Capital Asset Pricing Model: A renewed application on S&P 500 index

Abstract

This study employs the Capital Asset Pricing Model (CAPM) to analyze the risk-return relationship of a diversified portfolio of nine companies from technology, finance, and health sectors within the S&P 500 index sourced from Yahoo Finance. Utilizing daily stock returns data from January 01, 2019, to December 31, 2023, we estimate beta coefficients and expected returns for each company, shedding light on their performance and risk characteristics. Our findings reveal that technology and finance sector stocks generally exhibit higher beta values and expected returns compared to healthcare sector stocks. Notably, NVIDIA Corporation emerges as the most volatile stock with the highest expected return of 23.095%, reflecting its position in the innovation-driven technology sector. Conversely, healthcare sector stocks demonstrate lower beta values and expected returns. Through an in-depth analysis, we underscore the importance of balancing risk and return in portfolio construction, considering investors' risk tolerance and objectives. While acknowledging the limitations of CAPM, our study contributes to a deeper understanding of its applicability in portfolio management and asset pricing, providing valuable insights for investors and financial practitioners.

Key words: Capital Asset Pricing Model (CAPM), risk-return relationship, diversified portfolio, technology sector, finance sector, health sector, beta coefficients, expected returns, NVIDIA Corporation, portfolio construction

1 Introduction

The Capital Asset Pricing Model (CAPM), developed by William Sharpe, John Lintner, and Jan Mossin in the 1960s, serves as a fundamental tool in finance for determining the expected return on investments [1, 2]. This model, grounded in the assumption of rational, risk-averse investors, calculates an asset's expected return based on its risk and the overall market's risk, with implications extending to portfolio management and asset pricing theory. CAPM posits that an asset's expected return comprises a risk-free rate and a risk premium, proportional to the asset's beta, indicative of its sensitivity to market movements [1, 3]. A study by [4] investigated the predictive power of the components of the Capital Asset Pricing Model (CAPM) in forecasting stock returns in the Ghana Stock Exchange. The empirical analysis validated the hypotheses, revealing that the Risk-Free Rate, Beta of the Security, and Market Risk Premium significantly influence the Expected Rate of Return in a positive manner. This study aims to utilize CAPM to estimate the expected returns of nine companies from various industries within the S&P 500 index, thereby assessing the relationship between individual asset risk and return relative to the market index and identifying opportunities for portfolio optimization. The main objective involves applying the CAPM model to estimate beta and expected returns for the selected companies, analyzing their risk-return relationship using the

S&P 500 index. To achieve this, specific objectives include company selection, data collection, beta coefficient calculation via regression analysis, expected return estimation using CAPM, comparison of calculated and actual returns, and portfolio risk-return analysis in comparison to the market index.

2 Literature Review

The stock market plays a vital role in driving a country's economic growth by facilitating the allocation of funds from individuals or firms with surplus capital to those with investment opportunities. This process enhances the overall economic efficiency of a nation. However, the stock market is inherently volatile, influenced by various factors that can affect investors' returns on their stock investments [5]. The Capital Asset Pricing Model (CAPM) serves as a fundamental framework in finance, elucidating the relationship between systematic risk and expected asset returns through the assessment of return variances and risk metrics within a well-diversified portfolio. Widely utilized in estimating firm's cost of capital and assessing portfolio performance, CAPM's development by Sharpe (1964), Lintner (1965), and Mossin (1966) has entrenched its significance, particularly in corporate finance and investment valuation [2]. [6] elaborate on its applications. Industries globally rely on CAPM for various financial decisions, including discount rate determination for firm valuations, pricing regulations in utilities, and performance benchmarking for fund managers, among other uses [1].

Despite the extensive research and widespread application of the Capital Asset Pricing Model (CAPM) in empirically analyzing market behavior, criticisms have been directed towards its assumptions [7]. While some studies [8, 9, 10] have demonstrated a clear relationship between firms' betas and asset return outcomes, concerns persist regarding the model's effectiveness. The beta of an asset, which measures its risk relative to the market, is obtained by dividing the covariance of the asset with the market portfolio by the variance of the market portfolio. Assets with betas higher than 1 are considered riskier than the market average. Despite these empirical findings, challenges remain regarding the adequacy of CAPM in accurately capturing the complexities of real-world market dynamics [11].

Further advancements in CAPM literature include the introduction of the Arbitrage Pricing Theory (APT) [12], which provides an alternative asset pricing model based on multifactor models and no-arbitrage conditions. The APT framework allows for the incorporation of multiple risk factors beyond market beta, accommodating diverse sources of systematic risk. [13] expanded CAPM by incorporating international factors, leading to the development of the International Capital Asset Pricing Model (ICAPM), which accounts for cross-country differences in asset returns.

