Analysis and forecasting of consumer price index (CPI) in Kenya and South Africa using Holt Winter Model

Original Article

Abstract

In this paper, Holt-Winters exponential smoothing approach is applied to model and forecast monthly CPI in Kenya and South Africa. Monthly data from January 2000 to December 2023 obtained from Central Bank of Kenya and South Africa department of statistics. Time series decomposition showed that the trend component is the most dominant in both countries. Kenya Holt-Winters estimated model has parameters 0.6756, 0.0077 and 1 for level smoothing, trend smoothing and seasonal smoothing respectively. On the other hand, South Africa estimated model has parameters 0.8917, 0.1057 and 1 for level smoothing, trend smoothing and seasonal smoothing respectively. The estimated models are efficient and effective as on average the fitted values are less than one percent off the observed values. The initial values for level smoothing, trend smoothing and seasonal smoothing are approximately equal in both countries. The estimated models are used to predict CPI in the year 2024 months, where an rise in CPI is expected. Over the forecast period, South Africa will experience a lower index as compared to Kenya. In both countries, it's expected that monthly CPI will rise over the forecast period.

Keywords:Holt-Winters; Kenya; South Africa; CPI; Forecasting.

1 Introduction

Consumer price index (CPI) is a measure of consumer price weighted aggregate change based on a representative basket of goods such as food fuel etc and services within an economy over time. CPI is an index, which is constructed by comparing aggregate costs with the costs of the same basket in a selected base period price. CPI is used to calculate inflation, which is defined as the percentage rate of change in CPI over one-year period. CPI is important macroeconomic variable that has interactions with other economic variables. CPI and purchasing power parity (PPP) are two price statistics that interact. PPP is a spatial price index while CPI is a temporal price index. There exist a cointegration relationship between the CPI and PPP with a short-run dynamic relationship caused by CPI on PPP equilibrium status. On the other hand, PPP exhibits price pass-through effect onto CPI (Chen and Hu [1]). Cost of living is the expense incurred by a given household to buy good and services in order to maintain a certain standard of living. Increase in CPI affects cost of living by inducing an upward pressure (Jacobs et al. [2]). Empirical studies have shown bidirectional causality between inflation and different money proxies. The causal link from inflation to money supply is greater than the causal link from money supply to inflation (Göçmen [3]). There exist a long-run relationship between inflation and lending rates, with inflation having a significant positive effect both in short term and long term (Nitescu and Anghel [4]). Returns on stocks and bonds are significantly impacted in negative direction by steady high rate of inflation (Feldstein [5]). CPI also affect several economic activities. Business firms CPI estimates to establish good and services prices as well as plan production schedule. The aim of this study is to model monthly CPI in Kenya and South Africa, compare the two countries performance and predict values for year 2024 and 2025.

Several techniques have been applied to model and forecast CPI in different countries, country blocks or regions within a country. These techniques includes; ARIMA analysis, exponential smoothing techniques and multivariate regression. Espasa et al. [6] carried out a disaggregated study of monthly CPI in US. The CPI component considered were food, energy, rest of commodities and services. The study proved that food, services and rest of commodities are of integration order 2 ie I(2) while energy is of integration order one ie I(1). Contegration analysis of I(2) variables indicated there are several sources of non stationary in the CPI component. Jere and Siyanga [7] used Holt's exponential smoothing to forecast inflation rate in Zambia using monthly data from May 2010 to May 2014. A Holt's model with parameter α (0.988) and β (0.04123) was the optimal fit over the sample period. Kuhe and Egemba [8] analyzed annual CPI in Nigeria for the sample period 1950 to 2014 using univariate ARIMA technique. They study noted that CPI is non-stationary in level, but stationary after the first difference. ARIMA (3,1,0) best describes CPI data modeling in Nigeria for the sample under study. The forecast of CPI for a period of six years from 2015 also showed a steady increase.

Norbert et al. [9] modeled CPI using Box and Jenkins methodology in Rwanda. The data was well fitted by an ARIMA (4,1,6). Twelve forecast values were estimated for the year 2016, which showed an expected continuous increase in CPI. Lidiema [10] compared SARIMA and Holt-Winters exponential smoothing in analyzing Kenya's inflation rate from November 2011 to October 2016. The study proved that SARIMA (1,1,0)$(1,0,0)_{12}$ was a better fit in forecasting inflation rate. Gjika et al. [11] studied CPI in Albania using time series and multiregression technique. The time series models was used to forecast CPI while the multiregression was used to simulate forecast for macroeconomic sub component with significant correlation with CPI. Number of Albanian traveling abroad and exchange rate had a significant positive relationship with CPI. The study observed that the time series model forecast were more satisfactory in predicting short term CPI. Mia et al. [12] analyzed annual CPI in Bangladesh from 1986 to 2018 using econometric models. The study provided evidence of CPI being of integration order two ie I(2). An ARIMA (2,2,0) best fitted the dataset and was used to forecast 2019 to 2025 CPI values. CPI values are expected to continue increasing with time.

