A detailed physical explanation of an aircraft flutter mechanism

Abstract.
Each dynamic mode (aeroelastic) is made up of torsional and rotational movements. These two movements in each mode were dissociated and the phase, amplitude, damping and frequency of each of these movements were analyzed. The structural resistances of torsion and bending, as well as the bending movement itself, have a damping effect and torsion has effect son the oscillations. After a certain speed, bending becomes out of phase with the applied forces. At this point, the bending has an amplifying effect on the oscillations and only the structural stiffness dampens the movement. From the speed at which the torsional movement is out of phase with the applied aerodynamic loads, the damping of the mode decreases with speed, until flutter occurs. The type of analysis presented here was only possible due to the dissociation of torsion and bending movements in each mode. This is a novelty of this article. And this dissociation was made possible due to the useof the strain-based formulation, also called here as methodology NFNS_. The use of this methodology for this type of analysis was another contribution. The article presents the proposal of a new way of analyzing the aeroelastic stability of aircraft.

Keywords:aeroelasticity, strain based formulation, flexible airplane, flutter mechanism

1. Introduction

The term aeroelasticity designates the field of study interested in evaluating the interactions that are established between the disciplines of aerodynamics, elasticity and dynamics (Wright and Cooper, 2007). The multidisciplinary nature of this field can be synthesized by the prediction of the forces acting on the structure by using the aerodynamic theories, the deformations being predicted by the elasticity and the dynamics introducing inertial forces in the system (Hodges and Pierce, 2011). Inside the field of dynamic aeroelasticity, one of the phenomena that carries most attention is flutter. This phenomenon is considered one of the most relevant in aeroelastic studies and one of the most difficult to predict (Garrick and Reed III, 1981).

In recent decades, the coupling between the rigid modes normally studied in the discipline of flight dynamics and the structural modes in newly developed aircraft has been noted. This has motivated the development of mathematical models that integrate the disciplines of flight dynamics and aeroelasticity (Cesnik, 2023, Palacios and Cesnik 2023).

According to Bisplinghoff and Ashley (1975), the insights that aeroelasticity specialists have are largely mathematical. Although it was pronounced more than forty years ago, it is not common to find a detailed physical explanation that links the mathematical models with the physics of the problem. Generally, the approaches that use eigenvalues to find the instability are focused on plotting frequency and damping ratio charts (Wright, Cooper, 2007). None of the actual analysis methods allows a clear, and detailed physical interpretation of what is happening. Although some efforts and insights have already been developed to explain the physics involved in flutter (Biot and Arnold, 1948; Rheinfurthn and Swift, 1965; Bisplinghoff and Ashley, 1975; Patil, 2001), these works do not incorporate the data that can be extracted from eigenvectors, i.e., phase and amplitude. Only the model shape uses to be plotted in aeroelasticity analyses. Bisplinghoff and Ashley (1975) have already mentioned that mode shape and **phase variations** play a fundamental role in the physical mechanism, since these changes have a great influence on how and where the instability of a system with multiple degrees of freedom begins, but they did not explore in details the use of eigenvectors to explain the flutter mechanism. The new approach presented here considers the data of eigenvalues and eigenvectors to diagnose what is happening on the structure. These analyses show an initial development that was made for an aeroelastic mode, found in a numerical model of a highly flexible aircraft, that exhibited flutter. This is the first step in supporting future analysis aimed at creating a new general theory about the flutter. The analysis commonly performed in which the different modes are considered is called here as intermodal analysis. The analysis commonly performed not always considers what happens "inside" the mode. The separation and quantification of bending and torsion deformations allows one deeper and detailed analysis. This type of analysis is called here as intramodal analysis.

2. Methodology

A numerical platform developed initially in Ribeiro (2011), called AEROFLEX, was adapted in Sousa (2013) for modeling a medium size jet airplane with similar properties to Embraer EMB-190/195 and Boeing 737-200/300. The AEROFLEX uses a methodology that is named here as NFNS_s **(N**onlinear**F**light Dynamics– **N**onlinear **S**tructural Dynamics- **s**train based formulation).