Recent literature on CAPM focuses on refining the model to address its limitations and incorporate new insights from behavioral finance and empirical findings. [14] extended the Fama-French Three-Factor Model to the Fama-French Five-Factor Model, incorporating profitability and investment factors. Additionally, empirical studies by [15] and [16] explore the implications of time-varying risk premia and investor sentiment on asset pricing, providing valuable insights into the dynamics of asset returns within the CAPM framework. Overall, the extensive literature on CAPM underscores its significance as a foundational framework in asset pricing theory while highlighting the ongoing efforts to refine and extend its applicability in modern financial markets.

This works aims to explore and analyze the Capital Asset Pricing Model (CAPM) and its implications in modern finance, particularly focusing on the S&P 500 index. The study will delve into the foundational principles of CAPM, examining its theoretical underpinnings and practical applications in estimating asset returns and assessing portfolio performance.

3 Data and Methods

3.1 Experimental Design for the study

The experimental design for this project involves several key steps. Firstly, data collection will be conducted, focusing on obtaining historical stock prices for nine companies representing diverse industries from the S&P 500 index, along with market index data. Next, the data will be preprocessed, including adjusting for corporate actions and calculating daily returns. Subsequently, the Capital Asset Pricing Model (CAPM) will be applied to estimate the expected returns of individual stocks using market data and risk-free rates. Beta coefficients will be computed to measure each stock's sensitivity to market movements. We will also explore the relationship between beta and the expected return to get the insight about the CAPM model. Python programming language was used for all computations and visualization in this study.

3.2 Data

The data set used in this study includes historical adjusted close prices of the selected nine companies from different industries within the S&P 500 index, as well as the adjusted close prices of the S&P 500 index itself.

The study period covers a span of 5 years, from January 1, 2019, to December 31, 2023. The data for the listed companies in table 1, along with the S&P 500 index, was obtained from Yahoo Finance.

Abbreviation	**Company**	**Industry**
MSFT	Microsoft Corporation	Technology
AAPL	Apple Inc	Technology
NVDA	NVIDIA Corporation	Technology
ABBV	AbbVie Inc	Healthcare
MRK	Merck & Co Inc	Healthcare
LLY	Eli Lilly and Company	Healthcare
SPGI	S&P Global Inc	Finance
GS	The Goldman Sachs Group Inc	Finance
AXP	American Express Company	Finance
CGPI	S&P 500 Index	Market Index

Table 1: Selected Companies from S&P 500 and their Industries

3.3 Methods

The methodology involves several steps:

3.3.1 Compute Daily Returns for each stock/security

Firstly, compute the returns for each stock/security. Returns are calculated using the formula:

$$\text{Return}_t = \frac{\text{Adjusted Close Price}_t - \text{Adjusted Close Price}_{t-1}}{\text{Adjusted Close Price}_{t-1}} \tag{1}$$

where t represents the time period.

3.3.2 Compute Daily Returns for the Markert Index

Daily market return can be calculated similarly using the closing prices of the market index (e.g., S&P 500):

$$r_{m,t} = \frac{I_t - I_{t-1}}{I_{t-1}} \tag{2}$$

Where I_t is the index value at time t, and I_{t-1} is the index value at time $t-1$.

3.3.3 Set Risk-Free Rate

Next, set the risk-free rate, typically represented by the yield on government bonds, as a benchmark for risk-free return. The risk-free rate is denoted as R_f.

3.3.4 Compute Beta using Ordinary Least Square Method

To compute the beta coefficient for each stock/security in the portfolio, we use Ordinary Least Squares (OLS) regression. The regression model for CAPM is given by:

$$r_i = \alpha + \beta \times r_m + \varepsilon \tag{3}$$

where:

- r_i is the return of the asset,
- r_m is the return of the market index,
- α is the intercept,
- β is the beta coefficient (systematic risk),
- ε is the error term.

The objective is to minimize the sum of squared errors (SSE) between the observed returns of the asset and the returns predicted by the model. The SSE is given by:

$$\text{SSE} = \sum_{i=1}^{n}(r_i - \hat{r}_i)^2$$

where $\hat{r}_i$ is the predicted return of the asset based on the model.

To minimize the SSE, we differentiate it with respect to α and β, and set the derivatives equal to zero.

