Review Article

Simulating Allergic Rhinitis infections in Adults with HIV-AIDS using Runge-Kutta Fourth Order Method in SIR Model

Comment [SAM1]: Simulating Allergic Rhinitis infections in Adults with HIV-AIDS usingFourth Order Runge-Kutta Method in SIR Model

ABSTRACT

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Simulating Allergic Rhinitis infections in Adults with HIV-AIDS using Runge-Kutta Fourth Order (RK4) Method in SIR Model is considered. Allergic rhinitis patients living with HIV-AIDS disease in adults is regarded as the target population. This condition is modeled through an SIR model with solutions solved using the RK4. The applied parameters and initial conditions in numerical simulations are carried out in this study in the literature. Application of the Runge-Kutta solutions provides the behaviour and prediction of the spread of allergic rhinitis infection for the patients living with HIV-AIDS disease in adults. The results suggest that after a given number of days, most of the population under investigation becomes infected with allergic rhinitis, which displaces the virus. Results are computed ad analyzed byMATLAB computer software.

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Keywords: Allergic rhinitis with HIV-AIDS, SIR model, Fourth-Order Runge-Kutta method.

1. INTRODUCTION

A set of explicit and implicit iterative techniques used in numerical analysis are known as the Runge–Kutta methods. Among them is the well-known Euler Method, which is applied to temporal discretization in order to obtain approximate solutions to ODEs [2]. These methods were developed about 1900 by two German mathematicians, Carl Runge and Wilhelm Kutta. Differential equations are used in today's environment and technology almost everywhere [14].

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HIV-positive individuals may exhibit respiratory symptoms as well as chronic pulmonary illnesses, such as asthma, cardiac dysfunction, and chronic obstructive pulmonary disease [10]. Longevity has been linked to an increased load of comorbidities, or "multimorbidity," including chronic disorders [11]. Known as HIV-associated non-AIDS, or "HANA," conditions, many of these comorbid disease@
s are not directly caused by HIV infection or immunosuppression; however, they are independently linked to HIV infection and may manifest more frequently, earlier, or more severely in HIV-positive individuals than in non-infected individuals. Human

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immunodeficiency virus (HIV) infection has been shown to increase a person's susceptibility to allergy reactions and infectious diseases. Numerous medical professionals have observed that chronic nasal symptoms, sinusitis, and otitis media are very typical issues as the number of HIV-positive individuals rises. [9].
These days, the majority of scientists and researchers use the SIR and SEIR models as an efficient mathematical method to forecast the global infectious rate of disease, particularly COVID-19, as well as the trend and spread of epidemics [6, [12], and [15].

Sneezing, nasal congestion, nasal itching, and rhinorrhea, in any combination, are symptoms of rhinorrhea, which is an inflammation of the nasal membranes that is most usually caused by allergic rhinitis [21]. The morbidity from allergic rhinitis can be substantial, even though the illness itself is not life-threatening (unless it is combined with severe asthma or anaphylaxis).

High levels of allergic antibody (IgE) are linked to HIV infection, particularly when CD4+ T-cell counts decline. Raising IgE levels can be linked to deteriorating immunodeficiency because of B-cell malfunction, although they are not always connected with allergies getting worse. IgE antibodies may be nonspecific or directed toward a variety of allergens [23]. These modifications might be brought on by an immune system imbalance, which can affect the body's natural defenses against allergies and, as a result, cause symptoms of allergic disease to manifest [22]. Additionally, a number of research reveal that, in comparison to non-HIV patients, individuals with HIV infection had higher frequencies of positive results on allergy skin tests [7].
Clinically significant pulmonary diseases are often noninfectious, however chronic pulmonary illness and respiratory symptoms are common in HIV-positive persons. Chronic obstructive pulmonary disease (COPD), asthma, gas exchange abnormalities (i.e., diffusing capacity impairment), and cardiopulmonary dysfunction are non-infectious pulmonary conditions that are common in HIV-positive individuals. HIV infection is linked to numerous of these disorders and is a separate risk factor for COPD [1], [16], [17].

