Approximate Solutions For Non-Linear Evolution Stochastic Equations With Variations Of Drift Parameters

Abstract

This paper considered system of stochasticdifferential equations with emphasis on variations of drift parameters as it affects financial markets.These problems were solved analytically by adopting the Ito's methodof solution and three different investment solutions were obtained accordingly. The necessary conditions were achieved which govern various drift parameters in assessing financial markets.Therefore, the impressions on each solution of investors in financial markets were analyzed graphically. Finally, the influences of the relevant parameters of stochastic variables were effectively discussed all in this paper.

Keywords: Stock Prices, Financial markets, Stochastic analysis , investors and volatility

1.1 Introduction

In general sense, investments are ventures linked with risk which cannot be written off.
The human lives and day-to-day activities, are associated with risk; thus, risk is a determinant to effectively manage investment portfolios, because it is instrumental to the ascertainment of fluctuation or variations of returns on the stock and portfolio, which furnishes the investor a mathematical framework for investment decisions [1] Bonds, stocks, property, etc, are all prototypes of the risk associated to securities.

Nevertheless, because of the risk involved in the management of investment portfolios, insurance companies deemed it pertinent for lives, properties, etc, to be insured. In point of fact, insurance companies share third party in the management and control of their financial results. Risk transfer or risk sharing is the methodology employed by insurance firm on financial outcomes of its coverage duty in a number of ways with risk transfer agreement, risk among numerous insurance firms globally. therefore, in a situation of astronomical loses from financial situation as insurance company will not encounter risk, particularly, reinsurance means the division and distribution of risk. In general, risk is an established factor as long as humans are concerned, since we secure risky or riskless assets properly. A finer way to model these factors is as the trajectory or path of a diffusion process defined on many basic or fundamental probability space, possessing the geometric Brownian motion, used as the standard reference model [2] Modelling financial concepts cannot be exaggerated because of its numerous applications in science and technology.For instance, [3] analyzed the maximization of the exponential utility and the minimization
of the ruin probability and the results obtained displayed the same or kind of investments scheme or approach for zero interest rate [4] studied an optimal reinsurance and investment problem for insurer with jump diffusion risk process. [5] examined the risk reserved for an insurer and a reinsurer to follow Brownian motion with drift and applied optimal probability of survival problem under proportional reinsurance and power utility preference. Similarly, [7] studied the excess loss of reinsurance and investment in a financial market and obtained optimal strategies. [8] employed a problem of optimal reinsurance investment for an insurer having jump diffusion

risk model when the asset price was control by a CEV model. [9] studied strategies of optimal reinsurance and investment for exponential utility maximization under different capital markets. [10] considered investment problem having multiple risky assets. [1] examined an optimal portfolio selection model for risky assets established on asymptotic power law behaviour where security prices follow a Weibull distribution. The research of [11] analyzed the stability of stochastic model of price fluctuation on the floor of the stock market, where exact steps were derived, which aided the determination of the equilibrium price and growth rate of stock shares. [12] examined the unstable property of stock market forces, making use of proposed differential equation model. [13] studied a stochastic analysis of stock prices and their characteristics and obtained results which showed efficiency in the use of the proposed model for the prediction of stock prices. Similarly,[14] considered the stochastic formulations of some selected stocks in the Nigerian Stock Exchange (NSE), and the drift and volatility measures or quantities for the stochastic differential equations were obtained and the Euler-Maruyama technique for system of SDEs was applied in the stimulation of the stock prices. [8] produced the geometric Brownian motion and assessment of the correctness or exactness of the model, using detailed analysis of stimulated data. Furthermore, [15] looked stochastic problem of unstable stock market prices obtained conditions for determining the equilibrium price, required and adequate conditions for dynamic stability and convergence to equilibrium of the growth rate of the valued function of stocks. Nevertheless, [16] examine a stochastic problem of unstable prices at the floor of the stock market. From their evaluation, the equilibrium price and the market growth rate of shares were found out. Therefore, so many scholars have written extensively on stock market prices such as [11-24], etc.

Previous studies have therefore investigated similar problems but did not consider the variations of drift parameters as it affects financial markets. In particular, some studies, for instance [19],[20] and [24], etc.To the best of our understanding, this is the first study that has assessed disparities of drift parameters and its influences in financial markets. Therefore, this paper compliments that of [22] as it widens the scope of applicability in this dynamic area of mathematical finance.

