ON APPLICATION OF PARTITIONS OF ODD NUMBERS AND THEIR ODD SUMSTO PROVE THENONEXISTENCE OF ODD PERFECT NUMBERS

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APPLICATION OF PARTITIONS OF ODD NUMBERS AND THEIR ODD SUMS TO PROVE THE NONEXISTENCE OF ODD PERFECT NUMBERS

Abstract

Comment [H2]: Abstracts generally contain the objectives, methods, and results of the research. However, in this abstract only the results of the research appear.

Perfect numbers, which are integers equal to the sum of their proper divisors, excluding themselves, have intrigued mathematicians for centuries. While it is established that even perfect numbers can be expressed as $2^{p-1}(2^p - 1)$, where p and $2^p - 1$ are prime numbers (Mersenne primes), the existence of odd perfect numbers remains an unsolved problem.This study utilizes an algorithm which demonstrates that a positive even integer can be partitioned into all pairs of odd numbers. Using this approach, it is proven that any positive odd number $2n + 1$ can be partitioned into all pairs of both odd and even numbers and from the set of these partitions, we show that there exist a proper subset containing all proper divisors of $2n + 1$. Using these results, and the facts that there exist infinitely many odd numbers and the odd sums of odd numbers is always odd, we prove the nonexistence of odd perfect numberscontributing to the conjecture that they do not exist.

Keywords: Perfect Numbers, Odd numbers, Even numbers, Natural Numbers, Partition

Original Research Article

1. INTRODUCTION

The study of perfect numbers traces its origins back to ancient Greek mathematicians, particularly Euclid and Pythagoras, who were among the first to explore the properties of these intriguing integers. Euclid, in his seminal work "Elements," provided some of the earliest known results on perfect numbers, including the proof that if 2^p-1 is prime, then $2^{p-1}(2^p-1)$ is a perfect number [1].This foundational work laid the groundwork for further exploration into the properties of perfect numbers and their relationships with prime numbers. Pythagoras, known for the Pythagorean theorem, also made contributions to the study of perfect numbers, viewing them as mystical and divine [2].

The study of perfect numbers continued to evolve over the centuries, with mathematicians such as Leonhard Euler making significant contributions to the field. Euler, in the 18th century, furthered the understanding of perfect numbers by developing various factorization methods, including the factorization of even perfect numbers, and creating generating functions for expressing even perfect numbers [3].The contrast between even and odd perfect numbers has captured the imagination of mathematicians and remains an unsolved problem in number theory. The existence of even perfect numbers is well understood; they are all of the form $2^{p-1}(2^p-1)$, where p and 2^p-1 are prime numbers known as Mersenne primes [4]. However, despite centuries of exploration, no odd perfect numbers have ever been discovered, leading to the intriguing question: do odd perfect numbers even exist?

While even perfect numbers have been extensively studied and are well understood, the existence of odd perfect numbers remains an open question. Euler's factorization pattern for odd perfect numbers provides a framework for understanding these elusive numbers [5], but despite extensive computational searches, no odd perfect numbers have ever been discovered. This has led to the conjecture that odd perfect numbers do not exist, a conjecture that has yet to be proven but has sparked significant interest and research in the field of number theory.

This manuscript adopts a multifaceted approach to explore the properties of perfect numbers and partitions of odd numbers. By utilizing an algorithm that demonstrates how a positive even integer can be partitioned into all pairs of odd numbers [6], the study extends this approach to show that any positive odd number can be partitioned into all pairs of both odd and even numbers. From this set of partitions, a proper subset containing all proper divisors of the odd number are identified.The contributions of this exploration extends beyond the specific investigation of perfect numbers and odd partitions. By providing further evidence for the non-existence of odd perfect numbers, the studycontributes to the broader understanding of number theory and its foundational concepts. The study also highlights the importance of computational methods and algorithms in exploring complex mathematical problems.