Studies on adult HIV patients have reported a significant frequency of allergy disorders, particularly allergic rhinitis [8]. There are currently few reports of pediatric patients, and it is unclear how common allergies are among kids living with HIV/AIDS. Reducing morbidity and infectious consequences, the course of treatment won't be finished until it is determined that some symptoms are allergic in character.

Researchers [3] et al. conducted a comparison of filtering approaches for the modeling and retrospective forecasting of influenza epidemics. The best filter for any system of interest is determined by the model's nonlinearity and complexity, the quantity and quality of observations being entrained, and the intended use. The findings indicated that while the iterated filtering (MIF) and Markov chain Monte Carlo (pMCMC) do not perform as well for multimodal outbreaks, the ensemble filters and the basic particle filter (PF) are better at accurately recreating historical influenza incidence time series when using the humidity-forced susceptible-infectious-recovered-susceptible (SIRS) model.

[4] et al. investigated a numerical solution of the SIR model on the Runge-Kutta technique of tuberculosis transmission. The outcome was a generic solution using the fourth-order Runge-Kutta method for the SIR model of tuberculosis transmission. It was found that tuberculosis cases rose temporarily before declining, and that the fourth-order Runge-Kutta method can be used to anticipate transmission and should be taken into consideration for tuberculosis prevention.

A comparison was conducted [5] between the Euler Method and the 4th Order RungeKutta method for the numerical simulation of epidemiological models. A system of differential equations was developed in their method to comprehend the development of the coronavirus disease 2019 in order to comprehend the disease's dissemination inside a population. This was accomplished by solving the system using the SIR model and the fourth order Runge-Kutta and Euler methods. After solving it, it was discovered that the Euler technique outperforms the 4th order Runge-Kutta method in terms of speed.

This article will provide a brief overview of infectious lung disease in relation to HIV, describe respiratory symptoms that people with HIV experience, and review the literature on the most common chronic noninfectious lung diseases in HIV. The main focus of this article is antiretroviral therapy (ART) era data. In order to better prevent and treat chronic lung disease in this particular population, we will go over risk factors, the pathophysiology of the disease, the data that is currently available describing the differences in disease characteristics between HIV-infected and HIV-uninfected groups, gaps in the literature, and opportunities for future research.

2. MATHEMATICAL FORMULATION

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Considering one popular and efficient technique for resolving differential equation initial-value problems is the Runge-Kutta method, which builds precise numerical methods at high orders without requiring the usage of high order function derivatives.

The SIR model's primary presumptions are as follows:

1. The total population stays constant.
2. The rate of infectious agents is directly related to the susceptible.
3. The percentage of infected people who live or die is always the same.

Consider the initial value problem;

$\frac{dy}{dt} = f(t, y)$, where $y(t_0) = y_0$ (1)

$y(t)$ is unknown function

Using iterative formula, equation (1) is solved as follows

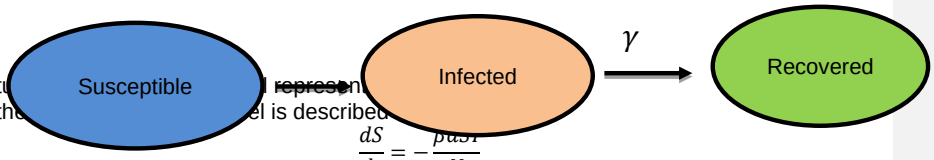
$y_{n+1} = y_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ where

$$k_1 = f(t_n, y_n)$$
$$k_2 = f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{k_1 \Delta t}{2}\right)$$
$$k_3 = f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{k_2 \Delta t}{2}\right)$$
$$k_4 = f(t_n + \Delta t, y_n + k_3 \Delta t)$$

$t_{n+1} = t_n + \Delta t$, $n = 1,2,3, \ldots$ (2)