The organization of this paper is set as follows: Section 2.1 presents the mathematical formulations, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2.1 Mathematical Formulation

A Stochastic Differential Equation is a differential equation with stochastic term. Therefore, assume that $(\Omega, \mathrm{F}, \wp)$ is a probability space with filteration $\{f_t\}_t \geq 0$ and

$W(t) = \left(W_1(t), W_2(t), ..., W_m(t)\right)^T$, $t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g *as* follows ;

$$dX(t) = f\left(t, X(t)\right)dt + g\left(t, X(t)\right)dZ(t),\ 0 \leq t \leq T,$$

$$X(0) = x_0,$$

Where, $T > 0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f : [0,\mathrm{T}] \times \mathbb{R}^n$ *and* $g : [0,T] \times \mathbb{R}^n \rightarrow \times \mathbb{R}^{n \times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f\left(S, X(S)\right)dS + \int_0^t g\left(S, X(S)\right)dZ(S)$$

Where, dX, dZ are terms known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$.

Theorem 1.1: let $T > 0,$ be a given final time and assume that the coefficient functions $f : [0,\mathrm{T}] \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ *and* $g : [0,T] \times \mathbb{R}^n \rightarrow \times \mathbb{R}^{n \times n}$ are continuous. Moreover, $\exists$ finite constant numbers λ *and* β such that $\forall t \in [0,\mathrm{T}]$ and for all $x, y \in \mathbb{R}^n$, the drift and diffusion term satisfy

$$\| f\left(t,x\right) - f\left(t,y\right) \| + \| \mathrm{g}\left(t,x\right) - g\left(t,y\right) \| \leq \lambda \| x - y \| \ ,$$

$$\| f\left(t,x\right) \| + \| \mathrm{g}\left(t,x\right) \| - g\left(t,\mathrm{x}\right) \| \leq \beta\left(1 + \| x \|\right).$$

Suppose also that x_0 is any $\mathbb{R}^n$ -valued random variable such that $E\left(\| x_0 \|^2\right) < \infty.$ then the above SDE has a unique solution X in the interval $[0,T].$ Moreover, it satisfies $E\left(\sup_{0 \leq t \leq T} \| X(t) \|^2\right) < \infty$. the proof of the theorem 1.1 is seen in [23].

Theorem 1.2:(Ito's lemma). Let $f\left(S,t\right)$ be a twice continuous differential function on $[0,\infty) \times \mathrm{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0 \ ,$$

Applying Taylor series expansion of F, gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} \left(dS_t\right)^2 + \text{higer order terms } \left(h.\mathrm{o}, \mathrm{t}\right) \ ,$$

So, ignoring h.o.t and substituting for dS_t ,we obtain;

$$dF_t = \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}\left(a_t dt + bdz(t)\right)^2$$

$$= \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2 dt,$$

$$= \left(\frac{\partial F}{\partial S_t}a_t + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2\right)dt + \frac{\partial F}{\partial S_t}b_t dz(t)$$

More so, given the variable $S(\text{t})$ denotes stock price, then following GBM implies (5) and hence, the function $F\left(S,t\right)$,Ito's lemma gives:

$$dF = \left(\mu S\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 F}{\partial S^2}\right)dt + \sigma S\frac{\partial F}{\partial S}dz(t)$$

Nevertheless,the stochastic analysis on the variations stock drift and it influences in financial markets is considered. The volatility dynamics and other drift coefficients of stock prices was taken to be constant throughout the trading days. The initial stock price which is assumed to follow different trend series was categorized the entire origin of stock dynamics is found in a complete probability space $\left(\Omega, \mathrm{F}, \wp\right)$ with a finite time investment horizon $T > 0$. Therefore, we have the following system of stochastic differential equations below;

$$dX_t = -\beta\mu X_t dt + \sigma X_t dW_t^1 \tag{0.1}$$

$$dX_\phi = K\tanh X_\phi dt + \sigma X_\phi dW_t^2 \tag{0.2}$$

$$dX_\varpi = \left(-\beta\alpha + K\tanh\right)X_\varpi dt + \sigma X_\varpi dW_t^3 \tag{0.3}$$

2.2 Method of Solution

The propose model (1.1) - (1.3) consist of a system of variable coefficient system of stochastic differential equations whose solutions are not trivial. we solve equations independently as followsusing Ito's theorem 1.2:

