \quad (3)$$

where C denotes the $R\times T$ rectangular loop and $\langle ...\rangle$ means the statistical average over the gauge configuration. The Abelian part of the interquark potential is given by-

$$V_{abelian}(r)=-\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C[A(s,\mu)]\rangle \quad (4)$$

The off diagonal factor gives the off diagonal potential

$$V_{offdiagonal}(r)=-\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C[O(s,\mu)]\rangle \quad (5)$$

From the simulation results [11] based on this decomposition it was observed that SU(3) non-Abelian potential and its Abelian part have almost same slope at large distances. Observations confirmed that Lattice QCD supports the concept of Abelian dominance [3,6,11-14]. From the analysis of static interquark potential $V(r) \approx V_{abelian}(r) + V_{offdiagonal}(r)$ also has been verified [11].

2.2. Decomposition of Static potential into Monopole and Modified part:

In another approach the non-Abelian lattice gauge field $X(s,\mu)$ decomposed into the Abelian part $A(s,\mu)$ and the coset part $C(s,\mu)$ [15,16] given as

$$X(s,\mu) = C(s,\mu) A(s,\mu) \quad (6)$$

The Abelian part can further decomposed into the monopole and photon part [17] given as

$$A(s,\mu) = A_{monopole}(s,\mu) A_{photon}(s,\mu) \quad (7)$$

The coset part and photon part together form modified gauge field and is defined as

$$\widetilde{X}(s,\mu) = C(s,\mu) A_{photon}(s,\mu) \quad (8)$$

$A_{photon}(s,\mu)$ is the Abelian projection of modified non-Abelian field $\widetilde{X}(s,\mu)$ and does not involve monopole.

The original non-Abelian potential from the non Abelian Wilson loop is calculated as given in equation (3).

Similarly, $$V_{monopole}(r) = -\lim_{T \to \infty} \frac{1}{T} \ln \langle W_C[A_{monopole}(s,\mu)] \rangle \quad (9)$$

also, $$V_{modified}(r) = -\lim_{T \to \infty} \frac{1}{T} \ln \langle W_C[\widetilde{X}(s,\mu)] \rangle \quad (10)$$

From the simulation it can be observed that monopole part of the decomposed potential $V_{monopole}(r)$ has small curve at small distances but shows almost linear behavior at large distances [15,16,18,19] . Potential analysis informed that the non-Abelian potential V(r) is well matched by the summation of monopole and modified parts of potential $V(r) \approx V_{monopole}(r) + V_{modified}(r)$ [19].

3. Methodology:

3.1 Calculation of deviation of Decomposed potentials using average relative deviation method :

To compare the sum of potential $V_{abelian}(r) + V_{offdiagonal}(r)$ and $V_{monopole}(r) + V_{modified}(r)$ to original SU(3) static potential V(r) more precisely, we calculated the average relative deviation percentage of the sum of decomposed potentials with respect to V(r).

The average relative deviation of the sum of potential $V_{abelian}(r) + V_{offdiagonal}(r)$ is given as

$$(\Delta d)_1=\frac{1}{N}\sum_{i=1,2...N}\frac{V_i(r)-[V_{abelian}(r)+V_{offdiagonal}(r)]_i}{V(r)_i} \tag{11}$$

where the total numbers of observations represented by ‘N’ .

Similarly , the average relative deviation for the sum of potential $V_{monopole}(r)+V_{modified}(r)$ is calculated as

$$(\Delta d)_2=\frac{1}{N}\sum_{i=1,2...N}\frac{V_i(r)-[V_{monopole}(r)+V_{modified}(r)]_i}{V(r)_i} \tag{12}$$

3.2 Calculation of string tension of interquark potential :

In the context of quantum chromodynamics the string tension is a parameter used to describe the behavior of the interquark potential at large distances .The string tension for the given graph is calculated as

$$\sigma=\frac{\Delta(r_0 V(r))}{\Delta(R/r_0)} \tag{13}$$

where $\Delta(r_0 V(r))$ is change in interquark potential and $\Delta(R/r_0)$ is change in distance between two observation points at large distances.

4. Results and Discussion:

From calculations it is noted that the average relative deviation percentage for the sum of Abelian and off diagonal potential $(\Delta d)_1$ is approximately 7.404 % . As from Figure 1 it can be observe that the green curve represents sum of the potentials $V_{abelian}(r)+V_{offdiagonal}(r)$ and the blue curve represent original SU(3) interquark potential. Lesser the deviation from original potential will give better decomposition result. Our calculation of average relative deviation informs that there is approximately 92.596 % similarity of $V_{abelian}(r)+V_{offdiagonal}(r)$ with original interquark potential V(r). Again in Figure ‘1’ the orange curve represent sum of the decomposed potentials $V_{monopole}(r)+V_{modified}(r)$ .The average relative deviation percentage for sum of monopole and modified potential $(\Delta d)_2$ is approximately 2.572 % .