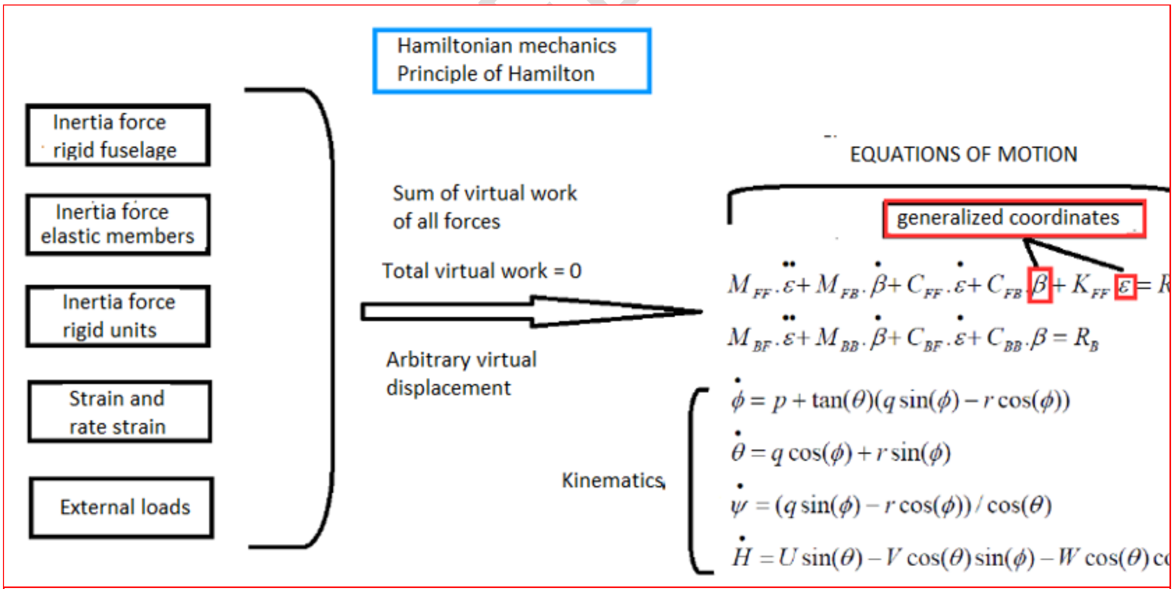


Figure 1 – Equations of motion that describe the dynamics of a flexible aircraft

The methodology NFNS_s was developed by Cesnik and his co-workers (Brown, 2003, Shearer, 2006, Su and Cesnik, 2008). NFNS_s uses a beam formulation to capture nonlinearities of the structural deformations and that is also capable to compute large deformations and inertial coupling between elastic and rigid generalized coordinates (Shearer, 2006; Su, 2008, Ribeiro, 2011). Figure 1 presents the equations of motion of a flexible airplane. More details about the model developed can be achieved in Sousa et al. (2017).The aircraft model presented here is the same presented in Sousa et.al., (2017), but with two differences: the wing flexural axis is located at 75% mean aerodynamic chord and the structural stiffnesses are six times lower. Figure 2 presents the aircraft plan view and the aerodynamic plan view, and structural elements of wing and horizontal tail. There are five structural elements on each wing, names as E1, E2, E3, E4, E5. The engines were modeled as rigid units appended on Element 2 (E2).

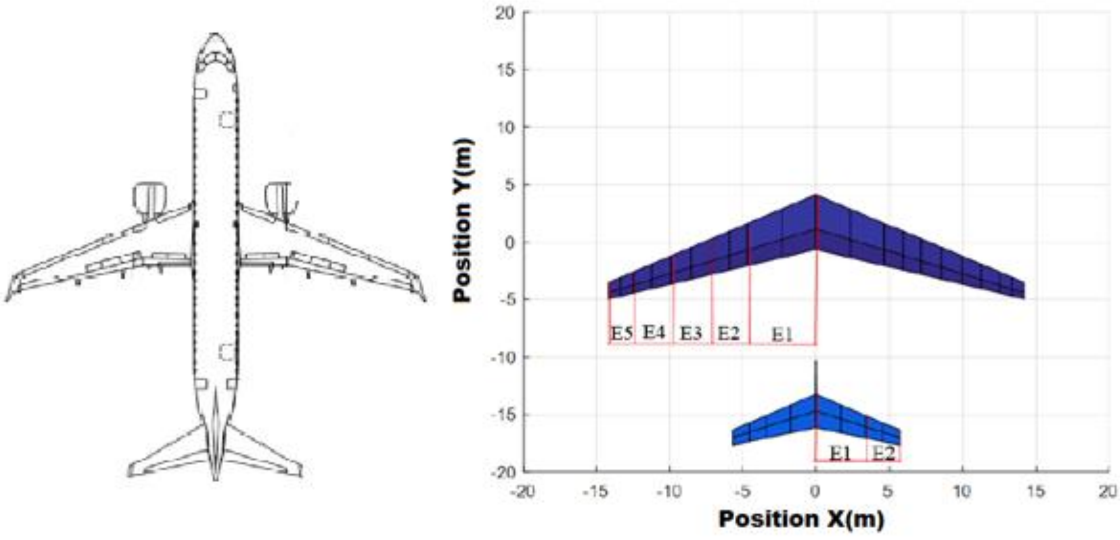


Figure 2 Aircraft plan view (Doc846, w.d), and Structural elements on the wing (Siqueira, 2019)

Figure 3 present the type of structural deformations presented in each element: The bending ky and the torsion kx.

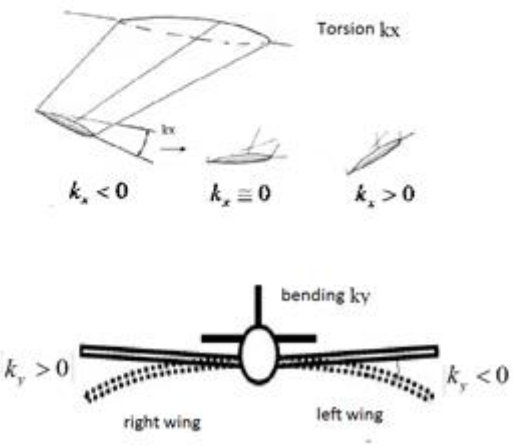


Figure 3 – Structural deformations (bendingky and twistkx) (AEROSPACEWEB, 2000, Sofla, et. Al., 2010, Sousa, 2013)

Figure 4 presents the flexural axis and mass axis located at 75%mac (mean aerodynamic chord) of each aerodynamic profile and the axis with aerodynamic centers located at 25% mac.