2. PRELIMINARIES

To establish the nonexistence of odd perfect numbers, we draw upon foundational concepts in number theory. The exposition of these concepts provides the necessary framework for the subsequent proof. Central to our discussion are key definitions and theorems that underpin the core arguments presented in this scholarly work.

Definition 1: Perfect Numbers are integers that are equal to the sum of their proper divisors, excluding themselves [7].

Definition 2: Even Perfect Numbers are Perfect numbers that can be expressed in the form $2^{p-1}(2^p-1)$, where p and 2^p-1 are prime numbers [8].

Definition 3: Odd Perfect Numbers are hypothetical perfect numbers that are not yet proven to exist [7].

Definition 4: A composite number is a positive integer greater than one that is not prime, meaning it has divisors other than 1 and itself [9].

Definition 5: (Union of two Sets)

The union of two sets *A* and *B*, denoted by $A \cup B$, is the set containing all elements that are in *A*, in *B*, or in both *A* and *B*. In other words, the union of two sets *A* and *B* is the set of all elements that belong to either *A* or *B* or both [10].

Theorem 1

Let p_1 and $p_2 \in P$, where P is the set of all primes, and d be the difference between p_1 and p_2 such that $d = p_2 - p_1 > 0$ where $p_2 > p_1$. Let $z_i \in 1 \leq O \leq \frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n)$ be the set of odd numbers for $i \in 1 \leq O \leq (\frac{1}{2}(p_1 + p_2) + (p_2 - p_1)^n)$, then any multiple of d in the half-open interval [1, $[\frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n]]$ can be used to partition $(p_1 + p_2) + (p_2 - p_1)^n$ into all pairs of odd numbers[11].

Theorem 2

The difference between an odd number and an even number is odd [12]

Proof

Let a be an odd number and b be an even number. By definition, an odd number can be expressed as $a = 2n + 1$ for some integer n, and an even number can be expressed as $b = 2m$ for some integer m.The difference between a and b is $a - b = (2n + 1) - 2m$.Simplifying, we get: $a - b = 2n + 1 - 2m$implying that $a - b = 2(n - m) + 1$. Since n and m are integers, $n-m$ is also an integer. Therefore, $2(n-m)$ is an even number. Adding 1 to an even number results in an odd number.

Theorem 3

The difference between any two different odd numbers is even[12].

Proof

Let A be an odd number, B another odd number $A = 2c + 1, B = 2d + 1$ (where both c and d are integers) $A - B = (2c + 1) - (2d + 1) = 2c - 2d + (1 - 1) = 2(c + d)$ Since c and d are integers, $c+d$ is an integer too, and A-B is even (since it can be expressed as 2*integer)

2.1 SUM OF ODD NUMBERS

The sum of the odd numbers from 1 to infinity can be determined using the concept of Arithmetic Progression. Odd numbers are those that are not divisible by 2, such as 1, 3, 5, 7, 9, 11, and so on. To find the sum of these numbers, let's consider the sum of the first n odd numbers as *Sn*:

$Sn = 1 + 3 + 5 + 7 + 9 + \cdots + (2n - 1)$(1)

According to the formula for the sum of an Arithmetic Progression, which states that the sum of $n$terms is given by:

$s_n = \frac{n}{2} \times [2a + (n - 1)d]$

(2)

where n is the number of terms in the series, aa is the first term of the series, and d is the common difference between the terms. Applying this formula to equation (1), with a=1 and d=2, and considering the last term to be l=(2n−1), we get: $s_n = \frac{n}{2}\left(1 + (2n - 1)\right) = \frac{n}{2} \times (2n) = n^2$ [13].

3. GENERATING PARTITIONS OF ODD NUMBERS

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Odd numbers, being integers that are not divisible by 2, have unique properties that distinguish them from even numbers. One interesting aspect of odd numbers is that they can be partitioned in various ways. A partition of a number is a way of expressing it as a sum of other numbers, where the order of the summands does not matter. For example, the number 5 can be partitioned as 5, 3+2, 2+2+1, and 1+1+1+1+1. This property of odd numbers allows for a diverse range of combinations, making them a rich area of study in number theory.