To describe the spread of the Allergic Rhinitis disease as an SIR model, Roda et al (2020) model that contains three components;

a. Susceptible population denoted by $S(t)$ at time t
b. Infected population denoted by $I(t)$ at time t and
c. $R(t)$ represents Recovered population at time t



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$$\frac{dS}{dt} = -\frac{\beta dSI}{N}$$

$$\frac{dI}{dt} = \frac{\beta dSI}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I \quad (3)$$

Where β is the the transmission rate, γ is the recovery rate, N is the fixed population. Since population is fixed, $\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$. Consequently, just two of the three ODEs are sufficient to solve the ODEs and are independent.

3. NUMERICAL TECHNIQUE

The SIR model's ordinary differential equations (ODE) system served as the foundation for the development of this model. Since the RungeKutta 4th Order approach yields more precise results than Euler's approach does, it must be used in lieu of the other method in the SIR Model. [5].

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Using iterative formula, equations (2) and (3) is written as;

$$S_{n+1} = S_n + \frac{\Delta t}{6}(k_1^S + 2k_2^S + 2k_3^S + k_4^S)$$

$$k_1^S = f(t_n, S_n, I_n) = -\frac{\beta S_n I_n}{N}$$

$$k_2^S = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^S \Delta t}{2}\right) = -\frac{\beta}{N}\left(S_n + \frac{k_1^S \Delta t}{2}\right)\left(I_n + \frac{k_1^S \Delta t}{2}\right)$$

$$k_3^S = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^S \Delta t}{2}\right) = -\frac{\beta}{N}\left(S_n + \frac{k_2^S \Delta t}{2}\right)\left(I_n + \frac{k_2^S \Delta t}{2}\right)$$

$$k_4^S = f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t) = -\frac{\beta}{N}(S_n + k_3^S \Delta t)(I_n + k_3^I \Delta t) \quad (4)$$

$$I_{n+1} = I_n + \frac{\Delta t}{6}(k_1^I + 2k_2^I + 2k_3^I + k_4^I)$$

$$k_1^I = f(t_n, S_n, I_n) = \frac{\beta S_n I_n}{N} - \gamma I_n$$

$$k_2^I = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}\right) = \frac{\beta}{N}\left(S_n + \frac{k_1^S \Delta t}{2}\right)\left(I_n + \frac{k_1^I \Delta t}{2}\right) - \gamma\left(I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$k_3^I = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}\right) = \frac{\beta}{N}\left(S_n + \frac{k_2^S \Delta t}{2}\right)\left(I_n + \frac{k_2^I \Delta t}{2}\right) - \gamma\left(I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$k_4^I = f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t) = \frac{\beta}{N}(S_n + k_3^S \Delta t)(I_n + k_3^I \Delta t) - \gamma(I_n + k_3^I \Delta) \quad (5)$$

$$R_{n+1} = R_n + \frac{\Delta t}{6}(k_1^R + 2k_2^R + 2k_3^R + k_4^R)$$

$$k_1^R = f(t_n, I_n) = \gamma I_n$$

$$k_2^R = f\left(t_n + \frac{\Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}\right) = \gamma\left(I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$k_3^R = f\left(t_n + \frac{\Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}\right) = \gamma\left(I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$k_4^R = f(t_n + \Delta t, I_n + k_3^I \Delta t) = \gamma(I_n + k_3^I \Delta t) \quad (6)$$

Here, $R(t)$ is computed as $R(t) = N - S(t) - I(t)$ only using the iterative formula for $S(t)$ and $I(t)$ are used and
Through this process, differential equations can be obtained and through the solutions to these equations, the model's corresponding outputs can be obtained, such as the incidence of infection, the frequecy of infection and the basic reproductive number (R0). After the model is constructed, its outputs should be fitted to the reported numbers from the actual epidemic. The SIR model should be modified given the variation in factors associated with HIV/AIDS transmission [18], [19]. Dynamic models can provide a comprehensive understanding of the epidemic process through simulation and can reflect its transmission by obtaining population-level outcomes from individual-level inputs [20].