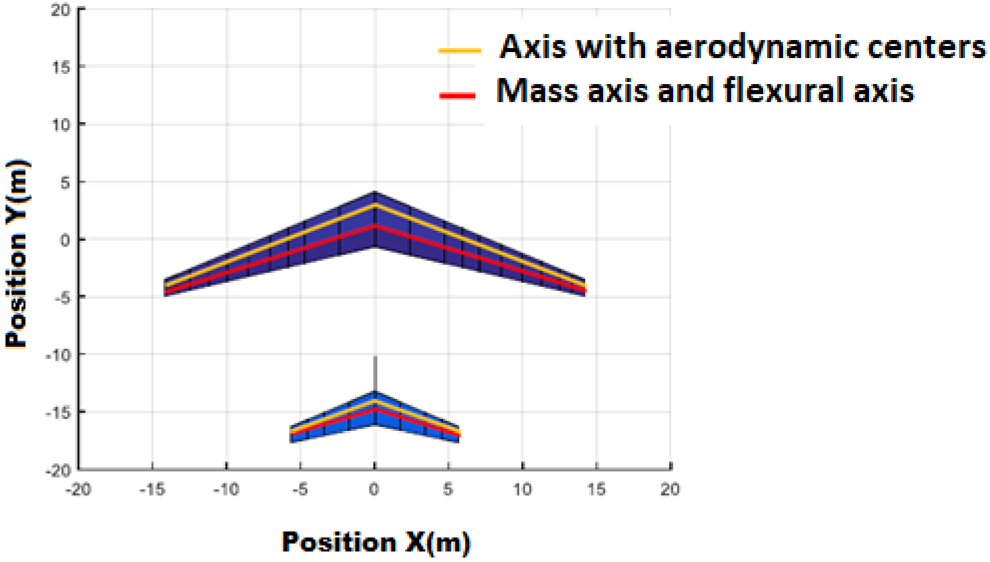


Fig.4 – Flexural axis, mass axis and aerodynamic center axis positions

The AEROFLEX software produced in Ribeiro (2011) needed some modifications in order to be ready to capture the eigenvalues and eigenvectors matrices and manage all the data properly. The modifications are better explained on Siqueira, et. al. (2019) and Siqueira, (2019). The modifications were based on changes and developments of new routines to treat the data that

AEROFLEX code itself already provides at each trimmed condition. A detailed explanation can be found in Siqueira (2019). The modifications were made to add functionalities that would make it possible to acquire and process the data obtained in each trimmed condition.Originally, the model required a single airspeed, from which the dynamic simulation was done and the results were generated for flight dynamics analysis and applications of flight control techniques. So, the eigenvalues and eigenvectors were already calculated by the code. However, it would be essential to obtain them at a large range of airspeeds in order to plot charts to use as a basis for the physical analysis. In this way, the original code was modified to include aairspeed loop that permits the input of the initial and final airspeeds to simulate the aircraft. The initial and final airspeeds are set before the loop, as well as the speed increment that will be used. At each step of the simulation, the aircraft model is loaded and trimmed to generate the new eigenvalues and eigenvectors matrices, which are stored for future analysis data management.

One of the processes developed that has to be emphasized is the data filters. This process involved the selection of aeroelastic modes based on the real and imaginary parts of the eigenvalues. It was necessary to implement a process of mode tracking to order the eigenvalues and the eigenvectors. This was necessary because at each increment of airspeed a new order of eigenvalues/ eigenvectors was generated. In other words, there was randomness in the ordering of eigenvalues and eigenvectors. Although the merge of eigenvalues and eigenvectors was not too large, a method had to be implemented to put the data into the correct sequence that would allow the future treatment to generate the charts. The development of the data filters included four distinct subroutines:

- Data cleaning: subroutine used to select the aeroelastic modes of interest;
- Eigenvalues and eigenvectors ordering: responsible for the primary ordering of the data based on the real part of an eigenvalue;
- Extraction: subroutine used to extract eigenvalues and eigenvectors of conjugate complex pairs;
- Ordering check: subroutine used to check and reorder eigenvalues and eigenvectors, if necessary.

In short, this code makes a consistency calculation that involves eigenvalues and eigenvectors so that they are in an optimal permutation that guarantees the best sequence between the results.

After the selection of the aeroelastic modes, the code was updated to calculate the values for amplitude, phase, frequency and damping ratio. The first two quantities are extracted from the eigenvectors, while the two latter are from the eigenvalues. The frequency and damping ratio are relative to a specific mode and calculated by Equation 1 and 2, respectively. The phase and amplitude are obtained for each element in each member of the aircraft by Equations 3 and 4, respectively.

$$Frequency = \frac{\sqrt{\left({Re_{va}}^2 + {Im_{va}}^2\right)}}{2\pi} \quad (1)$$

$$Damping\ Ratio = \frac{-100\, Re_{va}}{\sqrt{\left({Re_{va}}^2 + {Im_{va}}^2\right)}} \quad (2)$$

$$Phase = \tan^{-1}\left(\frac{Im_{ve}}{Re_{ve}}\right) \qquad (3)$$

$$Amplitude = \sqrt{\left({Re_{ve}}^{2} + {Im_{ve}}^{2}\right)} \qquad (4)$$

where:

- Reva: eigenvalue real part;
- Imva: eigenvalue imaginary part;
- Reve: eigenvector real part;
- Imve: eigenvector imaginary part;

3. Results

Having all the results found, it is possible to plot the data. The frequency and damping ratio are related to the mode, whereas the phase and amplitude are associated with each element at each member (wings, horizontal tail or vertical tail). If a member with five elements is chosen, five phase curves will be shown in one graph and five amplitude curves in another graph.

The idea used to explain the physical mechanism was to find a flutter condition and understand what is happening in each part of the aircraft structure. Therefore, simulations with a highly flexible aircraft were performed. The airspeed range analyzed was between 100 and 600 m/s. More than 50 aeroelastic modes were obtained, but just three are presented. So, the three first aeroelastic modes were selected for the physical analysis. Figure 5 shows the frequency and damping ratio of these three modes analyzed.