3.3.5 Partial Derivatives

The partial derivative of SSE with respect to α is:

$$\frac{\partial \text{SSE}}{\partial \alpha} = -2\sum_{i=1}^{n}(r_i - \alpha - \beta \times r_m)$$

The partial derivative of SSE with respect to β is:

$$\frac{\partial \text{SSE}}{\partial \beta} = -2\sum_{i=1}^{n}(r_i - \alpha - \beta \times r_m) \times r_m$$

3.3.6 Setting Partial Derivatives to Zero

Setting the partial derivatives equal to zero, we get the normal equations:

$$\sum_{i=1}^{n} r_i - \alpha \sum_{i=1}^{n} 1 - \beta \sum_{i=1}^{n} r_m = 0$$

$$\sum_{i=1}^{n}(r_i \times r_m) - \alpha \sum_{i=1}^{n} r_m - \beta \sum_{i=1}^{n} r_m^2 = 0$$

3.3.7 Solving for α and β

Solving the normal equations simultaneously, we get:

$$\hat{\alpha} = \bar{r}_i - \hat{\beta} \times \bar{r}_m$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n}(r_i - \bar{r}_i) \times (r_m - \bar{r}_m)}{\sum_{i=1}^{n}(r_m - \bar{r}_m)^2}$$

where:

- $\bar{r}_i$ is the mean return of the asset,
- $\bar{r}_m$ is the mean return of the market index.

This $\hat{\beta}$ is the estimated beta coefficient obtained through OLS regression.

$$\beta = \frac{\text{Covariance}(r_i, r_m)}{\text{Variance}(r_m)}$$

where r_i is the return of the asset, r_m is the return of the market index, Covariance(r_i, r_m) is the covariance between the asset's returns and the market returns, and Variance(r_m) is the variance of the market returns.

3.3.8 Interpretation of Beta

Beta measures the sensitivity of an asset's returns to movements in the overall market.

- A beta greater than 1 indicates that the asset tends to amplify market movements, meaning it is more volatile than the market.
- A beta less than 1 suggests that the asset moves less than the market, indicating lower volatility compared to the overall market.
- A beta equal to 1 implies that the asset moves in line with the market.

3.4 Compute the Expected Return on market

When we multiply the mean daily return by 252 to annualize it, we are assuming that the daily returns are representative of the market's performance over a full year. This assumption is based on the notion that the market operates on approximately 252 trading days in a year.

The mathematical representation of annualizing the return is expressed as follows.

$$\text{Annualized Expected Market Return} = \text{Mean Daily Return} \times 252 \tag{4}$$

3.5 Compute Expected Return for the stock/security

Finally, compute the expected return for the portfolio using the CAPM formula, which incorporates the risk-free rate, the beta of the portfolio, and the expected return for the market portfolio:

$$\textbf{Expected Return}_{\textbf{stock/security}} = R_f + \beta \times (\textbf{Expected Return}_{\text{market}} - R_f) \tag{5}$$

where R_f is the risk-free rate, β is the beta of the portfolio, and **Expected Return**$_{\textbf{market}}$ is the expected return for the market portfolio.

4 Results and Discussion

We discuss the visualisation of daily stocks of the nine companies , daily market index, daily stock returns, daily market returns and the plots of daily stock returns with the market returns. The betas and expected returns for each stock are also discussed. We also present the portifolio return results for each stock.

4.1 Visualization of Adjusted Daily stock prices for the 9 companies

Figure (12) depicts the plot of daily stock based on the adjusted price. The stocks exhibit a coherent movement, characterized by a decline in daily adjusted closing price in April 2020, followed by a subsequent increase. Overall, the trend of the daily adjusted closing prices demonstrates a discernible upward trajectory.