From(1.1) let $f\left(X_t, t\right) = \ln X_t$

Taking the partial derivative yields

$$\frac{\partial f}{\partial X_t} = \frac{1}{X_t}, \quad \frac{\partial^2 f}{\partial X_t^2} = \frac{-1}{X_t^2}, \quad \frac{\partial f}{\partial t} = 0 \tag{0.4}$$

According to Ito's ,gives:

$$df\left(X_t, t\right) = \sigma X_t \frac{\partial f}{\partial X_t} dW_t^1 + \left(-\beta\mu X_t \frac{\partial f}{\partial X_t} + \frac{1}{2}\sigma^2 X_t^2 \frac{\partial^2 f}{\partial X_t^2} + \frac{\partial f}{\partial t} \right) dt \tag{0.5}$$

Substituting (1.4) into (1.5),gives;

$$df\left(X_t, t\right) = \sigma X_t \frac{1}{X_t} dW_t^1 + \left(-\beta\mu X_t \frac{1}{X_t} + \frac{1}{2}\sigma^2 X_t^2 \left(-\frac{1}{X_t^2} \right) + 0 \right) dt \tag{0.6}$$

$$= \sigma \frac{X_t}{X_t} dW_t^1 + \left(-\beta\mu X_t \frac{X_t}{X_t} - \frac{1}{2X_t^2}\sigma^2 X_t^2 \right) dt = \sigma dW_t^1 + \left(-\beta\mu - \frac{1}{2}\sigma^2 \right) dt = \left(-\beta\mu - \frac{1}{2}\sigma^2 \right) dt + \sigma dW_t^1$$

Integrating the above expression;

$$\int_0^t d\ln X_t = \int_0^t df\left(X_t u, u\right) = \int_0^t \left(-\beta\mu - \frac{1}{2}\sigma^2 \right) du + \int_0^t \sigma dW_t^1 \tag{0.7}$$

$$\ln X_t - \ln X_0 = \left[-\beta\mu u - \frac{1}{2}\sigma_u^2 \right]_0^t + \left[\sigma W_u^1 \right]_0^t = \ln\left[\frac{X_t}{X_0} \right] = \left[-\beta\mu - \frac{1}{2}\sigma^2 \right] t + \sigma W_t^1$$

Taking ln of the both sides, gives;

$$X_t = X_0 e \left(-\beta\mu - \frac{1}{2}\sigma^2 \right) t + \sigma W_t^1 \tag{0.8}$$

Where, W_t^1 is a Brownian Motion

From(1.2),let $f\left(X_\phi, t\right) = \ln X_\phi$

Taking the partial derivative, yields;

$$\frac{\partial f}{\partial X_{\phi}} = \frac{1}{X_{\phi}},\ \ \frac{\partial^2 f}{\partial X_{\phi}^2} = \frac{-1}{X_{\phi}^2},\ \ \frac{\partial f}{\partial t} = 0 \tag{1.9}$$

According to Ito's,gives:

$$df\left(X_{\phi},t\right) = \sigma X_{\phi}\frac{\partial f}{\partial X_{\phi}}dW_t^2 + \left(K\tanh X_{\phi}\frac{\partial f}{\partial X_{\phi}} + \frac{1}{2}\sigma^2 X_{\phi}^2\frac{\partial^2 f}{\partial X_t^2} + \frac{\partial f}{\partial t}\right)dt \tag{1.10}$$

Substituting(1.9) into (1.10), gives

$$df\left(X_{\phi},t\right) = \sigma X_{\phi}\frac{1}{X_{\phi}}dW_t^2 + \left(K\tanh X_{\phi}\frac{1}{X_{\phi}} + \frac{1}{2}\sigma^2 X_{\phi}^2\left(-\frac{1}{X_{\phi}^2}\right) + 0\right)dt \tag{1.11}$$

$$= \sigma\frac{X_{\phi}}{X_{\phi}}dW_t^2 + \left(K\tanh X_{\phi}\frac{X_{\phi}}{X_{\phi}} - \frac{1}{2X_{\phi}^2}\sigma^2 X_{\phi}^2\right)dt = \sigma dW_t^2 + \left(K\tanh - \frac{1}{2}\sigma^2\right)dt = \left(K\tanh - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t^2$$