Muhammed et al. [13] used exponential smoothing method to model and forecast Nigeria CPI using monthly data from January 1995 to December 2017. A double exponential smoothing model with parameters $\alpha(0.933)$ and $\beta(0.07)$ best fitted the data set. In the year 2018, the forecast values showed a gradual increase over time. Mwanga [14] used ARIMA technique to analyze Monthly CPI in Uganda from 2010 to 2020. The study provided evidence showing monthly CPI has integration order one ie(1). A Seasonal ARIMA (1,1,1)$(0,1,1)_{12}$ without a constant best fit the data. The fitted model was used to estimate CPI forecast for the next twelve months where a fluctuation between 4.7 and 6 percent was projected. Mohamed [15] studied Somaliland monthly CPI from 2013 to 2020 using ARIMA approach. The most suitable model for the modelling CPI was ARIMA (0,1,3). The CPI forecast's predicted an upward trend in coming years. Ibrahim and Olagunju [16] conducted a study to model monthly CPI in Nigeria for the period 2009 to 2019 using ARIMA modeling approach. CPI was observed to be integrated of order one and ARIMA (2,1,2) best fitted the data set. The study predicted a long upward memory trend in forecast values.

Franz et al. [17] compared time-varying parameter VAR, factor-augmented VAR and structural break models to analyze core inflation in South Africa using quarterly data from 1981Q1 to 2013Q4.

The study showed time varying parameter model having consistence performance over constant coefficient model. Muthu [18] studied CPI for selected factors in India rural and urban areas from 2013 to 2020. The selected factors were education,fuel and light, tobacco and intoxicants, and food and beverages. A seasonal ARIMA model proved to be the most suitable model for predicting CPI at disaggregated level. Konarasinghe [19] modeled monthly CPI in Malaysia using exponential smoother approach for the sample period 2012 to 2022. The CPI forecasts from double exponential smoothing (DES) were more suitable as compared to Winter additive and multiplicative models. The model provided evidence of an increasing trend in CPI. Gasper and Ernest [20] used Holt Winter's approach to model monthly CPI for the period from 2010 to 2022 in Tanzania. A Holt model with parameters 0.9,0.12, and 0.003 for level, trend and seasonal respectively best fit the CPI data.