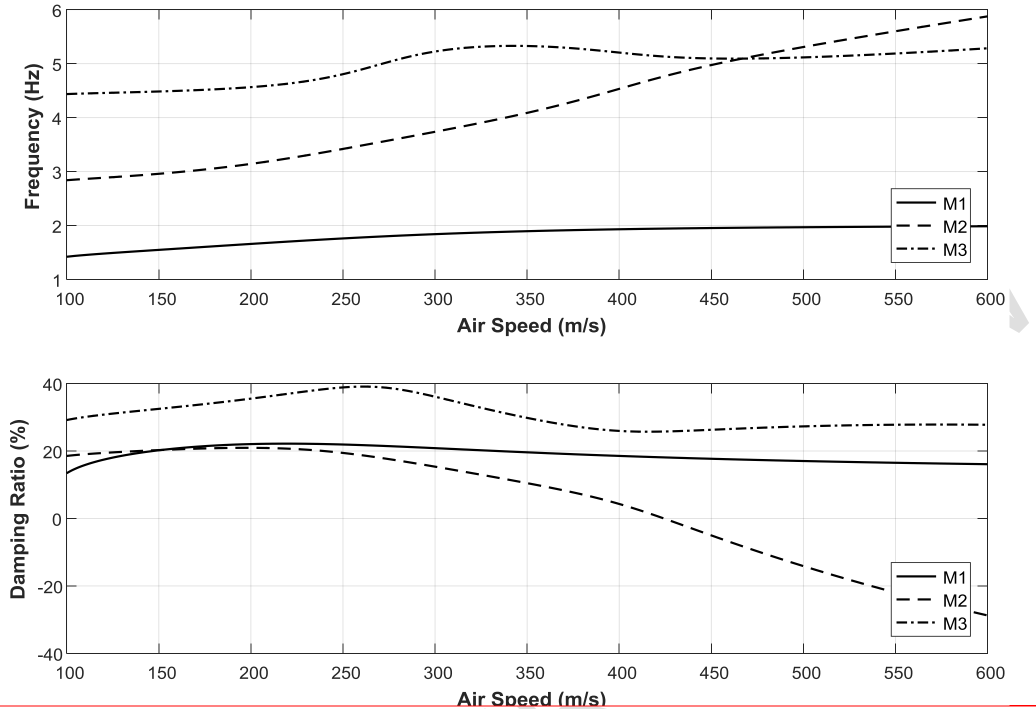
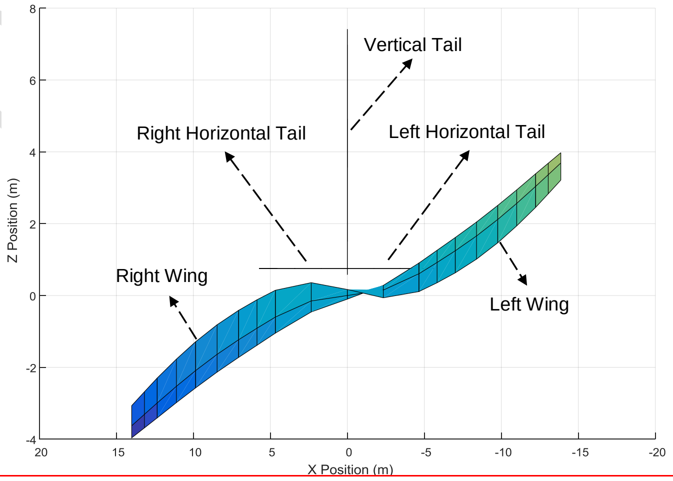
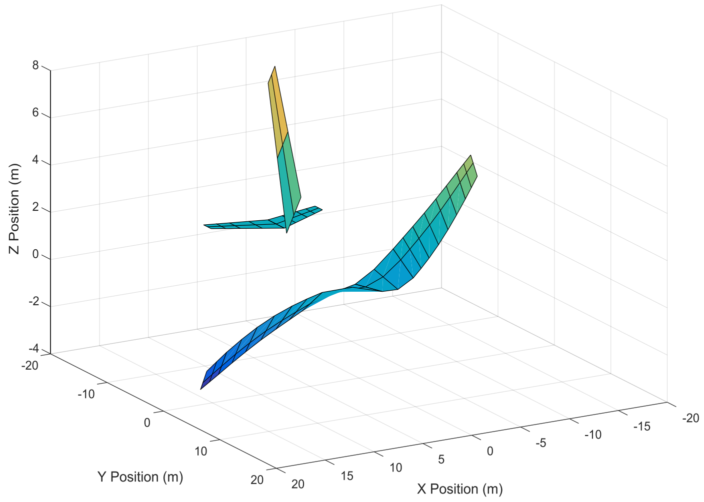


Figure 5 - Frequency and Damping Ratio of the aeroelastic modes.

It is clear from Figure 5 that the second mode, M2, is the one that has negative damping ratio and, consequently, flutter. Since this mode is the one which presented flutter, the intramodal analysis was totally focused on it. Figures 6 shows a 3D representation of the second mode. Both figures allow the visualization of one positive bending (Up) and negative twist on the left wing and positive bending (down) and positive twist on the right wing. Structural deformations on the horizontal and vertical tail were not observed.



(a)



(b)

Figure 6: (a) Front view and (b) perspective view of mode 2.

Figure 6 illustrates a qualitative format of the aeroelastic mode, demonstrating how a certain part of the aircraft deforms in relation to the equilibrium calculated at the trimmed condition. It is possible to observe the two wings, the horizontal and vertical tail. They do not consider the phase lag present between the elements. The fuselage is not illustrated because the numerical model considers it as a rigid unit. Figure 7 shows the amplitudes and phases for both components, torsion kx and bending ky, for mode 02 and specifically for the right wing, what is analogous to the left one. There are five curves in each chart because each wing is composed of five elements. Element 1 is the first wing element attached to the fuselage. Element 5 is on the wing tip, and Elements 2, 3 and 4 are intermediary (See Fig.2). The engine is attached to the Element 2 (Siqueira, 2019).