Integrating the above expression;

$$\int_0^t d\ln X_{\phi} = \int_0^t df\left(X_{\phi}u,u\right) = \int_0^t\left(K\tanh - \frac{1}{2}\sigma^2\right)du + \int_0^t \sigma dW_t^2 \tag{1.12}$$

$$\ln X_{\phi} - \ln X_0 = \left[K\tanh u - \frac{1}{2}\sigma_u^2\right]_0^t + \left[\sigma W_u^2\right]_0^t = \ln\left[\frac{X_{\phi}}{X_0}\right] = \left[K\tanh - \frac{1}{2}\sigma^2\right]t + \sigma W_t^2$$

Taking ln of the both sides,gives;

$$X_{\phi} = X_0 e\left(k\tanh - \frac{1}{2}\sigma^2\right)t + \sigma W_t^2 \tag{1.13}$$

Where, W_t^2 is a Brownian Motion

From (1.3) let $f\left(X_{\varpi},t\right) = \ln X_{\varpi}$

Taking the partial derivative, yields;

$$\frac{\partial f}{\partial X_{\varpi}} = \frac{1}{X_{\varpi}},\ \ \frac{\partial^2 f}{\partial X_{\varpi}^2} = \frac{-1}{X_{\varpi}^2},\ \ \frac{\partial f}{\partial t} = 0 \tag{1.14}$$

According to Ito's,gives:

$$df\left(X_{\varpi},t\right)=\sigma X_{\varpi}\frac{\partial f}{\partial X_{\varpi}}dW_t^3+\left((-\beta\alpha+K\tanh)X_{\varpi}\frac{\partial f}{\partial X_{\varpi}}+\frac{1}{2}\sigma^2X_{\varpi}^2\frac{\partial^2 f}{\partial X_{\varpi}^2}+\frac{\partial f}{\partial t}\right)dt \quad (1.15)$$

Substituting(1.14) into (1.15), gives;

$$df\left(X_{\varpi},t\right)=\sigma X_{\varpi}\frac{1}{X_{\varpi}}dW_t^3+\left((-\beta\alpha+K\tanh)X_{\varpi}\frac{1}{X_{\varpi}}+\frac{1}{2}\sigma^2X_{\varpi}^2\left(-\frac{1}{X_{\varpi}^2}\right)+0\right)dt \quad (1.16)$$

$$=\sigma\frac{X_{\varpi}}{X_{\varpi}}dW_t^3+\left((-\beta\alpha+K\tanh)X_{\varpi}\frac{X_{\varpi}}{X_{\varpi}}-\frac{1}{2X_{\phi}^2}\sigma^2X_{\varpi}^2\right)dt=\sigma dW_t^3+\left((-\beta\alpha+K\tanh)-\frac{1}{2}\sigma^2\right)dt$$

$$=\left((-\beta\alpha+K\tanh)-\frac{1}{2}\sigma^2\right)dt+\sigma dW_t^3$$

Integrating the above expression;

$$\int_0^t d\ln X_{\varpi}=\int_0^t df\left(X_{\varpi}u,u\right)=\int_0^t\left((-\beta\alpha+K\tanh)-\frac{1}{2}\sigma^2\right)du+\int_0^t\sigma dW_t^3 \quad (1.17)$$

$$\ln X_{\varpi}-\ln X_0=\left[(-\beta\alpha+K\tanh)u-\frac{1}{2}\sigma_{\varpi}^2\right]_0^t+\left[\sigma W_u^3\right]_0^t=\ln\left[\frac{X_{\phi}}{X_0}\right]=\left[(-\beta\alpha+K\tanh)-\frac{1}{2}\sigma^2\right]t+\sigma W_t^3$$

Taking ln of the both sides, gives;

$$X_{\varpi}=X_0e\left(\left(-\beta\alpha+k\tanh-\frac{1}{2}\sigma^2\right)t+\sigma W_t^3\right) \quad (1.18)$$

Where, W_t^3 is a Brownian Motion

The expected value of the solutions (1.8),(1.13) and (1.18), gives;

$$\left.EX_t\left(t\right)=X_0e\left(-\beta\mu-\frac{1}{2}\sigma^2\right)t+\sigma W_t^1=X_t(0)=e-\beta\mu t\right\} \quad (1.19)$$

$$EX_{\phi}(t) = X_0 e\left(K\tanh - \frac{1}{2}\sigma^2\right)t + \sigma W_t^1 = X_{\phi}(0) = e - K\tanh t\Big\} \tag{1.20}$$

$$EX_{\varpi}(t) = X_0 e\left((-\beta\alpha + K\tanh) - \frac{1}{2}\sigma^2\right)t + \sigma W_t^1 = X_{\varpi}(0) = e(-\beta\alpha + K\tanh)t\Big\} \tag{1.21}$$