3 Methodology

3.1 Data

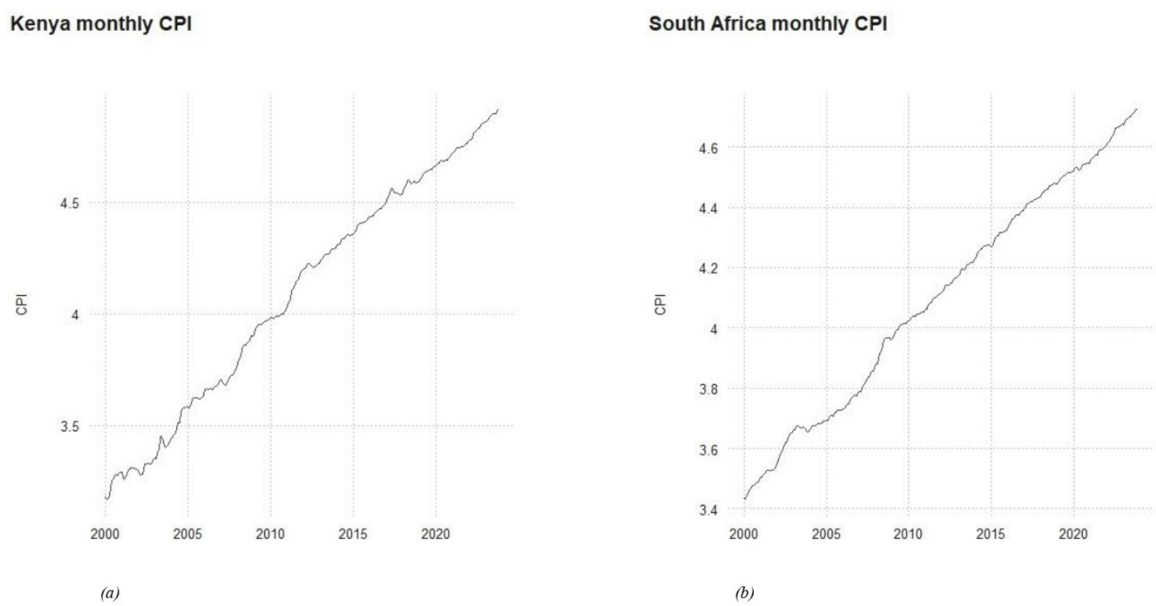


Figure 1: Consumer price index plot

This study use monthly consumer price index (CPI) data form January 2000 to December 2023. The data was obtained from Central Bank of Kenya economic review reports and South Africa department of Statistic. Kenya's CPI has a mean of 68.21 and standard deviation of 32.97. On the other hand, South Africa's CPI has a mean of 64.70 and a standard deviation of 23.51. A log transformation was performed to reduce the effect of outliers and make the variance constant. A plot of monthly CPI are as shown in Figure 1, which shows a continuous increase over time in both countries.

3.2 Model

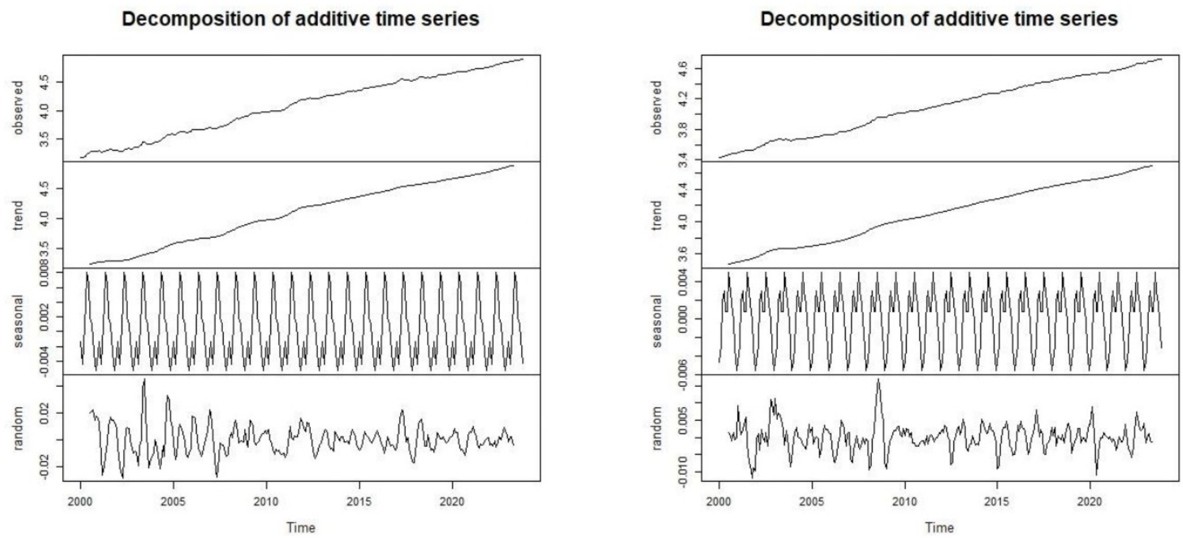
3.2.1 Decomposition

Time series data are mostly composed of three major components, namely; trend-cycle component, seasonal component and random component. Decomposition is a technique of splitting a time series into it's several components, representing the underlying category pattern. Assuming additive decomposition, a time series is represented as

$$y_t = S_t + T_t + R_t \tag{3.1}$$

Where T_t is the trend component, S_t is the seasonal component and R_t is the random component. Figure 2 (a) and (b) shows the different underlying components of Kenya and South Africa CPI respectively.

There is a continuous increase in trend component in Kenya and South Africa. Trend is the most dominant component in both countries, with the highest scale of approximately 1.0 and 0.8 for Kenya and South Africa respectively.



(a) *(b)*

Figure 2: Time series decomposition

3.2.2 Holt winter model

Exponential smoothing is a process of assigning exponential decreasing weights on observed data while fitting a model. There are three major exponential smoothing techniques, namely; single exponential smoothing, double exponential smoothing (Holt) and triple exponential smoothing (Holt-Winter's).