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4. RESULTS AND DISCUSSION

The impact of values of the parameters in this model is observed and investigated. The simulation results indicate that one parameter that needs to be taken into account when tracking the spread of allergic rhinitis is the infection rate, β. R(t), which stands for Removed or Recovered, is the total number of people who have either been removed (and are immune) or have passed away. Infected people are those who have been infected and have the potential to infect susceptible people. [13].
Further complicating the assessment, patients living with HIV-AIDS infected populations are enriched for confounders of the relationship between infection and Allergic rhinitis

Figure (1) shows the parameters that define the rates at which individual transition between the different compartments of the model occur. For instance, beta_HIV/AIDS and beta_HIV Allergic Rhinitis represent rates at which individual transition between the different. The initial condition takes the initial population sizes for each compartment of the model. Time intervals and step size variable defines the time interval for which the simulation will run with the rates of change for each compartment as a function of time (t) and the current population values (x). Therefore, applying the ode45 function to numerically solve the system of differential equations defined by RK4. The resulting plot visualizes the changes in population sizes for each compartment over time, providing insights into the spread and recovery Allergic rhinitis patients living with HIV/AIDS diseases based on the specified model parameters.

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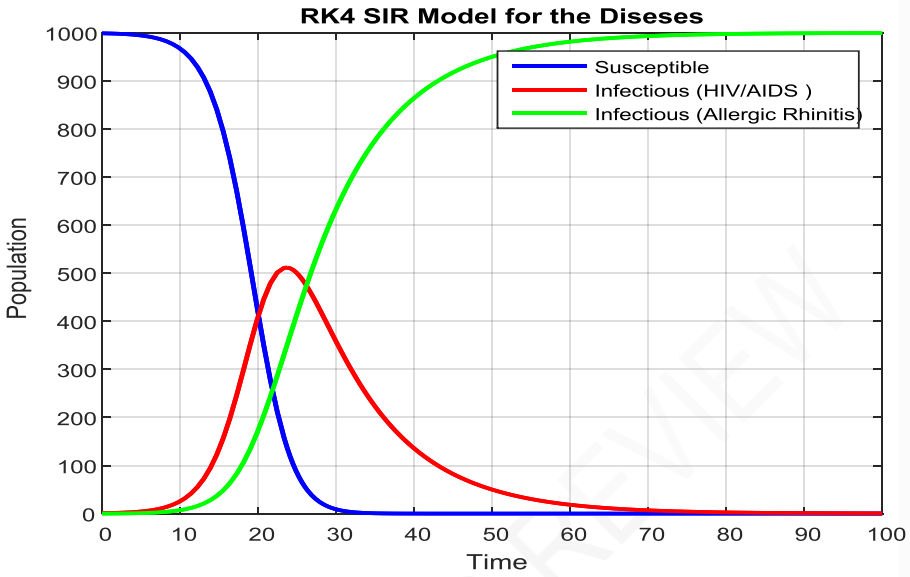