3.1 Results and Discussion

This Section presents the graphical results for whose solutions are in (1.8), (1.13), (1.18) and (1.19-1.21) respectively. Hence, the following parameter values were used in the simulation study:

$X_0 = 52.25,\ \beta = 25.6\ ,\sigma = 0.03,\ \mu = 0.88,\ \mathrm{t} = 1,\ \mathrm{W}_t^1 = \mathrm{W}_t^2 = \mathrm{W}_t^3 = 1, \alpha = 0.95, K = 30.7$ and $h = 0.75$

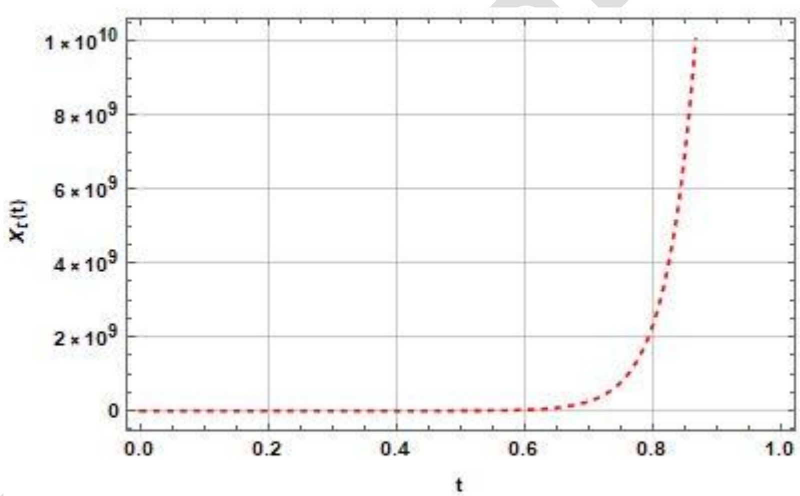

Figure 1: The effect of negative drift coefficient on financial market against time

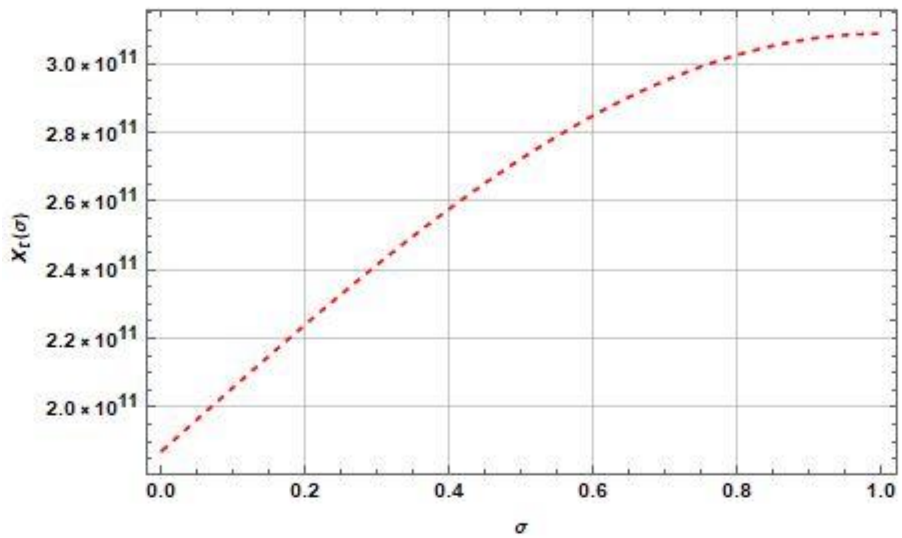

Figure 2: The effect of negative drift coefficient on financial market against volatility