In 2023 Sankei et al., presented a research showing that using the new formulation of a set of even numbers as $(P_1 + P_2) + (P_2 - P_1)^n$,[14]it is always possible to partition any even number into all pairs of odd numbers using the following algorithm[6]:

Let P be the set of all prime numbers, $\mathbb{N}$ be the set of all natural numbers and O the set of all odd numbers.

Step 1 : Let P_1 and $P_2 \in P$,then $(P_1 + P_2) + (P_2 - P_1)^n$ is even, $\forall n \in \mathbb{N}$, and $p_2 > p_1$.

Step 2: Let $d$be even and belong to the half-open interval [1, $[\frac{1}{2}(\,(p_1 + p_2) + (p_2 - p_1)^n]]$.

Step 3: Let z_i *and* $y_i \in 1 \le O \le \frac{1}{2}((P_1+P_2)+(P_2-P_1)^n)$ for $i \in O$ and belong to the half-open interval [1, $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n]]$.

With p_1, p_2, d and z_i, we partition $(p_1+p_2)+(p_2-p_1)^n$ as follows:

Partition 1: $((P_1+P_2)+(P_2-P_1)^n)$ - $(d+z_1)=y_1$

Partition 2: $((p_1+p_2)+(p_2-p_1)^n)$- $(d+z_3)=y_3$

Partition 3: $((p_1+p_2)+(p_2-p_1)^n)$- $(d+z_5)=y_5$

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Partition i: $((p_1+p_2)+(p_2-p_1)^n)$- $(d+z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)})=y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}$

The set of pairs $(d+z_1,y_1),(d+z_3,y_3),(d+z_5,y_5),\ldots,$ $(d+z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)},y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)})$ of odd numbers are all partitions of the even number $(p_1+p_2)+(p_2-p_1)^n$. Since prime numbers greater than 2 are subsets of odd numbers, from these set of pairs of odd numbers, the possibility is that, there exist at least one pair of primes[6].

In this research, we utilize this algorithm to show its application in partitioning any odd number of the form $2n+1$ in pairs of both even and odd number. The algorithm starts by assuming an even integer value and from it sets of even and odd numbers are generated in the range [1, $[\frac{1}{2}($

$(p_1+p_2)+(p_2-p_1)^n$]][14], these two sets are used in partitioning the even number $(p_1+p_2)+(p_2-p_1)^n$. The same approach is used to partition the odd number $2n+1$ by letting the generator $d=\left(\frac{1}{2}\times((2n+1)-1)\right)$ that so that d is even and in the interval $[1,\left(\frac{1}{2}\times(2n+1)\right)]$, if $d=\left(\frac{1}{2}\times((2n+1)-1)\right)$ is not even , for instance when $2n+1=7$ then $d=\left(\frac{1}{2}\times((7)-1)\right)=3$, which is not even, we then decrement it by 1 to make it even so that $d=\left(\frac{1}{2}\times((2n+1)-1)\right)-1$ can be used to partition the odd number $2n+1$ as follows:

Step 1: Let $2n+1$ be any odd number

Step 2: Let $d=\left(\frac{1}{2}\times((2n+1)-1)\right)$ or $d=\left(\frac{1}{2}\times((2n+1)-1)\right)-1$

Remark 1

The value of d will be determined by the first parity after division which can either be odd or even. If it is even after division, then $d=\left(\frac{1}{2}\times((2n+1)-1)\right)$ and if odd then $d=\left(\frac{1}{2}\times((2n+1)-1)\right)-1$.