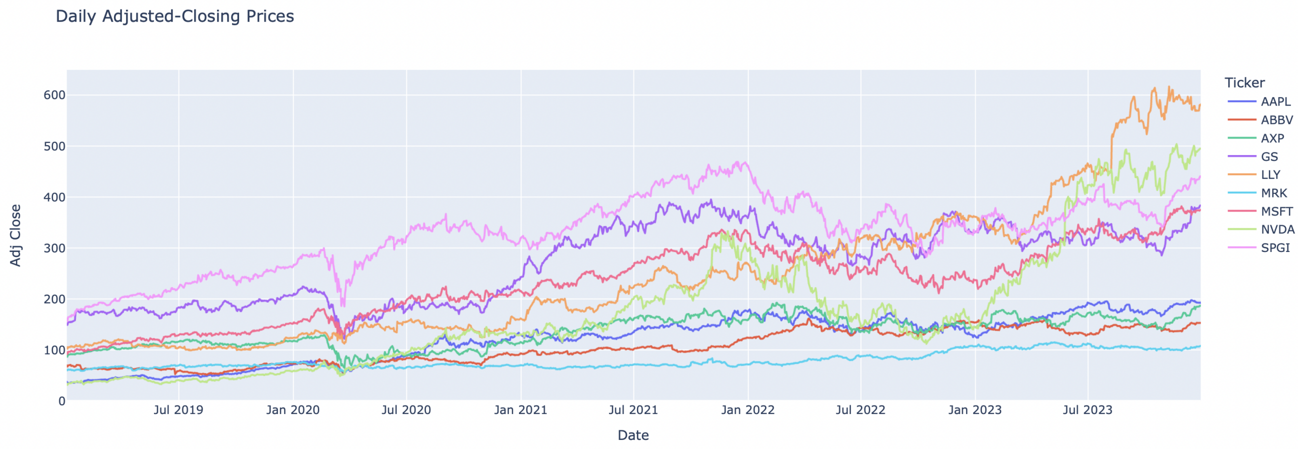

Figure 1: Daily stock/security prices

4.2 Visualization of the daily market Index

The daily market index, as visualized in Figure 12, demonstrates an upward trajectory with intermittent fluctuations. Notably, there is a discernible surge in the index followed by a sharp decline in April 2020. Subsequently, the index exhibits a pattern of consistent growth interrupted by occasional downturns. These fluctuations reflect alternating trends of expansion and contraction in the daily market index over the observed period.

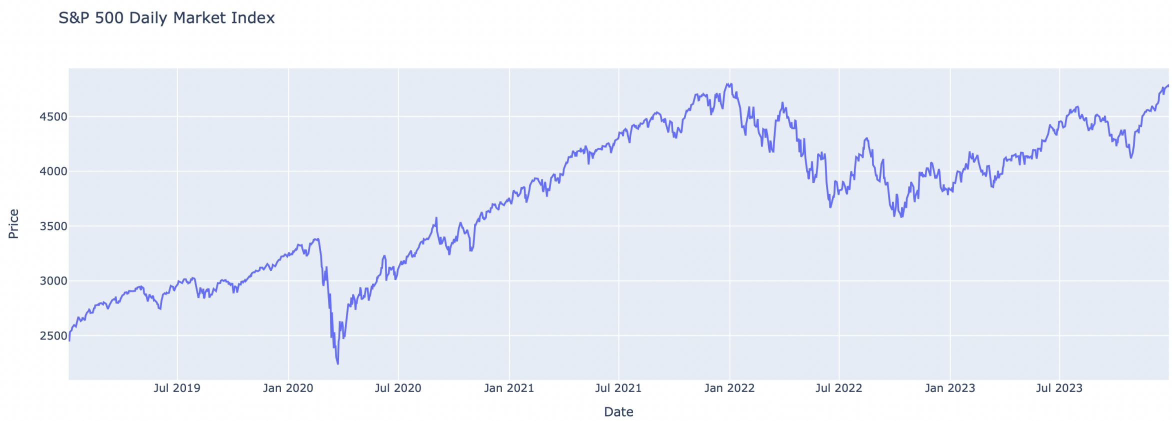

Figure 2: Daily SP500 Market Index

4.3 Visualisation of Daily Stock/security Returns

Daily returns were computed and all missing values which reflect as Nan after python computation were replaced by zero.. There is only one missing value which results as of computation of daily returns for stock and the market index daily returns. As shown in figure 3 we observe clusters of high and low volatility. Due to decrease in closing price in April 2020, the volatility in this time as shown by daily returns is very high.

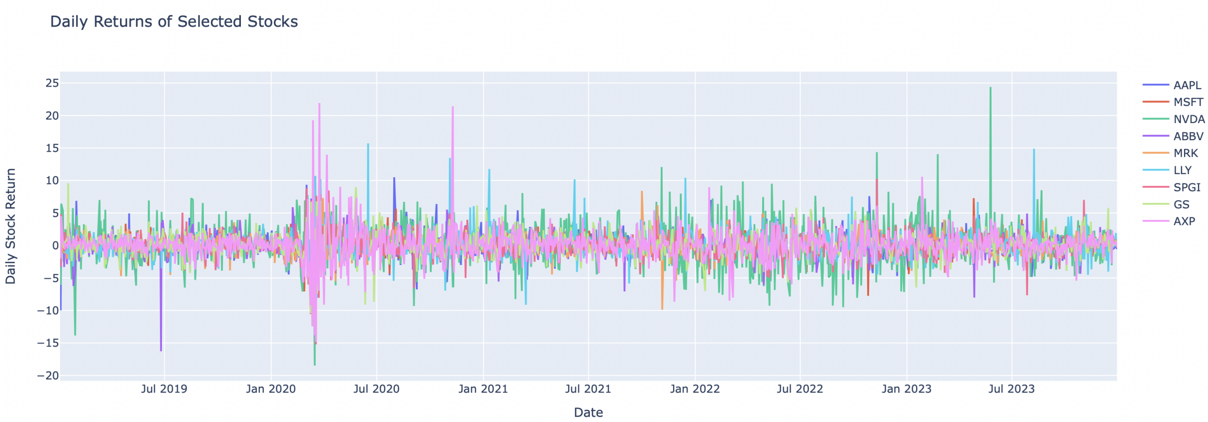

Figure 3: Daily Returns for stock/security

4.4 Visualisation of Daily Market Index Returns

In figure 4 of daily returns of market index, we observe high volatility in April 2020. The returns shows stationary time series characteristic around the mean =0.