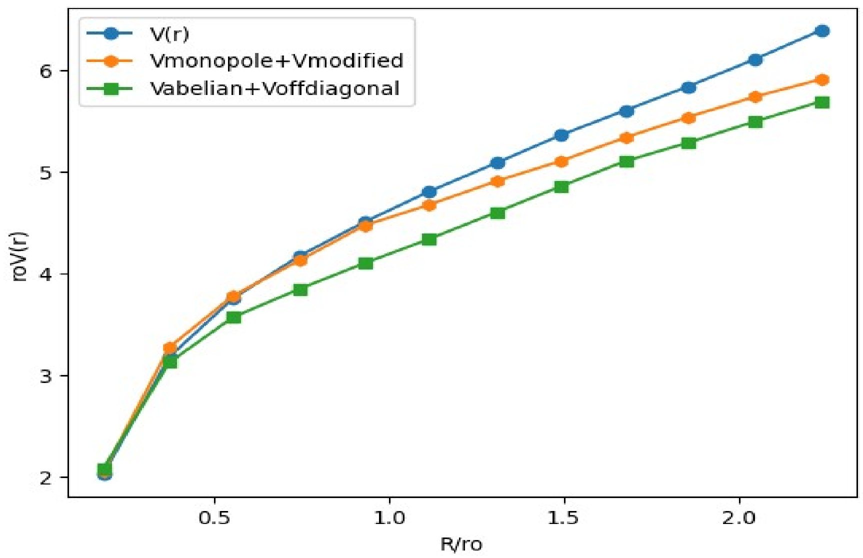

FIGURE 1 : The curve of V(r) , $V_{abelian}(r)+V_{offdiagonal}(r)$ *and* $V_{monopole}(r)+V_{modified}(r)$ *for lattice* 24^4 *at* $\beta = 6.0$

Hence our calculations indicates that the decomposed potential $V_{monopole}(r)+V_{modified}(r)$ resembles approximately 97.28% with non-Abelian potential V(r) for the given lattice volume.

The concept of confinement becomes particularly evident at large distances, where attempts to separate quarks results an increase in potential energy, forming a flux like structure between the quark and antiquark.

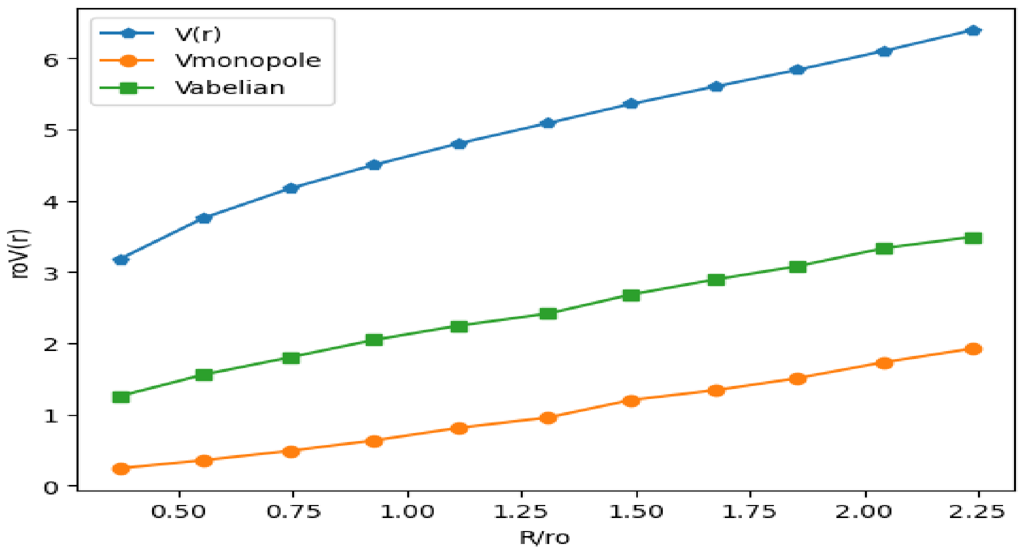

FIGURE 2: Slope of V(r), $V_{monopole}(r)$ *and* $V_{abelian}(r)$ *for* 24^4 *lattice at* $\beta = 6.0$.

String tension σ controls the strength of quark confinement and represented by the slope of potential at large distances. Since in our study monopole part and Abelian part of of decomposed potential are responsible for confinement behavior so to calculate corresponding string tension the curve of interquark potential V(r) , monopole part $V_{monopole}(r)$ and Abelian part $V_{abelian}(r)$ shown separately in Figure 2. More the similarity of string tension of decomposed potential with original SU(3) interquark potential implies more closer results of quark confinement studies. The observation of string tension for the given lattice implies that the ratios are $\frac{\sigma_{monopole}}{\sigma} \approx 0.786$ and $\frac{\sigma_{abelian}}{\sigma} \approx 0.808$.

5. Conclusion:

By calculating average relative deviation we compared two different approaches of interquark potential decomposition for specific lattice in quantitative manner . Our results suggested that $V_{monopole}(r)+V_{modified}(r)$ decomposition is not only closer to original SU(3) interquark potential in comparison to the decomposition $V_{abelian}(r)+V_{offdiagonal}(r)$ but gives approximately 4.832% better results. But on this behalf we can not say that $V_{monopole}(r)+V_{modified}(r)$ decomposition is always preferable then $V_{abelian}(r)+V_{offdiagonal}(r)$, because when we look for string tension of flux tube , i.e. deal at large distance region our results indicates that $\sigma_{abelian}$ is in 0.022% more good approximation than $\sigma_{monopole}$ and hence indicates that Abelian part of the decomposed potential behave approximately 0.022% more better than monopole part in nonperturbative studies.

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