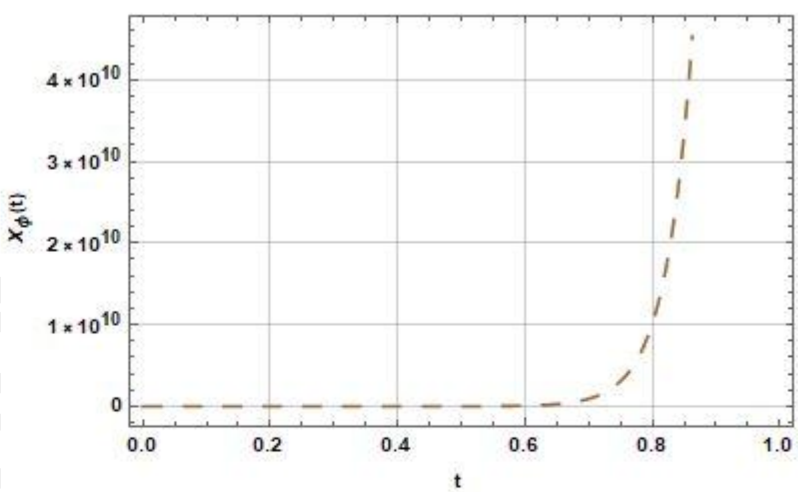

Figure 3: The effect of periodic drift coefficient on financial market against time

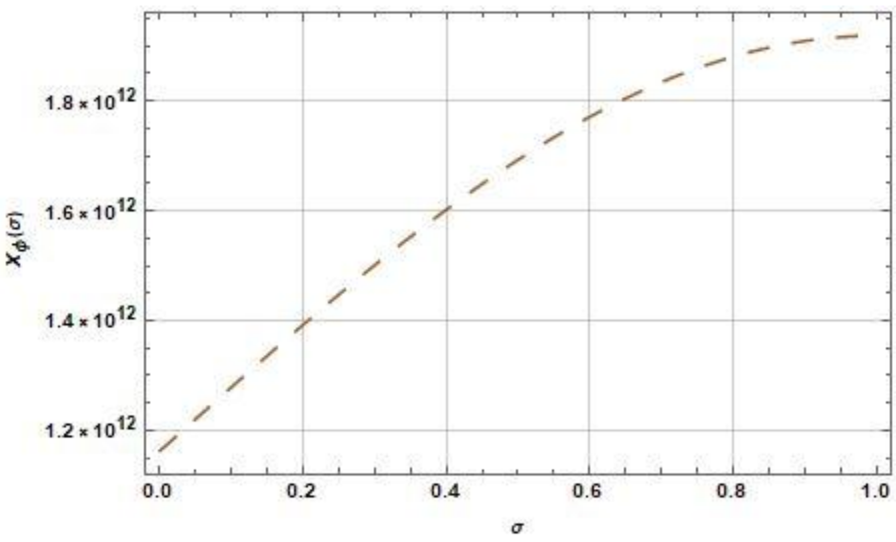

Figure 4: The effect of periodic drift coefficient on financial market against volatility

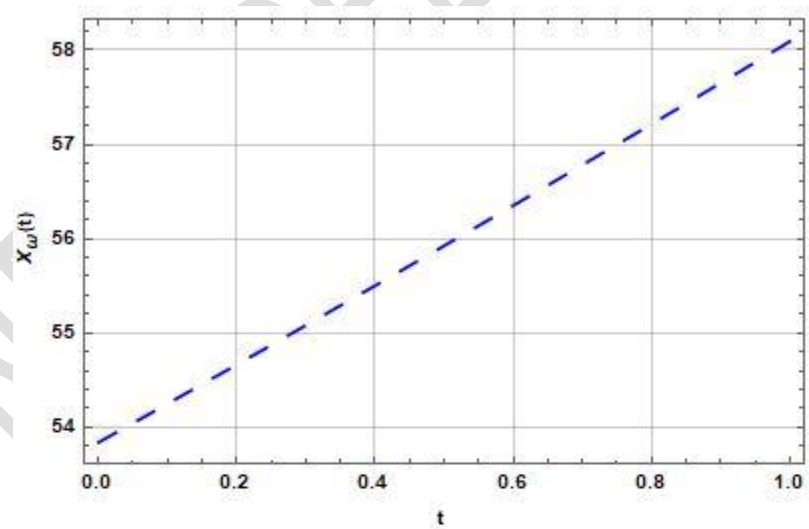

Figure 5: The effect of constant terms with periodic drift coefficient function on financial market against time