Figure 1: Transmission rate parameters

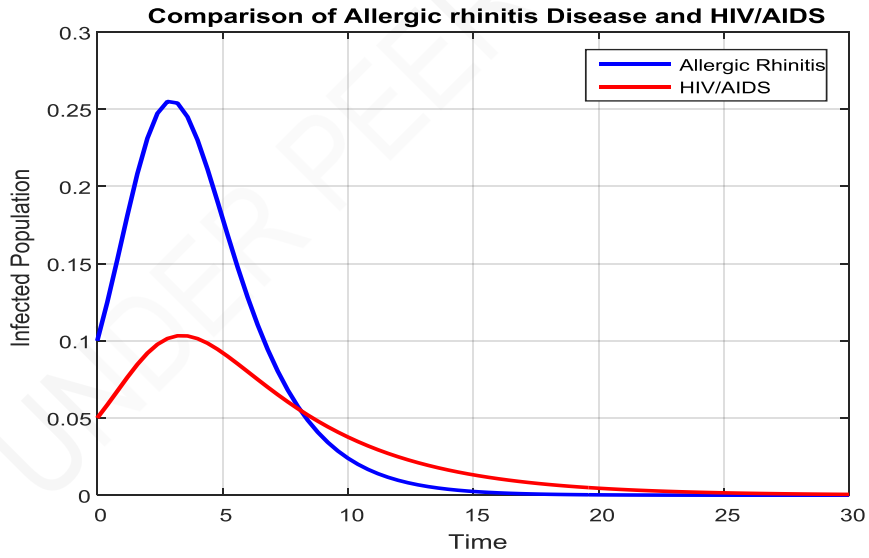

Figure 2: Simulate the spread and recovery of the dresses

The population dynamics for the infected individuals are plotted on the same graph, allowing for a visual comparison between the two diseases. Here a description on the spread of infectious diseases within a population is done. For instance beta_ Allergic rhinitis represents the probability of disease transmission from an infected individual to a susceptible individual per unit time.

Gamma_ Allergic rhinitis represents the rate at which infected individuals recover from the disease and become immune. Similarly, beta_ HIV/AIDS represents the probability of HIV transmission from an infected individual to a susceptible individual per unit time and gamma_ HIV/AIDS represents the rate at which infected individual's progress to AIDS from the infection. The susceptible population is updated based on the transmission rates and the number of infected individuals in both diseases, likewise, the infected populations are updated based on the transmission rates and the number of susceptible individuals. They are reduced based on the respective recovery rates. The recovered population is updated based on the number of individuals who have recovered from Allergic rhinitis disease.

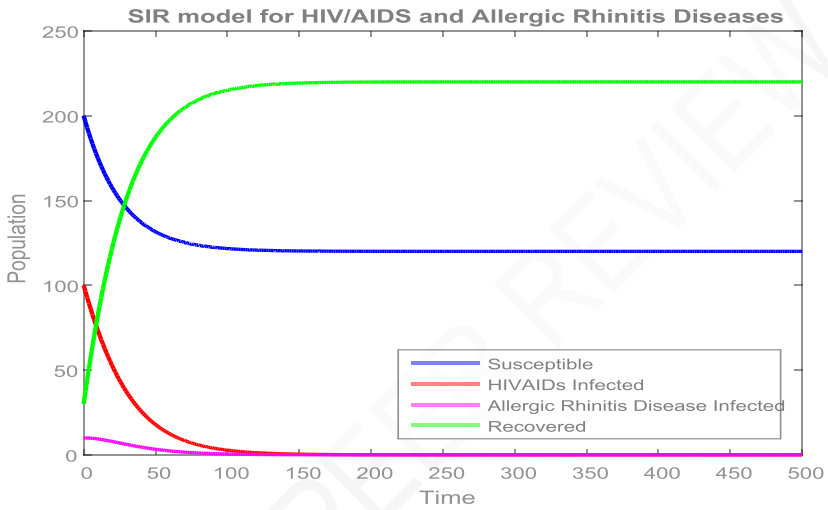

Figure 3: Parameters of the diseases

Figure (3) defines parameters and implements a basic SIR model for two diseases: HIV and respiratory disease. Parameters beta_ Allergic rhinitis and beta_ HIV/AIDS transmission rate represents the rate at which susceptible individuals become infected with Allergic rhinitis when they come into contact with infected individuals. Similarly, gamma_ Allergic rhinitis and gamma_ HIV/AIDS recovery represents the rate at which infected individuals recover from Allergic rhinitis and become immune to initial conditions. Also S0 representing the initial susceptible population or individuals who are susceptible to both diseases at the beginning of the simulation at a time intervals and step size.
The four lines representing the susceptible, Allergic rhinitis patients living with HIV-AIDS and recovered and those recovered from the Allergic rhinitis.