Single exponential smoothing

Under single exponential approach, the smoothing is produced by assigning weighted average of past observations a smoothing parameter, with the parameter decaying exponentially as the observation get older. This implies that new observations get relatively more weight in the average calculation than older observations. Consider a time series y_t , then let it's exponential smoothing output be s_t, which is an estimate for non-stationary mean of the time series at time t. Setting the initial condition:

$$s_0 = y_0$$

Then,

$$s_t = \alpha y_t + (1-\alpha)s_{t-1} \tag{3.2}$$

where s_{t-1} is the previous smoothed statistics and α $(0 \leq \alpha \leq 1)$ is a smoothing parameter.

Holt's exponential smoothing

Double exponential smoothing is an extension of single exponential smoothing that incorporates trend component. There are two smoothing components, namely; level smoothing output denoted by s_t and trend output denoted b_t. The trend component captures the rate of change and the direction of the time series y_t. Setting the initial conditions:

$$s_0 = y_0$$

$$b_o = y_1 - y_0$$

Then;

$$s_t = \alpha y_t + (1-\alpha)(s_{t-1} + b_{t-1}) \tag{3.3}$$

$$b_t = \beta(s_t - s_{t-1}) + (1-\beta)b_{t-1} \tag{3.4}$$

where s_{t-1} previous level smoothed statistics, b_{t-1} are previous trend smoothed statistics, β $(0 \leq \beta \leq 1)$ is a trend smoothing parameter and α $(0 \leq \alpha \leq 1)$ is a level smoothing parameter.

Holt-Winter's exponential smoothing

Holt-Winters (Holt [21] and Winters [22]) exponential smoothing techniques extends double exponential smoothing by including a seasonality component. In this approach, there are three output, namely: level smoothing output s_t, trend output b_t and seasonal output c_t. However, the trend and seasonal component can either be multiplicative or additive.

For a given time series y_t with seasonal period length p, the initial conditions are;

$$s_0 = y_0$$

$$b_0 = \frac{1}{p}\sum_{k=1}^{p}\frac{y_{p+k} - y_k}{p}$$

$$c_k = y_k - s_0$$

For $k = 1, 2, \ldots, p$ is the seasonal periods and $p = 12$ for this study. Then, the additive Holt-Winters model component are given as;

$$s_t = \alpha(y_t - c_{t-p}) + (1-\alpha)(s_{t-1} + b_{t-1}) \tag{3.5}$$

$$b_t = \beta(s_t - s_{t-1}) + (1-\beta)b_{t-1} \tag{3.6}$$

$$c_t = \gamma(y_t - s_t) + (1-\gamma)c_{t-p} \tag{3.7}$$

Where b_{t-1} are previous trend smoothed statistics, s_{t-1} previous level smoothed statistics, α $(0 \leq \alpha \leq 1)$ is a level smoothing parameter, β $(0 \leq \beta \leq 1)$ is a trend smoothing parameter and γ $(0 \leq \gamma \leq 1)$ is a seasonality smoothing parameter. The components of Holt-Winters under multiplicative scenarios are given as;

$$s_t = \alpha(\frac{y_t}{s_{t-p}}) + (1-\alpha)(s_{t-1} + b_{t-1}) \tag{3.8}$$

$$b_t = \beta(s_t - s_{t-1}) + (1-\beta)b_{t-1} \tag{3.9}$$

$$c_t = \gamma(\frac{y_t}{s_t}) + (1-\gamma)c_{t-p} \tag{3.10}$$

The parameters in Holt-Winters model are estimated by minimizing (Hyndman et al. [23])

$$L^*(\theta, Y_0) \tag{3.11}$$

where L^* is twice negative logarithm of conditional likelihood function without the constant terms, $\theta = (\alpha, \beta, \gamma))$ and $Y_0 = (s_0, b_0, c_0, c_{-1}, \ldots, c_{-p+1})$ are the initial conditions

Table 1 shows estimated Holt-Winters smoothing parameters and initial conditions coefficients $(s_0, b_0$ and c_k for$k = 1, 2, \ldots, 12)$ obtained from auto selection by stat package function called Holtwinter,in R software. The level smoothing parameter (α) is 0.6756 in Kenya and 0.8917 in South Africa. More weight is assigned to the latest CPI values in South Africa as compared to Kenya in the level smoothing output. The trend parameter (β) is 0.0077 and 0.1057 for Kenya and South Africa respectively. In both countries, the trend weight is assigned more to older values. The seasonal parameter (γ) is one in both countries. The initial conditions are approximately the equal in both countries.