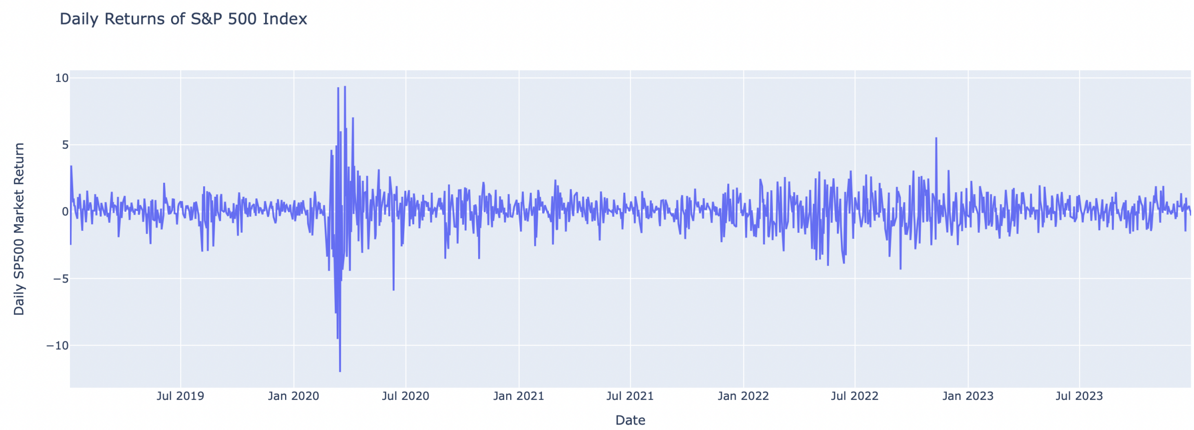

Figure 4: Daily Returns for SP 500 Market Index

4.5 Plot of the daily stock returns against, market returns

The plots depicting the daily returns of individual stocks against those of the market index reveal a distinct linear relationship as shown from Figure (5 to 13). This linear relationship signifies a consistent proportional change in stock returns corresponding to changes in the market returns. Essentially, as the market index moves, the stock returns tend to move in a similar direction, though with varying magnitudes. The degree of linearity observed in these plots reflects the correlation between the stock returns and the market returns. A higher correlation coefficient indicates a stronger linear relationship, implying that the stock returns closely align with the movements of the market index. Conversely, a lower correlation coefficient suggests a weaker linear relationship, indicating that the stock returns are less influenced by changes in the market index. Moreover, the slope of the line in each plot represents the beta coefficient, which quantifies the sensitivity of the stock returns to changes in the market returns. A beta coefficient greater than 1 suggests that the stock is more volatile than the market, with returns that tend to amplify market movements. Conversely, a beta coefficient between 0 and 1 indicates that the stock is less volatile than the market, while a negative beta coefficient implies an inverse relationship with the market. Overall, these observations provide valuable insights into the relationship between individual stock returns and market returns, aiding investors in portfolio management and risk assessment strategies.