Based on Figure 7(a), it is noted that the phase values of torsion depart slightly above zero in ascending order and tend to have the same negative phase value (around -10º). When this value is reached, there is a small leap around 225 m/s and a moderate one near to 250 m/s. After these leaps, the values were increased until the flutter velocity, around 425 m/s. The curves of torsion amplitudes have the same behavior among themselves and increase the values until 225 m/s. Reaching the maximum, the torsion amplitudes started to decay until the flutter airspeed.

The bending phases, shown in Figure 7(c), began around 0º and had a sudden variation of 180º around 200 m/s, the same velocity in which the minimum amplitude value was achieved. The bending phase values had a considerable changing and presented a smooth variation after the rapid changing. The element 01 did not present this variation, possibly because this element is attached to the fuselage. The element 02 had a different behaviour when compared with the elements03, 04 and 05, what can be explained by the presence of the engine in this element (Sousa, 2017).

The flutter is characterized by the signal change in the damping ratio (positive to negative). This is equivalent to say that a signal change occurs in the corresponding eigenvalue.

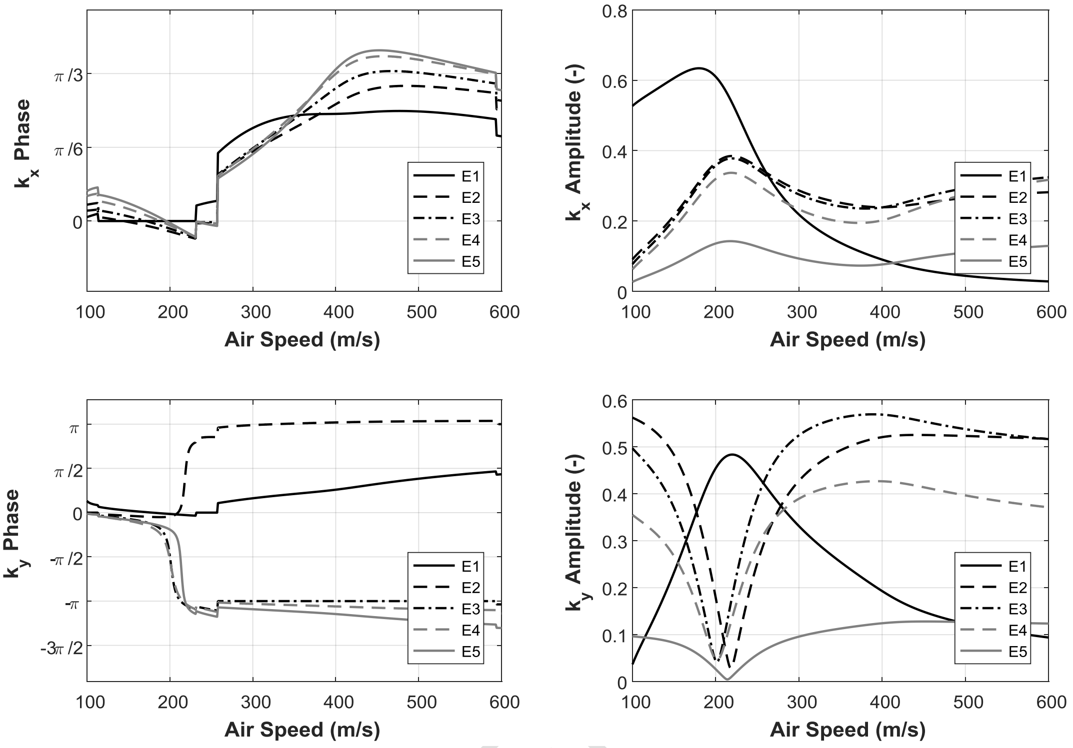


**Figure 7: (a) Phase for torsion kx, (b) amplitude for torsion kx (b),
(c) phase for bending ky, (d) amplitude for bending ky.**

It is evident by intramodal analysis that the torsion and bending presented a phase difference. Phase difference plays a fundamental role in the amount and direction of energy flux between the aerodynamic flow and the structure (Patil, 2001). This means that the mismatch between bending and torsion creates conditions for a flutter. Before presenting more results, it would be convenient to remember some basic physical concepts, that can be useful. Once the airplane receives one external aerodynamic perturbation, some aerodynamic forces and moments can be produced. Example: gusts produce modifications on the wing lift force (Fig.8).