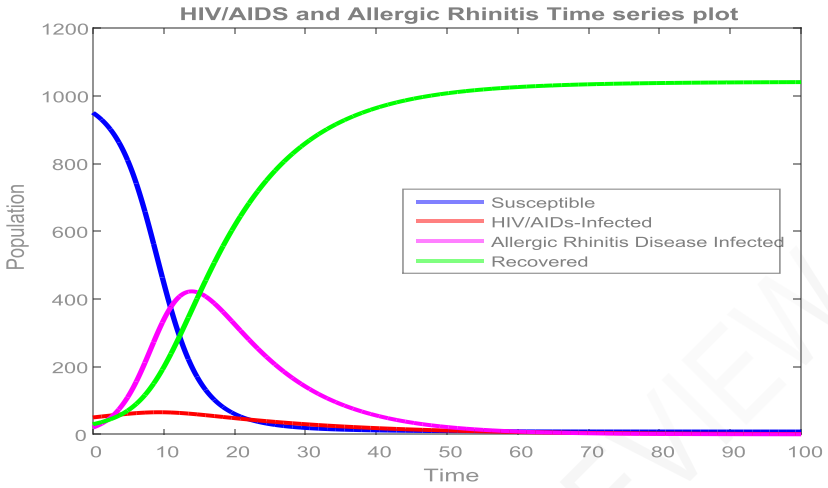

Figure 4: Time series plot for the diseases

Generally Figure (4) and (5) shows a time series plot that is a graphical representation of data that is collected over a sequence of time intervals as an andication on a particular variable or set of variables changes over time.

Figure 5: SIR Model for Time series plot for the diseases

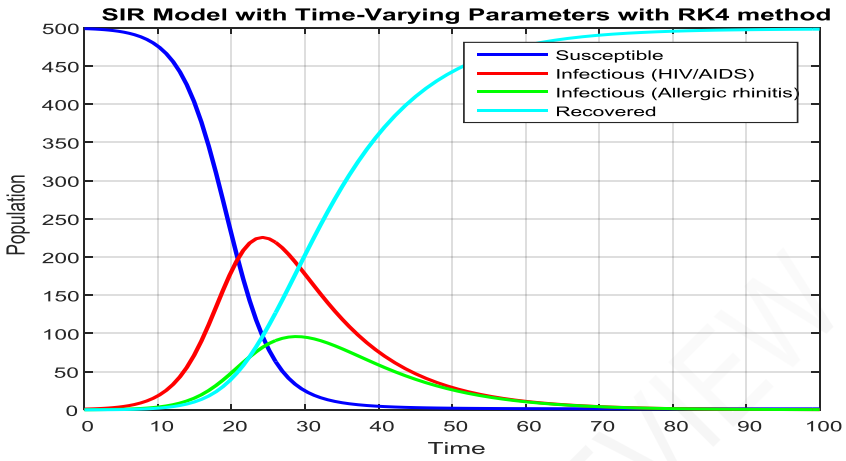

Figure 5: Equations defined as a function

Using equations (4)-(6), the SIR model equations (1)-(3) are defined as a plot function in Figure (5) that takes the current time t and the state variables$y$ as input and returns the derivatives of the variables. The ode45 function is used to numerically solve the differential equations using the RK4 method. Here, the values of parameters beta1, beta2, gamma1, and gamma2 can be modified to adjust the infection and recovery rates for each disease and the initial conditions to reflect the initial state of the population.Beta1 and beta2 parameters represent the infection rates for two diseases, and the gamma parameter represents the recovery rate.