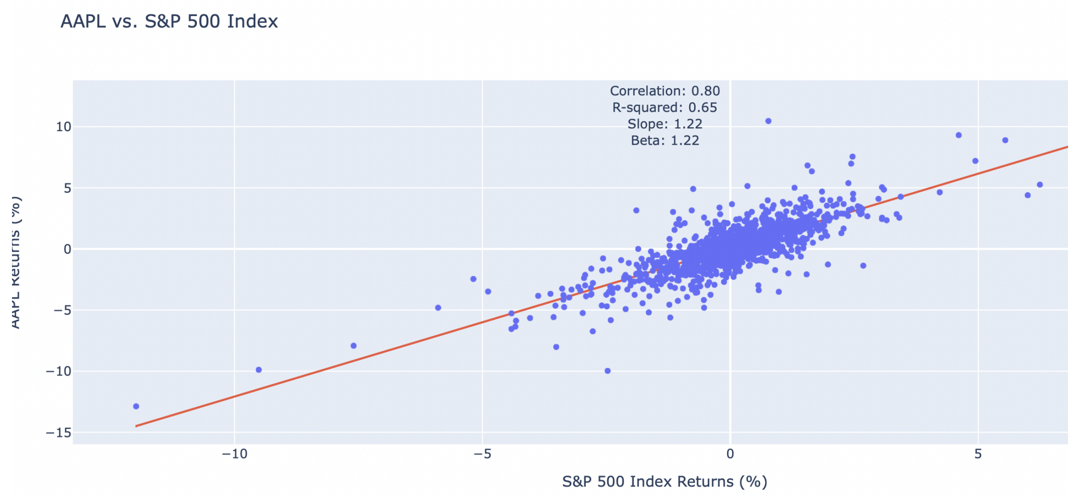

Figure 5: AAPL Daily Returns against Daily SP 500 Market Returns

Figure 6: MSFT Daily Returns against Daily SP 500 Market Returns

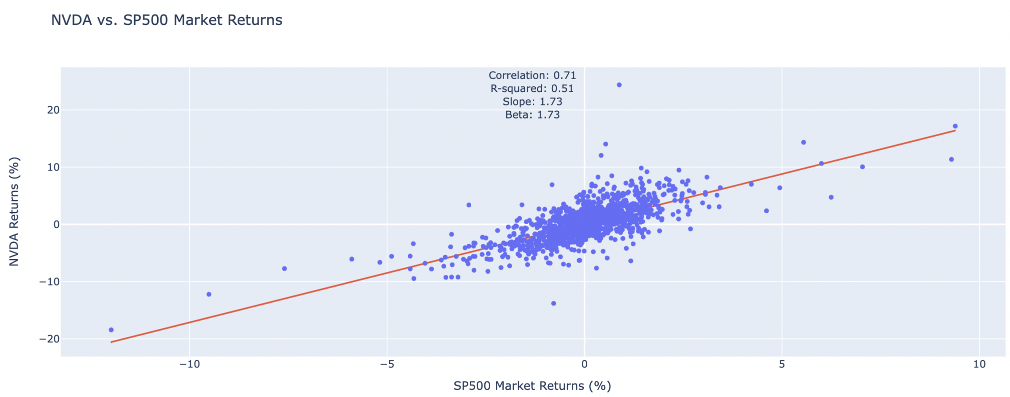

Figure 7: NVDA Daily Returns against Daily SP 500 Market Returns

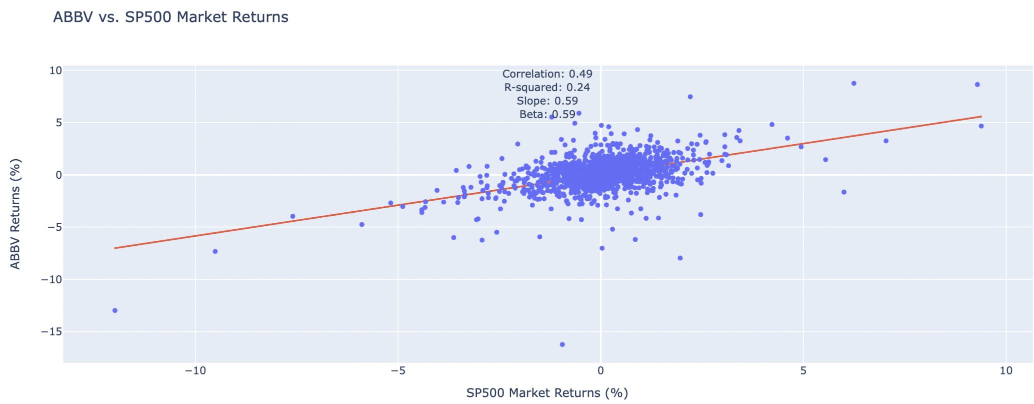

Figure 8: ABBV Daily Returns against Daily SP 500 Market Returns

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Figure 9: MRK Daily Returns against Daily SP 500 Market Returns