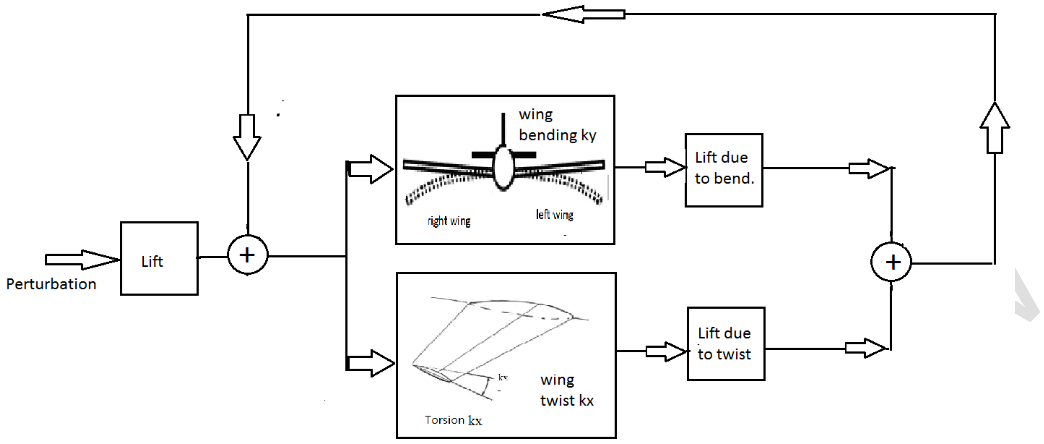


Figure 8 – Mechanism for amplification or damping of structural deformations

This modification on lift force (called hera as delta lift force) can produce structural deformations as wing bending and torsion. If the bending deformation has one phase difference smaller that 180 deg, the wing will bend Up, and if the twist has one phase lower than 180 deg, the wing will twist Up (leading edge Up). The consequence is different angles of attack, that will change the lift force.If the wing bending is Up, the local angle of attack will decrease, and the wing lift also (See Fig. 9). So, the bending reaction has one stabilizing effect (it decreases the initial delta lift force, and, as consequence it can dampen the structural oscilations). The bending act as one dynamic damping. It the wing twist Up, the angle of attack will increase, and the wing lift also. So, the wing twist has one destabilizing effect, when the aerodynamic lift force is ahead/ forward than the flexural axis. The angle of attack would increase more, and so, the delta lift force.

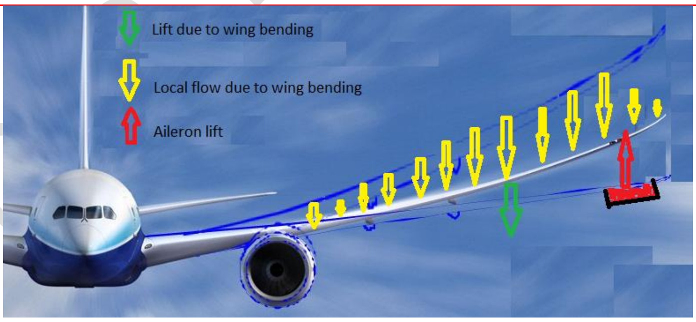


Figure 9 – Damping in external perturbations due to bending deformations (Sousa, 2013, Quora, 2016).

The situation can be different when **there are phase differences between the twist and bending** (See Fig.10).Both deformations acting together can have one destabilizing effect. The positive delta lift force due to the bending act at the same time as the positive delta lift force due to the torsion. Fig. 8-10 and the associated comments were presented in order to allow one better comprehension of these deformations and its effects.

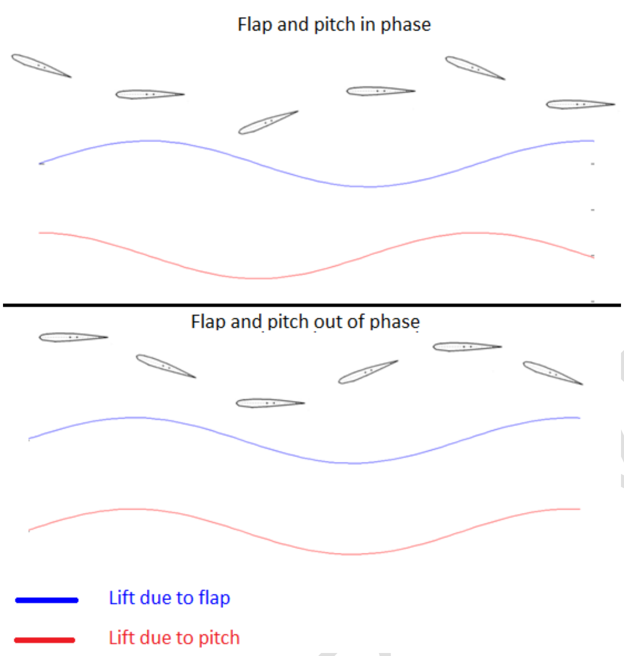


Figure 10– Example of phase difference between the pitch (twist) and flap (bending)

Before moving on, it is advisable to better comprehend the concept and meaning of phase difference. Concept: If it is considered one system under periodic perturbation, ex: sinusoidal input, the phase shows the time instant in which the system will respond to the external perturbation. This idea is commonly understood, but its meaning on aeroelastic analysis were not detailed understood during this study. If one plot of structural displacement (xo) versus the derivative of the displacement (relative to the time) (vo), divided by the frequency wn, and if all the possible combinations are plotted, one circle can be obtained (see Fig.11). If one point of this circle is chosen, the angle obtained is the phase. The phase presents the relation between the structural displacement, and its derivative (Inman, 2014). Its derivative vo is called here as velocity vo.Once these comments and explanations were presented, the analysis done can be presented.

With the results presented for the second mode, it is possible to begin the proposition of the flutter physical mechanism. First, it is essential to point out that the analysis considers five hypotheses:

1ª. The velocity (vo)is responsible for generating the impulse of the movement (bending and/or torsion), i.e., the movement begins with the initial velocity if the element in question is not cantilevered (attached to one fixed point);

2ª. The components of the movement, bending or torsion, with greater velocity module leads the oscillatory movement;

3ª. Only the action of an initial force F is considered and not its variation ΔF generated in the movement over time;

4ª. The proportion of the components (xo and vo) is maintained when the frequency changes;

5ª. The analysis shown is also applied for cases with structural damping.