3.2.3 Measures of Accuracy

To evaluate the accuracy of the forecast using additive Holt-Winter model, the error measure were considered, namely mean absolute error (MAE), root mean square error (RMSE) and mean average absolute error (MAPE). MAE is the average of absolute error of the forecast, given as;

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|e_i| \tag{3.12}$$

RSME shows how far predictions values are from the actual values by means of the Euclidean distance, given as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n}} \tag{3.13}$$

MAPE is used to tell how on average the forecast value is off from the actual values.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}|\frac{e_i}{y_i}| \tag{3.14}$$

Where n is the number of data values and e_i is the error for the ith data point. Table 2 shows the error measurements for both countries. MAPE for Kenya (0.2089) is higher compared to South Africa (0.0856). In both countries, on average the forecast values are less than 1% off the actual values. The fitted model for South Africa is more precise as compared to Kenya's model.

Table 1: Model estimation results

Smoothing parameters		
	Kenya	South Africa
α	0.6756	0.8917
β	0.0077	0.1057
γ	1	1
Coefficient		
	Kenya	South Africa
s_0	4.9288	4.7327
b_0	0.0057	0.0050
c_1	-0.0036	-0.0089
c_2	0.0011	-0.0024
c_3	0.0076	0.0036
c_4	0.0156	0.0044
c_5	0.0200	0.0022
c_6	0.0189	0.0022
c_7	0.0121	0.0070
c_8	0.0046	0.0038
c_9	0.0031	0.0016
c_{10}	0.0010	-0.0001
c_{11}	-0.0046	-0.0048
c_{12}	-0.0049	-0.0080

Table 2: Model error accuracy results

	RMSE	MAE	MAPE
Kenya	0.0112	0.0082	0.2089
South Africa	0.0048	0.0034	0.0856

3.2.4 Forecasting

Under additive Holt-Winters model, the exponential forecast for h steps ahead is given by the model (Nurhamidah et al. [24])

$$\hat{y}_{t+h|t} = s_t + hb_t + c_{t-p+h} \tag{3.15}$$

where s_t, b_t and c_t are as given in equations (3.5), (3.6) and (3.7) respectively

The forecast values of monthly CPI in Kenya and South Africa for the period 2024 and 2025 are as shown in table 3 and figure 3 (a) and (b) respectively. In both countries, it's expected that monthly CPI will continue with increasing trajectory over the forecast period. The expected CPI value for the month of December 2024 and 2025 in Kenya is 147.233 $\pm$ 8.232 and 157.598 $\pm$ 13.00 respectively. On the other hand, the expected CPI in December 2024 and 2025 in South Africa is 119.723 $\pm$ 5.680 and 127.184 $\pm$ 12.270 respectively. Over the forecast period, South Africa will experience a lower index as compared to Kenya.

Table 3: CPI forecast

	Kenya			South Africa		
Month	CPI	95% Lower bound	95% Upper bound	CPI	95% Lower bound	95% Upper bound
Jan 2024	138.504	135.501	141.573	113.169	112.108	114.241
Feb 2024	139.949	136.287	143.71	114.478	112.973	116.003
Mar 2024	141.661	137.408	146.046	115.747	113.83	117.696
Arp 2024	143.61	138.804	148.583	116.428	114.119	118.784
May 2024	145.07	139.757	150.584	116.758	114.066	119.514
Jun 2024	145.733	139.97	151.734	117.349	114.266	120.516
Jul 2024	145.57	139.412	152	118.516	115.018	122.12
Aug 2024	145.295	138.769	152.129	118.726	114.834	122.751
Sep 2024	145.906	138.988	153.168	119.069	114.771	123.528
Oct 2024	146.426	139.133	154.101	119.459	114.746	124.366
Nov 2024	146.444	138.812	154.495	119.503	114.381	124.854
Dec 2024	147.233	139.231	155.695	119.723	114.178	125.538
Jan 2025	148.254	139.545	157.506	120.222	114.187	126.575
Feb 2025	149.801	140.698	159.493	121.612	115.078	128.517
Mar 2025	151.634	142.119	161.786	122.96	115.912	130.436
Apr 2025	153.72	143.775	164.351	123.684	116.144	131.713
May 2025	155.282	144.941	166.361	124.034	116.016	132.607
Jun 2025	155.992	145.312	167.457	124.662	116.136	133.814
Jul 2025	155.817	144.862	167.601	125.901	116.813	135.697
Aug 2025	155.524	144.308	167.612	126.125	116.537	136.503
Sep 2025	156.177	144.635	168.641	126.489	116.381	137.475
Oct 2025	156.734	144.874	169.565	126.904	116.264	138.517
Nov 2025	156.753	144.619	169.905	126.95	115.802	139.172
Dec 2025	157.598	145.128	171.139	127.184	115.505	140.045