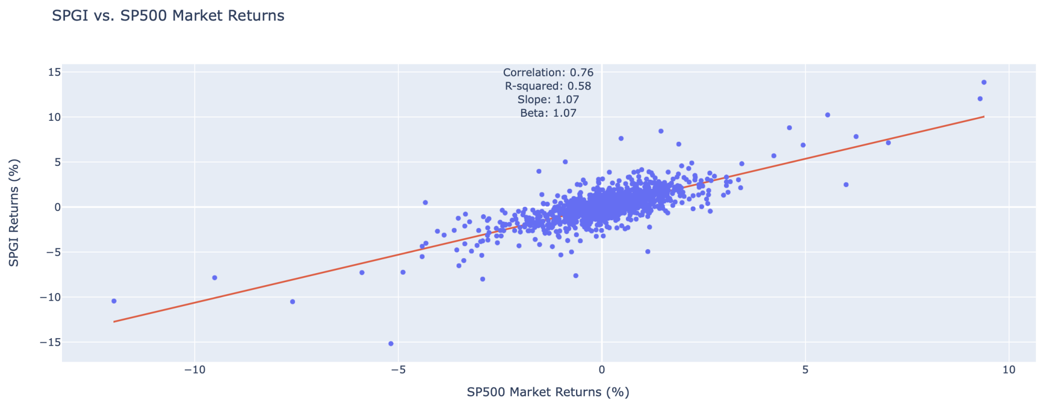

Figure 10: SPGI Daily Returns against Daily SP 500 Market Returns

Figure 11: LLY Daily Returns against Daily SP 500 Market Returns

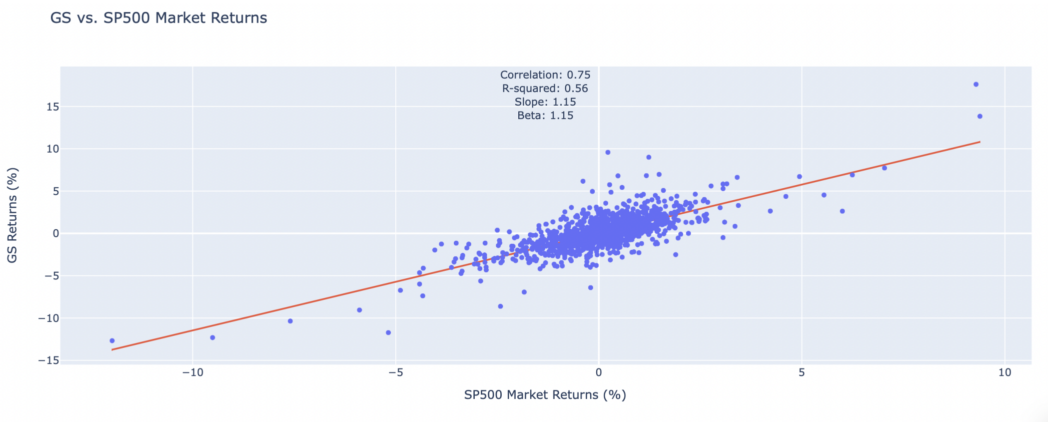

Figure 12: GS Daily Returns against Daily SP 500 Market Returns

Figure 13: AXP Daily Returns against Daily SP 500 Market Returns

4.6 Interpretation of Beta and Alpha and expected return for each Company

Table 2: Beta and Alpha Values for Each Company with Industries

Stock	**Beta**	**Alpha**	**Industry**
AAPL	1.215626	0.076828	Technology
MSFT	1.182137	0.055948	Technology
NVDA	1.728843	0.162474	Technology
ABBV	0.589005	0.040086	Healthcare
MRK	0.519138	0.024361	Healthcare
LLY	0.642156	0.115745	Healthcare
SPGI	1.065924	0.032870	Finance
GS	1.148924	0.025867	Finance
AXP	1.309564	0.008253	Finance

From table 2 we observe that among the companies with beta values greater than 1, NVDA stands out as the most volatile, with a beta of 1.728843. This indicates that NVDA's stock returns tend to amplify market movements more than any other company in the sample. MSFT and AAPL also exhibit high beta values, reflecting their sensitivity to market fluctuations, which is common in the technology sector. Investors in these companies should be aware of their higher risk levels compared to the broader market. Finance companies like SPGI,GS and AXP have beta greator than 1, suggests that their stock returns tend to amplify market movements, making them riskier investments.

Conversely, companies with beta values less than 1, such as ABBV, MRK, and LLY, are less volatile than the market, indicating that their stock returns are less influenced by market fluctuations.

The alpha values represent the excess return (or underperformance) of each company's stock compared to what would be predicted by the market index. All companies have positive alpha indicate that these companies have outperformed the market on average.

Analyzing the results across industries reveals interesting patterns. For example, technology companies like AAPL, MSFT, and NVDA exhibit higher beta values, indicating greater sensitivity to market movements, similary Finance companies, SPGI,GS, AXP have beta greator than 1, indicating greater sensitivity to market movements. This is consistent with the inherent volatility and rapid changes in the technology sector. Conversely, pharmaceutical companies like ABBV, MRK, and LLY tend to have lower beta values, reflecting the relative stability of these industries compared to the broader market. Understanding these industry-specific dynamics is crucial for portfolio diversification and risk management.

Table 3: Expected Returns for Each Stock with Industries

Stock	Expected Return (%)	Industry
AAPL	17.498	Technology
MSFT	17.132	Technology
NVDA	23.095	Technology
ABBV	10.664	Healthcare
MRK	9.902	Healthcare
LLY	11.243	Healthcare
SPGI	15.865	Finance
GS	16.770	Finance
AXP	18.522	Finance

Table 5 shows the expected return for each of the 9 companies. A risk free rate of 4.24% was used. This is based on the current risk-free rate on 10 year U.S. Treasury bonds. When the expected return of a stock exceeds the annualized expected market return, it suggests that the stock is anticipated to yield higher returns than the overall market. However, this higher expected return typically comes with increased risk. This risk can be assessed using the concept of beta, which measures a stock's volatility relative to the market. A high beta indicates that the stock is more volatile than the market, while a low beta suggests lower volatility.