Step 3: Let $z_i$be the set of all odd numbers $and\ y_i$ be the set of all even numbers belonging to the half-open interval $[1,\left(\frac{1}{2}\times((2n+1))\right)]$ of natural odd numbers, for $i\in O$, where O is the set of all odd numbers,

With$d$ andz_i, we partition $2n+1$as follows:

Partition 1: $(2n+1)-(d+z_1)=y_1$

Partition 2: $(2n+1)-(d+z_3)=y_3$

Partition 3: $(2n+1)-(d+z_5)=y_5$

Partition 4: $(2n+1)-(d+z_7)=y_7$

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Partition i: $(2n+1)-\left(d+z_{(\frac{1}{2}((2n+1)-1))-1}\right)=y_{(\frac{1}{2}((2n+1)-1))-1}$

The set of all partitions of $2n+1$: $(d+z_1,y_1),(d+z_3,y_3),(d+z_5,y_5),(d+z_7,y_7),\ldots,(d+z_{\left(\frac{1}{2}((2n+1)-1)\right)-1},y_{\left(\frac{1}{2}((2n+1)-1)\right)-1})$ are pairs of both even and odd numbers since according to theorem 3 , the difference between an odd number and an even number is always an odd number. In order to illustrate how the algorithm partitions any odd number 2n+1 into all pairs of even and odd numbers , we use example 1 for the value of $d=\left(\frac{1}{2}\times((2n+1)-1)\right)$, and example 2 for $d=\left(\frac{1}{2}\times((2n+1)-1)\right)-1$.

Example 1

Let $2n+1=$ 101 and $d=\left(\frac{1}{2}\times((2n+1)-1)\right)=\frac{1}{2}(101-1)=50$, which is even. We then partition 101 as follows:

Step 3: we generate the set of all odd numbers, belong to the half-closed-open interval [1, $\left(\frac{1}{2}\times\left((2n+1)\right)\right)$] of natural odd numbers. This set equals {1,3,5,7,9,11,13,15,17,19,21,23

$, 25,27,29,31,33,35,37,39,41,43,45,47,49\}$.

With $d = 50$ and the set of all odd numbers $\{1,3,5,7,9,11,13,15,17,19,21,23$

$, 25,27,29,31,33,35,37,39,41,43,45,47,49\}$, we partition 101 as follows:

Partition 1: $(101) - (50 + 1) = 50$

Partition 2: $(101) - (50 + 3) = 48$

Partition 3: $(101) - (50 + 5) = 46$

Partition 4: $(101) - (50 + 7) = 44$

Partition 5: $(101) - (50 + 9) = 42$

Partition 6: $(101) - (50 + 11) = 40$

Partition 7: $(101) - (50 + 13) = 38$

Partition 8: $(101) - (50 + 15) = 36$

Partition 9: $(101) - (50 + 17) = 34$

Partition 10: $(101) - (50 + 19) = 32$

Partition 11: $(101) - (50 + 21) = 30$

Partition 12: $(101) - (50 + 23) = 28$

Partition 13: $(101) - (50 + 25) = 26$

Partition 14: $(101) - (50 + 27) = 24$

Partition 15: $(101) - (50 + 29) = 22$

Partition 16: $(101) - (50 + 31) = 20$

Partition 17: $(101) - (50 + 33) = 18$

Partition 18: $(101) - (50 + 35) = 16$

Partition 19: $(101) - (50 + 37) = 14$

Partition 20: $(101) - (50 + 39) = 12$

Partition 21: $(101) - (50 + 41) = 10$

Partition 22: $(101) - (50 + 43) = 8$

Partition 23: $(101) - (50 + 45) = 6$

Partition 24: $(101) - (50 + 47) = 4$

Partition 25: $(101) - (50 + 49) = 2$

The partitions of 101 are $\{(51,50), (53,48), (55,46), (57,44), (59,42), (61,40), (63,38)(65,36),$

(67,34), (69,32), (71,30), (73,28), (75,26), (77,24), (79,22), (81,20), (83,18), (85,16), (87,14),

(89,12), (91,10), (93,8), (95,6), (97,4), (99,2)} and are all pairs of both even and odd numbers. From this pairs of odd numbers, we generate two proper subsets of even numbers and odd numbers as {2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42,44,46,48,50}and

{51,53,55,57,59,61,63,65,67,69,71,73,75,77, 79,81,83,85,87,89,91,93,95,97,99}.