The aeroelastic mode is composed by torsion and bending deformations, with phase difference. The "in phase" designation is usually made when the angle between the components is 0°. And the "out-of-phase" motion is normally referenced by the literature (Wright and Cooper, 2007) by a 90º difference (or more). When the components are in phase, it means that the bending movement reaches the maximum amplitude almost simultaneously with the torsion motion. In the first quarter of the cycle, the lift produced and the displacement of the structure are positive and then, positive work is done on the airfoil. In the second quarter of the cycle, the lift is positive and the displacement is negative. So, negative work is done on the airfoil. The same occurs for the last two-quarters of the cycle. In short, work close to zero is done (Kakkavas, 1998). When the phase difference between the motions is 90°, the maximum and minimum of the bending and torsion motions are not reached at the same time. In the two first quarters of the cycles, both displacement and lift are positive. In the last half of the cycle, both magnitudes become negative. So, a work different to zero is done on the airfoil. The motions do not attempt to stabilize, but rather to reinforce themselves (Kakkavas, 1998). When the movement with a phase difference (between torsion and bending) of 180º is evaluated, the maximum amplitude of the bending movement is counterposed by the minimum torsion and vice-versa. Then, returning and summarizing the data presented previously for the second mode, seen in Figures 7 and 11:

- Torsion: the phase begins close to 10º at 100 m/s, then change the direction and become approximately equal to -10º at 225 m/s. There are this leap and after that, the phase is increased until a maximum value around 60º is achieved at flutter speed, 425 m/s. The amplitude has a maximum value until 225 m/s and decays after this velocity;
- Bending: the phases were close to -20º before the phase leap, that occurs with the airspeed around 200 m/s. The phase values had a rapid changing and achieved a value close to ± 200º. After this leap, the phases had smooth behavior. The amplitudes of the elements 02, 03, 04 and 05 have a minimum value after the phase leap, that occurs with the airspeed close to 200 m/s. After this airspeed, the amplitudes start to increase.

The differences seen in the phase occurred practically at the same airspeed in which the amplitudes reach the maximum (torsion) and minimum value (bending). It is possible to subdivide the physical mechanism into two different moments: before and after the phase difference (phase leap). The behavior before and after the phase leap can be arranged like presented in Fig. 11.Each component has a velocity associated, which is found by the phase angle.

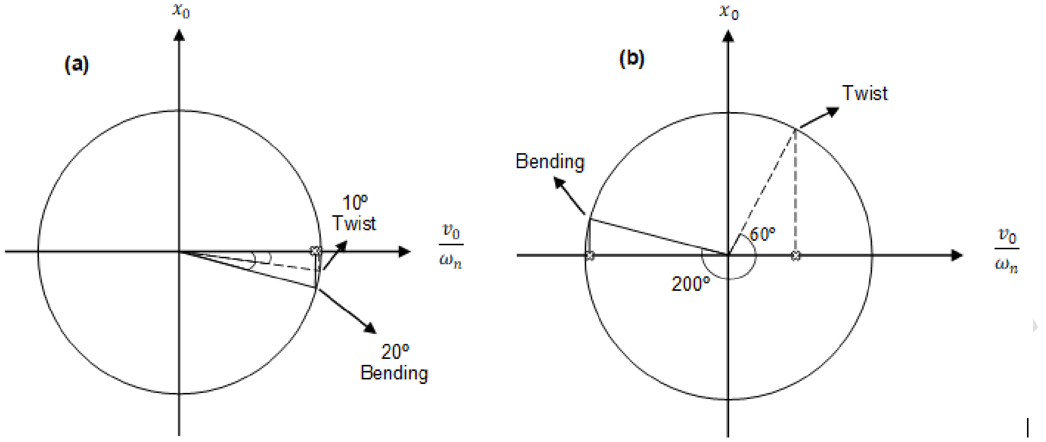


Figure 11 - Scheme for the components (a) before the phase difference and (b) after the phase difference.

In Fig.11, the x axis presents the values of vo/ wn, where vo is the time derivative of structural displacement and wn is the natural frequency. The y axis presents the structural displacement x0. The angle presents the phase (Inman, 2014). It is noticed by the Fig. 11(a) that initially both movements were practically in phase. Despite the similarity, there is a small "advantage" in the speed of torsion motion. That is, this movement leads the behavior of the aeroelastic mode. However, as both components are in phase, the bending stabilizing effect helps to prevent instability. But, the stability mechanism is completely changed when the phase difference occurs, which is demonstrated in Figure 11(b). The bending inverts the direction of movement (i.e., alters in 180º the phase angle) and the torsion that had a small advantage before and was countered by the stability effects, now has a minor instability effect. The magnitude of the bending speed is practically the same, but acts in the opposite direction, which physically changes the system behavior. When the lift is positive, the bending is negative. This increases the angle of attack, that increases the lift. In addition to this, the torsion component has a reduction in velocity magnitude since the phase is around 60°. The twist deformations keep maintaining one destabilizing effect, but, now the bending has also one strong instability effect. Only the structural bending and twist stiffnesses help to avoid the instability until one defined airspeed value. This stabilizing effect decreases with the airspeed increment. The damping ratio presented on Fig.5 seems to corroborate with this explanation. While the torsion acts as the mechanism to destabilize the movement, the stiffness of the structure and the aerodynamic effect produced by the bending tend to counteract the instability created. Until the phase difference occurs, the bending and torsion components move together and stability is ensured by the superiority of the stability effects. After the phase difference (phase leap), the bending starts to act in the direction of instability. The effect of instability increases with the airspeed because the delta aerodynamic force (delta lif force) created is greater than that ocurred before the phase difference. This is maintained until the moment that the structure is unable to counteract the instability, so flutter occurs (See Fig.12).