In the above table, several stocks, including AAPL, MSFT, NVDA, SPGI, GS and AXP, have expected returns that surpass the annualized expected market return of 15.146%. These stocks are expected to deliver returns higher than the market average. However, their higher expected returns also indicate higher levels of risk, especially since their betas are also high. This is in line with the CAPM model theory.

For instance, NVDA, with an expected return of 23.095%, stands out as having the highest expected return among the listed stocks. This suggests that investors expect NVDA to outperform the market significantly. However, NVDA's status as a technology stock, known for its innovation and growth potential, may justify its higher expected return. Nevertheless, investors should recognize that NVDA's high expected return has a high beta, indicating greater volatility and risk compared to the overall market.

Similarly, AAPL, MSFT, SPGI, GS and AXP also exhibit expected returns higher than the market return. As technology and finance sector stocks, respectively, these companies may offer growth opportunities and higher returns, but investors should be prepared for increased volatility and risk associated with these sectors.

Conversely, stocks like ABBV, MRK, and LLY have expected returns below the market average. While these stocks may offer stability and defensive characteristics typical of the healthcare sector, their lower expected returns suggest limited growth potential compared to the overall market.

In summary, while stocks with expected returns exceeding the market return may offer the potential for higher returns, investors should carefully consider their risk tolerance and investment objectives. High expected returns are often accompanied by higher risk, especially if the stock's beta is also high. Diversification and a thorough understanding of individual stock characteristics are essential for managing risk and optimizing investment returns.

5 Conclusion

Considering the Capital Asset Pricing Model (CAPM) and its application to a diversified portfolio of nine companies from different industries within the S&P 500 index, it is evident that CAPM serves as a valuable tool for estimating asset returns and assessing portfolio performance. The analysis began with an overview of CAPM's theoretical framework and its significance in modern finance, highlighting its role in estimating expected returns based on risk factors. Subsequently, the methodology for applying CAPM to estimate beta coefficients and expected returns for individual stocks was outlined, followed by a detailed discussion of the results and their implications.

The results revealed interesting insights into the risk-return relationship of the selected companies, with technology and finance sector stocks generally exhibiting higher beta values and expected returns compared to healthcare sector stocks. Notably, NVDA emerged as the most volatile stock with the highest expected return, reflecting its position in the technology sector known for innovation and growth potential. Conversely, healthcare sector stocks like ABBV, MRK, and LLY demonstrated lower beta values and expected returns, aligning with the defensive characteristics of this industry.

Furthermore, the analysis emphasized the importance of considering risk factors, as reflected in the relationship between expected return and beta. Assets with higher betas were found to have higher expected returns, consistent with CAPM's prediction of a positive linear relationship between risk and return. However, investors must carefully balance risk and return when constructing portfolios, considering their risk tolerance and investment objectives.

Overall, the study contributes to a deeper understanding of CAPM's applicability in portfolio management and asset pricing, providing valuable insights for investors and financial practitioners. This study however has limitations. Firstly, CAPM relies heavily on various assumptions, including the assumption of market efficiency, linearity of relationships, and constancy of betas, which may not always hold true in real-world financial markets. Additionally, CAPM utilizes historical data to estimate risk and expected return; however, past performance may not accurately predict future outcomes due to changing market dynamics and unforeseen events. Moreover, the quality and availability of data used in CAPM analysis can introduce uncertainties, as the accuracy and reliability of the data may vary depending on how it was collected, cleaned, transformed, or loaded. Furthermore, the continuity of time-series data is crucial for accurate analysis, yet limitations may arise when datasets have incomplete or insufficient historical records. For instance, while our analysis spans the last five years up to December 2023, the use of a 10-year US Treasury Bill rate from 2013 may not adequately reflect the current bill rate or ensure continuity in time-series data. These limitations underscore the need for cautious interpretation and contextualization of CAPM findings in financial decision-making processes. Despite its limitations and criticisms, CAPM remains a foundational framework in finance, guiding investment decisions and risk management strategies in modern financial markets. Further research and refinement of CAPM and related asset pricing models are essential for addressing its shortcomings and enhancing its utility in diverse market conditions.

Data

The data used in this study is sourced from Yahoo Finance. Daily stock returns data for the period from January 01, 2019, to December 31, 2023, was collected for nine companies representing technology, finance, and health sectors within the S&P 500 index.

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Conflict of Interest

The authors declare that they have no competing interests.

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