An odd number cannot be divided evenly by 2, meaning that any division of an odd number by a multiple of 2 will always result in a non-zero remainder. This definition implies that an odd perfect number, if it exists, cannot have any even numbers as proper divisors. In other words, every divisor of an odd perfect number must be an odd number. This restriction is a fundamental property of odd numbers and has significant implications for the search for odd perfect numbers. The inability of an odd number to be divided evenly by 2 is a defining characteristic that sets it apart from even numbers and forms the basis for its classification in number theory.

From the two proper subsets {2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42,44

, 46,48,50}and{51,53,55,57,59,61,63,65,67,69,71,73,75,77, 79,81,83,85,87,89,91,93,95,

97,99}obtained by partitioning the odd number 101, it is evident that the proper subset of even numbers, being distinct from the set of odd numbers, cannot contain any proper divisors of the odd number 101, since upon division there is always a nonzero remainder. Therefore, the potential proper divisors of the odd number 101 must lie solely within the generated proper subset of odd numbers. This observation highlights a fundamental property of partitions of integers and their implications for identifying potential divisors of odd numbers.

In order to obtain the set of all odd numbers less than 101 to help in identifying all potential proper divisors of the odd number, we take the union of the generated set of odd numbers and the set of all odd numbers in the interval $\left[1, \left(\frac{1}{2}(2n+1)-1\right)\right]$. The set of all odd numbers generated then becomes $\{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43,45,47,49\} \cup$ $\{51,53,$

$55,57,59,61,63,65,67,69,71,73,75,77, 79,81,83,85,87,89,91,93,95,97,99\}$ ={1,3,5,7,9,11,13,15,17,19,21,23 ,25,27,29,31,33,35,37,39,41,43,45,47,49, 51,53, 55,57,59,61,63,65,67,69,71,73,75 ,77,79,81,83,85,87,89,91,93,95,97,99}, the set of all odd numbers less than 101.

Since the odd number 101 is also prime, it follows that from the list of all odd numbers less than 101 we only obtain one proper subset {1} containing only one proper divisor of 101.

Example 2

Let $2n + 1 =\ 99$ and $d = \left(\frac{1}{2} \times \left((2n+1)-1\right)\right) - 1 = \left(\frac{1}{2}(99-1)\right) - 1 = 49 - 1 = 48.$

Step 3: we generate the set of all odd numbers, belong to the half-closed-open interval [1, $\left(\frac{1}{2} \times \left((2n+1)-1\right)\right)$] of natural odd numbers. This set equals {1,3,5,7,9,11,13,15,17,19,21,23 , 25,27,29,31,33,35,37,39,41,43,45,47}.

With $d = 48$ and the set of all odd numbers {1,3,5,7,9,11,13,15,17,19,21,23 , 25,27,29,31,33,35,37,39,41,43,45,47, }, we partition 99 as follows:

Partition 1: $(99) - (48 + 1) = 50$

Partition 2: $(99) - (48 + 3) = 48$

Partition 11: $(99) - (48 + 21) = 30$

Partition 12: $(99) - (48 + 23) = 28$

Partition 3: $(99) - (48 + 5) = 46$

Partition 4: $(99) - (48 + 7) = 44$

Partition 5: $(99) - (48 + 9) = 42$

Partition 6: $(99) - (48 + 11) = 40$

Partition 7: $(99) - (48 + 13) = 38$

Partition 8: $(99) - (48 + 15) = 36$

Partition 9: $(99) - (48 + 17) = 34$

Partition 10: $(99) - (48 + 19) = 32$

Partition 13: $(99) - (48 + 25) = 26$

Partition 14: $(99) - (48 + 27) = 24$

Partition 15: $(99) - (48 + 29) = 22$

Partition 16: $(99) - (48 + 31) = 20$

Partition 17: $(99) - (48 + 33) = 18$

Partition 18: $(99) - (48 + 35) = 16$

Partition 19: $(99) - (48 + 37) = 14$

Partition 20: $(99) - (48 + 39) = 12$

Partition 21: $(99) - (48 + 41) = 10$

Partition 24: $(99) - (48 + 47) = 4$

Partition 22: $(99) - (48 + 43) = 8$Partition 23: $(99) - (48 + 45) = 6$

We then obtain the union of the two proper subsets of odd numbers as in example 1 as $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59,$ $61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97\}$. From the set of all odd numbers less 99 we generate the proper subset of all proper divisors of 99. The proper divisors of 99 ,excluding 99 itself are$\{1, 3, 9, 11, 33\}$.

Perfect numbers are integers that equal the sum of their proper divisors, excluding the numbers themselves. Considering the odd number 99, if it were an odd perfect number, the sum of its proper divisors, namely 1, 3, 9, 11, and 33, should equal 99. However, the actual sum of these divisors is 57, leading to a contradiction. Thus, it is evident that 99 cannot be an odd perfect number, as 1 + 3 + 9 + 11 + 33 does not equal 99.

This result reinforces the absence of odd perfect numbers in the domain of positive integers and what is left is to show that this pattern will follow for any odd number $2n + 1$. It is therefore important to show that any odd number $2n + 1$ can be partitioned into all pairs of odd numbers.

3.1 UTILIZATION OF ONLINE DIVISORS CALCULATOR FOR LARGER ODD NUMBERS

The exploration of divisors plays a fundamental role in understanding the properties of integers. Proper divisors, which are all the positive divisors of a number excluding the number itself, are particularly important in various mathematical investigations. However, as numbers grow larger, calculating their divisors becomes increasingly complex and laborious, especially for odd numbers with potentially numerous divisors. In such cases, the utilization of online divisors calculators can significantly ease the computational burden and expedite the process of identifying and analyzing the divisors of large odd numbers. These tools leverage computational algorithms to efficiently generate and list all the proper divisors of a given odd number, aiding mathematicians and researchers in their exploration of number properties and relationships. This paper explores the practical application of online divisors calculators for larger odd numbers, demonstrating their utility in modern mathematical investigations and problem-solving.

Example 3

Let $2n + 1 = 89993933723$, then using the online divisors calculator [15], we obtain the following 7 proper divisors of 89993933723:

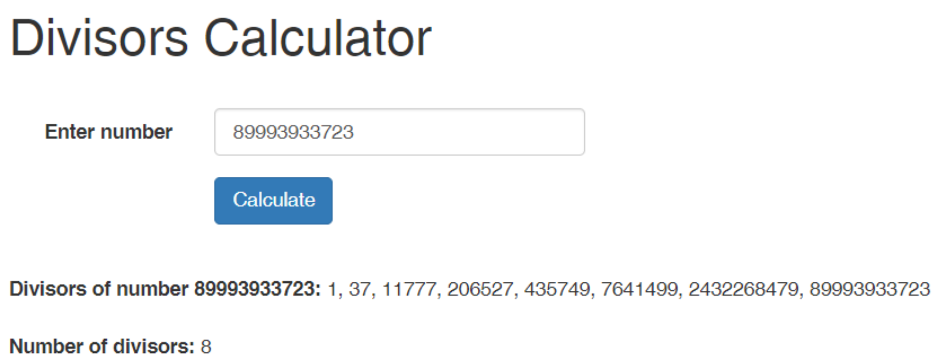

Figure 1: Divisors of 89993933723

Remark 2

Notice that the calculator identifies the number 89993933723 as its own divisor and therefore the set of its proper divisors are { 1, 37, 11777, 206527, 435749, 7641499, 2432268479}. Therefore the sum of the proper divisors of 89993933723 is 2440564069 which is not equal the number itself and therefore , 89993933723is not a perfect number.