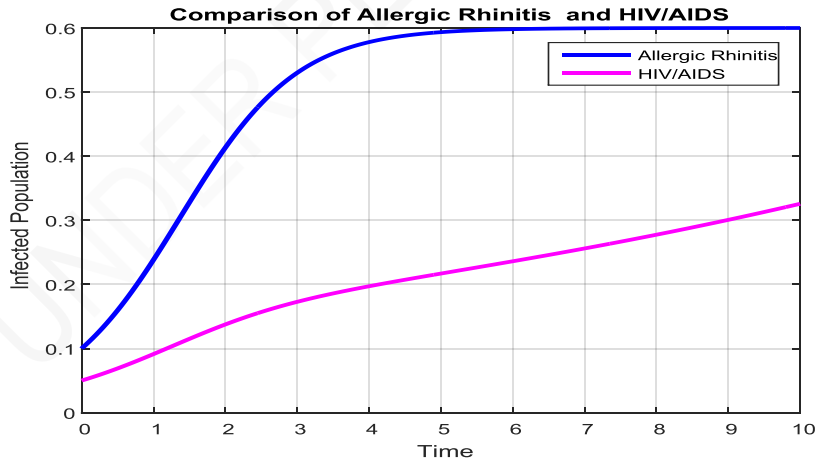

Figure 6: RK4 method

An increase in Recovery Rate (gamma1 > 0.1) for Allergic rhinitis disease means more individuals infected with HIV/AIDS would recover at a faster rate. This could lead to a decrease in the number of infectious individuals (Allergic rhinitis) over time, potentially

resulting in a smaller impact of HIV/AIDS disease in the population. The susceptible population may have a higher chance of avoiding infection.
Figure (6) contains the fourth-order Runge-Kutta (RK4) method to numerically solve the differential equations of the SIR model for both the Allergic rhinitis patients living with HIV-AIDS. The RK4 method provides a more accurate approximation of the population dynamics compared to the simple iterative approach. The RK4 method is implemented where the differential equations for both diseases are evaluated at each time step.

5. CONCLUSION

The RK4 steps for each disease are computed separately using the current population values and the corresponding transmission and recovery rates. These steps are then used to estimate the population values at the next time step, and the process is repeated until the desired time span is reached. The results are plotted on the same graph gives a visual comparison of the infected populations for the respiratory disease and HIV/AIDS over time. Therefore, the RK4 method provides a numerical approximation and the accuracy depends on the step size (dt) chosen. Adjusting the step size may be necessary to ensure accurate results for specific scenarios.

REFERENCES

Comment [SAM13]: This section is not justify. Justify.

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1. E. R. Weibel, “Morphometry of the Human Lung.” Academic Press, New York, NY, USA, 1963.
2. W. Kutta, "Beitrag zur naherungsweisen Integration von Differential gleichungen." Z. Math. und Phys. , vol. 46, pp. 435–453, 1901.
3. W. Yang , A. Karspeck , J. Shaman. “Comparison of Filtering Methods for the Modeling and Retrospective Forecasting of Influenza Epidemics.” *PLOS Computational Biology* 10(4) https://doi.org/10.1371/journal.pcbi.1003583, 2014.
4. S. Side, A. M Utami, Sukarna and M. I. Pratam, “Numerical solution of the SIR model on the spread of Tuberculosis by the Runge-Kutta method.” *Journal of Physics: Conference Series*, Volume 1040, International Conference on Mathematics and Natural Sciences (IConMNS 2017) 6–7 September 2017, Bali, Indonesia. 2017.
5. A. Rizky, A. Mochammad, , A. Pratama, Sri Purwani, “Comparison of Numerical Simulation of Epidemiological Model between Euler Method with 4th Order RungeKutta Method.” *International Journal of Global Operations Research*, Vol. 2, No. 1, pp. 37-44, 202, 2021.
6. W. C. Roda, M. B. Varughese, D. Han & M. Y. Li. “Why is it difficult to accurately predict the COVID-19 epidemic?” *Infectious Disease Modelling*, vol.5, pp. 271–281, 2020.
7. I. Rzewnicki, E. Olszewska, D. Rogowska-Szadkowska. “HIV infections in otolaryngology.” *Med SciMonit*, 18(3) pp.17–21, 2012.
8. D. L Linnemann de Martínez , G. López Pérez, L. Xochihua Díaz. “Allergic diseases and infection with human immunodeficiency virus (HIV)-AIDS in pediatric patients.” *National Library of Medicine, national Centre for Biotechnology Information*. 44(2), pp. 55-9, 1997.
9. A. Koutsonikolis, P. Robert. J. Nelson, F. C. Enrique, N. Emerita. Brigino MD, Mitchell Seleznick , Robert A. Good , DSc, Richard F. Lockey. “Serum total and specific IgE levels in children infected with human immunodeficiency virus.” *J Allergy ClinImmunol*, 97, pp. 692, 2005.
10. M.E. Fitzpatrick, K.M. Kunisaki, A.Morris. “Pulmonary disease in HIV-infected adults in the era of antiretroviral therapy.” AIDS, 32(3) pp.277-292, 2018.