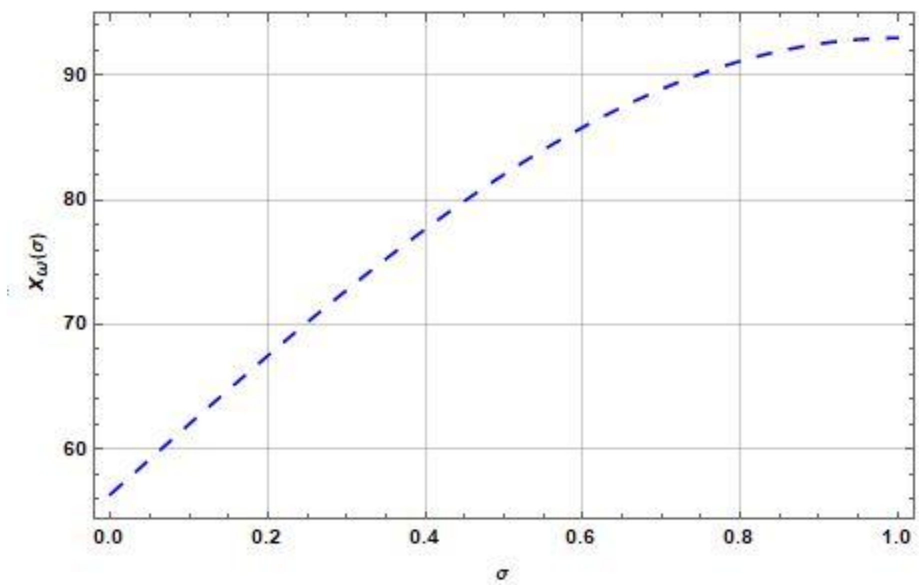

Figure 6: The effect of constant terms with periodic drift coefficient function on financial market against volatility

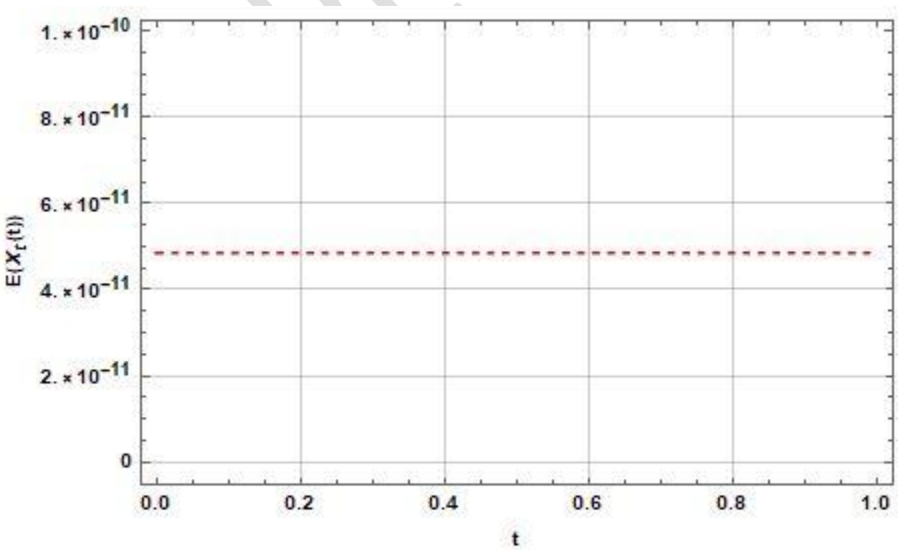

Figure 7: The effect of expected negative drift coefficient on financial market against time

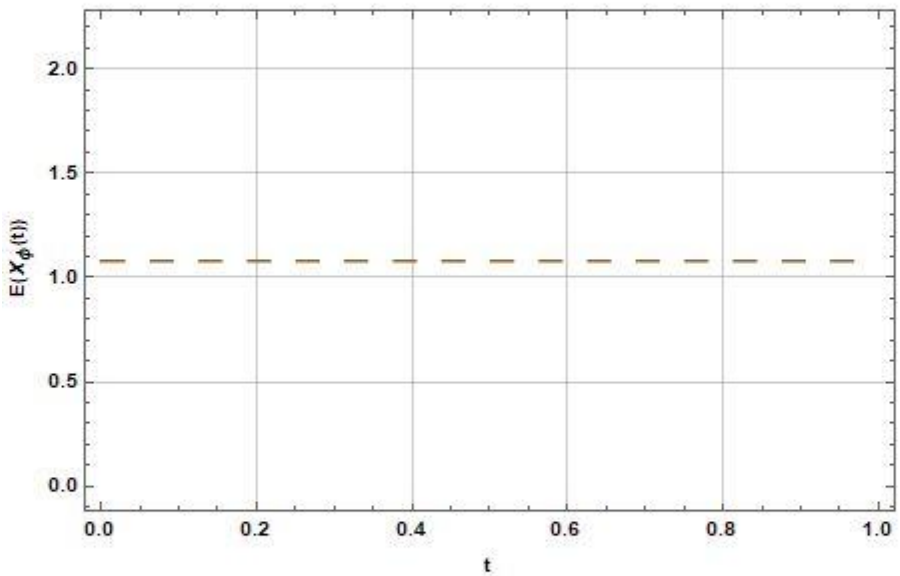

Figure 8: The effect of expected periodic drift coefficient on financial market against time