Qualitatively, the idea emphasized by this new approach to explain the flutter mechanism is that the component that leads the movement captures more energy input provided by the force and thus leaves a smaller amount to be used by the other component. The bending motion is naturally stable when there is not the phase lag acting upon the system. Therefore, considering only the bending movement, since it is dominant in this case, if it is assumed a force acting upwards, instead of the structure goes upwards too, it goes downwards. So, the effect originated by the bending component tends to destabilize the movement because it causes an opposite movement in relation to the force. And, in addition, there are amplitude variations. After the phase leap, the bending component amplitude starts to grow and the amplitude of the torsion to decrease. Consequently, the effect of instability provoked by bending gains dimension with the airspeed and the damping ratio decreases with the speed until the moment the flutter starts. Assuming a lift force F generated by the action of the flow, if the movement of the components were in phase, the structural movement of torsion and bending would be instantaneous.

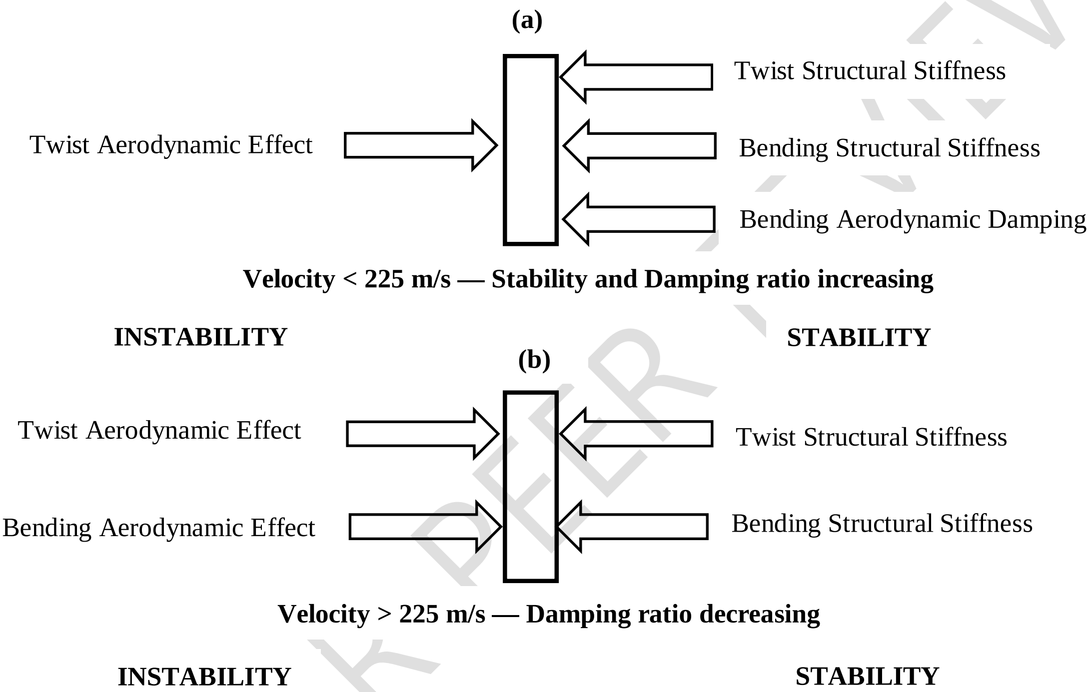


Figure 12:Instability versus Stability: (a) before the phase difference and (b) after the phase difference.

However, with the phase difference, a change in the structural behavior occurs, whereas there is a temporal lapse between the action of the force and the structure movement. The structure that previously moved upwards now is moving downwards and vice-versa. Although the movement is a combination of bending and torsion, the greater part of instability is directly associated with the bending mechanism. Figure 5 shows that the structural damping of mode 2 was approximately constant, close to the airspeed of 200 m/s and began to decrease close to 210 m/s. Figure 7 shows that the torsion phase significantly changed its value close to this speed. These facts seem to be related and the explanation presented seems reliable to the authors. There are important considerations that must be made in relation to the proposition of the physical mechanism. The first one is related to the need to improve/ extend the physical proposition for all the possible cases obtained, which includes other aircraft models. There is the necessity to create a general rule for the physical explanation, possibly incorporating other effects, like the damping ratio. The improvements should be incorporated in future analysis. Although the proposition fits well to mode 2, which presented flutter, element 01 presented a distinction of results when compared to the other elements. This fact cannot be ruled out and prove the urge to evaluate this cantilevered element (element attached to the fuselage). Another research that should be developed is the evaluation of the engine effect on the structure, because the element 02 showed a difference on its phase.

4. Conclusion

In this article a new explanation for the flutter mechanism in aircraft has been proposed. The explanation relies on information on the damping, frequency, amplitudes and relative phases of the torsional and bending movements of each aeroelastic mode, and in each structural element. The use of these parameters has been known and done for a long time. Many aeroelastic stability analyses depend on knowing the structural modes and the frequency and damping curves along the airspeed. The difference here lies in proposing the dissociation of each mode into torsional and bending movements. And in the analysis of each of these movements, as if each were a sub-mode within the structural modes already known. Here we can already use the term **intramodal analysis** (within modes). This is a novelty in this work. This dissociation was made possible by the use of the NFNS_s approach. And the use of this approach for this type of analysis was another contribution. The form of analysis proposed was only a first step in the development of an additional form in the study of the aeroelastic stability of aircraft. It is by no means intended to replace the criteria and form of analysis developed throughout the 20th century, but rather to provide a new tool and way of looking at such analysis.The methodology proposed here should be tested on other aircraft, and further research should be carried out to validate the hypotheses presented here. Also, the effect of structural damping should also be considered in future studies.The authors of this paper believe that these are valuable and important research topics for specialists/researchers in aeroelasticity.

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