Example 4

If weLet $2n + 1 = 345939477235$, then using the online divisors calculator [15], we obtain the following 15 proper divisors of 345939477235:

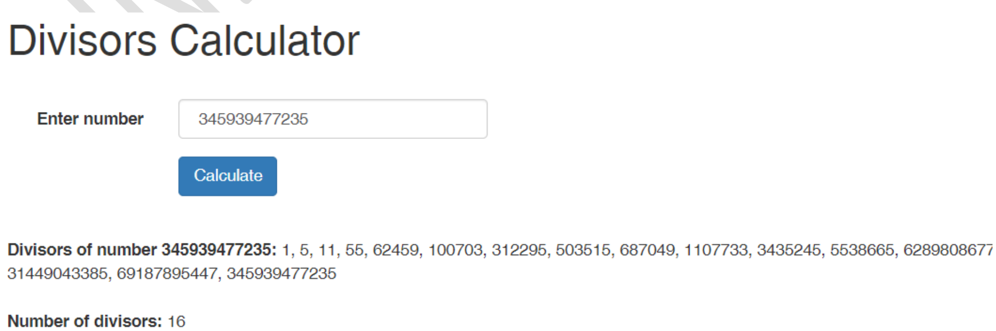

Figure 2: Divisors of 345939477235

Remark 3

The set of its proper divisors are {1, 5, 11, 55, 62459, 100703, 312295, 503515, 687049, 1107733, 3435245, 5538665, 6289808677, 31449043385, 69187895447}. Therefore the sum of the proper divisors is 106938495245 which is not equal the number itself and therefore , 345939477235

is not a perfect number.