4 Discussion and conclusion

The study sort to model monthly CPI in Kenya and South Africa using Holt-Winters approach. There is evidence of continuous increase trend in CPI in both countries. From time series decomposition ,the trend component scale is the highest, approximately 1.0 and 0.8 in Kenya and South Africa respectively. Kenya Holt-Winters model has a level smoothing parameters equal to 0.6756, trend smoothing parameter equal to 0.0077 and season smoothing parameter equal to 1. On the other hand, South Africa has a level smoothing parameter equal to 0.8917, trend smoothing parameter equal to 0.1057 and a season smoothing parameter equal to 1. The estimated models are efficient and effective as on average the fitted values are less than 1% off the observed values. The initial conditions for Kenya are 4.928 and 0.0057 for level smoothing and trend smoothing respectively. In South Africa, the initial values are 4.7327 and 0.0050 for level smoothing and trend smoothing respectively. The initial values for the seasonal smoothing are approximately equal in both countries.

From the study, there is evidence to conclude that trend component is the dominant component

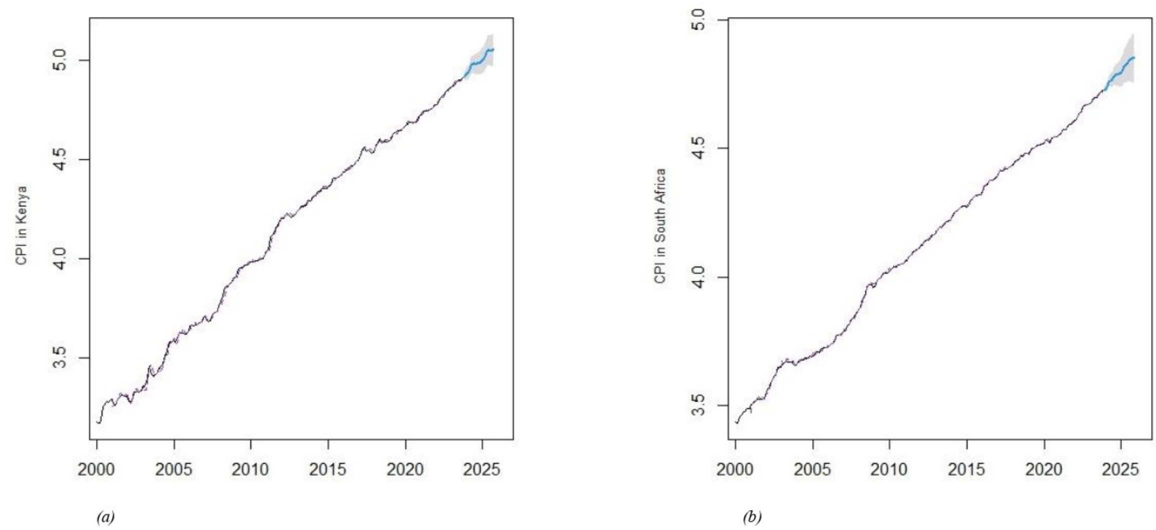


Figure 3: CPI forecast

of monthly CPI time series. Older CPI values have more weight determining in the trend component as compared to recent values. The expected CPI value for the month of December 2024 and 2025 in Kenya is 147.233 $\pm$ 8.232 and 157.598 $\pm$ 13.00 respectively. On the other hand, the expected CPI in December 2024 and 2025 in South Africa is 119.723 $\pm$ 5.680 and 127.184 $\pm$ 12.270 respectively. Over the forecast period, South Africa will experience a lower index as compared to Kenya. In both countries, it's expected that monthly CPI will rise over the forecast period. The implication of steady increase CPI will be increased cost of living (Jacobs et al. [2]), decline in money value (Göçmen [3]), increase in lending rates (Nitescu and Anghel [4]), low purchasing power (Chen and Hu [1]) and low returns from stock or bond investments (Feldstein [5]). Kenya and South Africa governments should employ fiscal measures targeting inflation and its interaction to manage the projected rise in CPI.

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