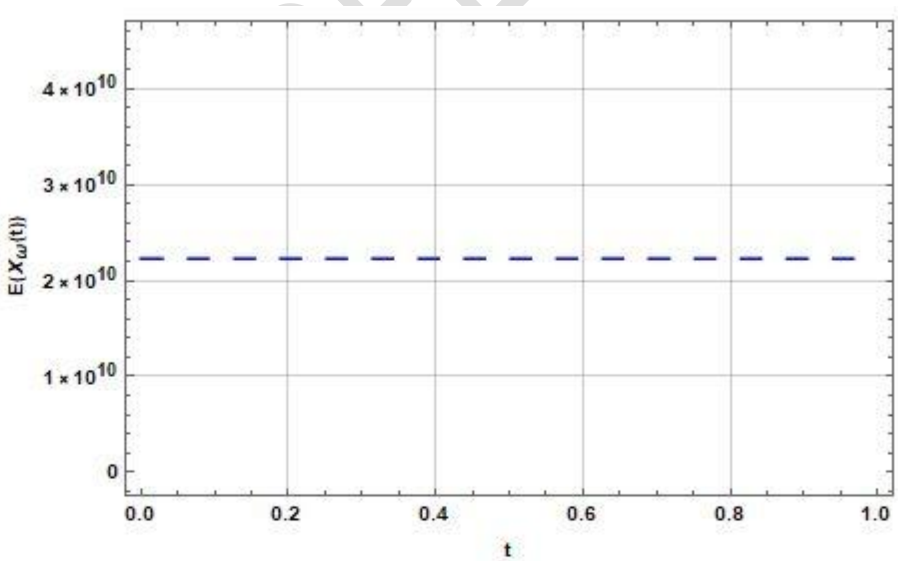

Figure 9: The effect of expected constant terms with periodic function drift coefficient on financial market against time

Figures 1 and 3, show that the market is experiencing exponential growth or decay. This type of growth or decay is often seen in financial markets when there is a strong feedback loop between prices and expectations. For instance, if investors expect prices to go up, they may start buying more, which can lead to an exponential increase in prices. Conversely, if investors expect prices to go down, they may start selling, which can lead to an exponential decrease in prices. It's important to note that exponential growth or decay in financial markets.

Figures 2,4 and 6 describe a market that is growing in value but is also highly volatile. This means that the value of the market is increasing over time, but there is also a lot of risk associated with investing in this market. Volatility is a measure of how much the markets value is changing over time and high volatility means that there is a lot of uncertainty in the market.

Clearly in Figure 5 connotes that the market is experiencing a constant drift. This means that the market is expected to move in a certain direction at a constant ratio.

However, Figures 7,8 and 9 describe the average value of the SDE over time. This value is calculated by taking the average of the SDE solution at different points in time. It can be thought of as the expected value of the market at any given time and it can be used to predict future market movement. For instance, if the expected mean is increasing over time, it means that the market is expected to move up, conversely, if the expected mean is decreasing, it means that the market is expected to move down.

4.1Conclusion

The success of any financial markettransactions hinges mainly on decision making which urgesinvestors to make improvements in their financial dealings andstochastic differential equations are well recognized predominant mathematical tools used for the prediction of stock market variables. Therefore, we considered systemof stochasticdifferential equations with disparities of drift parameters in the model. These problems were solved analytically by adopting the Ito's lemma method of solution and three different solutions with their expected mean solutions were obtained accurately.From the analysis of the graphical solutions we deduce that ; there is an effect of exponential growth during the period of trading, it shows a market that is growing in value but is also highly volatile, a market experiencing a constant drift and finally describes the average value of the SDE over time as it affect financial markets.

Consequently, we recommend that combining ordinary differential and stochastic differential equations in the assessment of drift variations in the next study.

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