4. PROOF OF NONEXISTENCE OF ODD PERFECT NUMBERS

Theorem 4

Odd perfect numbers do not exist

Proof

Let $2n + 1$ be any odd number, based on the algorithm used to partition 99 and 101, the odd number $2n + 1$ can be partitioned into all pairs of both even and odd numbers that makes it possible to generate a set containing all the odd numbers $\{1, ((2n + 1) - 1)\}$ less than $2n + 1$. Additionally, in section 2.1 on odd sums of odd numbers, it has been shown that the odd sums of odd numbers say$n_1 + n_3 + n_5 + \cdots + n_i = N, \forall i \in O$, where O is the set of all odd numbers, is always odd. However, if all the values of n forming $N$are all distinct prime numbers and N is also a prime, it then follows that N has only two divisors (1 and N), and for this case, N cannot be a perfect number.If all the values of n forming $N$are all distinct prime numbers or composite odd numbers and N is composite, then there exist at least one proper divisor of Nsay q such that $q\ (mod\ N) = 0$, the results in example1, example 2, example 3 and example 4, indicate that if N is composite, at least one of the ultimate odd numbers used in forming N, is not a proper divisor of N and this further shows that in this case N cannot be a perfect number. The proper divisors of even perfect numbers exhibit interesting patternsdefining characteristic of perfect numbers. The proper divisors are typically arranged in pairs that multiply to the perfect number, reflecting the underlying mathematical relationships. For example, in the case of 28, the proper divisors are 1, 2, 4, 7, and 14, and they are arranged in pairs (1, 28), (2, 14), and (4, 7), where each pair multiplies to 28. This pattern holds true for the proper divisors of all even perfect numbers. Additionally, the proper divisors of even perfect numbers exhibit another interesting relationship in terms of their multiples. Taking the example of 28, we can observe that 1 is a multiple of 2, 2 is a multiple of 4, and 7 is a multiple of 14. Interestingly, the multiples of each pair of divisors add up to the perfect number itself. Specifically, 1 is a multiple of 1, 2 is a multiple of 2, 4 is a multiple of 4, 7 is a multiple of 7, and 14 is a multiple of 14, and the sum of these multiples is 28. This pattern holds for all even perfect numbers, where the proper divisors form pairs whose multiples sum up to the perfect number, illustrating a unique mathematical property of these numbers. This clearly shows that if N is a perfect number whose proper divisors are say $p, q\ and\ m$ then $(N \div p) \in (p, q, m)$, $(N \div q) \in (p, q, m)$ and $(N \div m) \in (p, q, m)$. Taking the example of 28 whose proper divisors forms the set (1,2,4,7,14), we observe that $(28 \div 2 = 14) \in (1,2,4,7,14)$, $(28 \div 4 = 7) \in (1,2,4,7,14)$, $(28 \div 7 = 4) \in (1,2,4,7,14)$ and $28 \div 14 = 2) \in (1,2,4,7,14)$. Notice that $(28 \div 1 = 28)$, but the formation of the perfect numbers excludes the number itself. This property holds true for all even perfect numbers. Clearly if N is a perfect number whose proper divisors form the set $(n_1, n_3, n_5, \dots, n_i)$, then $n_i \big| N \in (n_1, n_3, n_5, \dots, n_i)$, $,\forall i \in O$,where O is the set of all odd numbers. Let M be an odd perfect number with proper divisors forming the set (n_1, n_3, n_5) where each proper divisor is a multiple of three of the next one, then each of the proper divisors divides N. To demonstrate this, let us consider the fact that if $n_1 = 2n + 1$, then $n_3, = 3(2n + 1) = 6n + 3$ and $n_5 = 3(6n + 3) = 18n + 9$.since M is a perfect number then $M = 2n + 1 + 6n + 3 + 18n + 9 = 26n + 13$, it then follows that each of the proper divisors should divide $M = 26n + 13 = 13(2n + 1)$. However, $2n + 1|\ 13(2n + 1)$ equals $13 \notin (n_1, n_3, n_5)$, $6n + 3 = 3(2n + 1) \nmid 13(2n + 1)$ since we obtain $\frac{13}{3} \notin (n_1, n_3, n_5)$, and $18n + 9 = 9(2n + 1) \nmid 13(2n + 1)$ since we obtain $\frac{13}{9} \notin (n_1, n_3, n_5)$, this shows that none of the proper divisors divides M which is a contradiction and therefore M is not an odd perfect number. If we consider the even perfect number 28 with proper divisors {1,2,4,7,14} , the proper divisors can be classified into two sets of multiples of 2 and 7 as {1,2,4} and {7,14} so that we can consider the odd numbers 1 and 7 as the seeds of the two sets respectively being multiplied by 2 to obtain the next proper divisor in the set. This pattern holds true for all even perfect number. Suppose Q is an odd perfect number formed by adding 5 multiple of three odd numbers with a seed 5 to 4 multiples of three odd numbers with a seed 7 , then $Q = 5 + 15 + 45 + 135 + 405 + 7 + 21 + 63 + 189 = 885$.Clearly $135 \nmid 885$ leading to a contradiction since 135 is a proper divisor and hence Q is not an odd perfect number. Based on this results we can conclude that if N is a potential odd perfect number with proper divisors such that $N = n_1 + n_3 + n_5 + \cdots + n_i$, then the possibility exit that at least one of the $n_i \nmid$ N leading to a contradiction and hence N is not a perfect odd number. ▪

CONCLUSION

The examination of partitions of odd numbers and their relationship to proper divisors has revealed distinct characteristics of odd perfect numbers. The results have illustrated that the properties and patterns observed in even perfect numbers, such as the formation of pairs of divisors whose multiples sum up to the perfect number, do not hold for odd perfect numbers. These findings culminate in a logical proof of the non-existence of odd perfect numbers, based on the inconsistencies in the divisibility and summation patterns of potential candidates. Overall, this manuscript contributes valuable insights into the elusive nature of odd perfect numbers and provides a foundation for further research in number theory.

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