11. A.C Justice, R.S. Braithwaite. “Lessons learned from the first wave of aging with HIV.” *Aids.* 26(Suppl 1), pp. S11–18, 2012.
12. T. TchavdarMarinov, S. Rossitza, Marinova, “Inverse problem for adaptive SIR model.” Application to COVID-19 in Latin America, Infectious Disease Modelling, vol. 7, Issue 1, pp. 134-148, pp. 2468-0427, 2022. https://doi.org/10.1016/j.idm.2021.12.001.
13. A. Kawther, M.A. Maha, Lashin. “An analytical study of the factors that influence COVID-19 spread.” *Saudi Journal of Biological Sciences,* vol. 28, Issue 2, pp. 1177-1195, 2021. ISSN 1319-562X, https://doi.org/10.1016/j.sjbs.2020.11.067
14. J. Roman J.Dwilewicz, A. Min. “MATerialsMATemàtics” no. 3, vol.17 pp. 1887-1097 Publicacióelectrònica de divulgació del Departament de Matemàtiques de la UniversitatAutònoma de Barcelona, 2013.
15. R.R. Marzo, A. Ahmad, K. Abid, A.P. Khatiwada, A. Ahmed, T.M. Kyaw, I.B.Z. Abidin, M. Srithar, S. Sinnathamby, A.P. Sarvasundram, S. Shrestha. “Factors influencing the acceptability of COVID-19 vaccination: A cross-sectional study from Malaysia.” *Vacunas,* pp. 1576-9887, 2021.
16. K. Crothers, A.A. Butt, C.L. Gibert, M.C. Rodriguez-Barradas, S. Crystal, “Increased COPD among HIV-positive compared to HIV-negative veterans.” *Chest.* 130(5), pp. 1326–1333, 2006.
17. K. Crothers, L. Huang, J.L. Goulet, M.B. Goetz, S.T Brown, M.C Rodriguez-Barradas. “HIV infection and risk for incident pulmonary diseases in the combination antiretroviral therapy era.” *American journal of respiratory and critical care medicine*. 183(3), pp. 388–395, 2011.
18. W. Qu, M. Robinson, F. Zhang. “Factors influencing the natural history of HIV-1 infection.” *Chin Med J*. vol. 121:pp. 2613–2621, 2008.
19. Z. Ma, Y. Zhou, W. Wang, Z. Jin. “Mathematical modeling dynamics of infectious diseases.” 1st. Beijing: Science Publishing House, pp. 1–24, 2004.
20. S. Cassels, S. Clark, M. Morris. Mathematical models for HIV transmission dynamics: tools for social and behavioral science research. J Acquire Immune Deficiency Syndrome, vol. 47, pp. 34–39, 2008.
21. E.O. Meltzer. “Quality of life in adults and children with allergic rhinitis.” *J Allergy ClinImmunol*. Vol. 108(Suppl 1), pp. 45–S53. doi: 10.1067/mai.2001.115566, 2001.
22. L.S. Linhar, J. Traebert, D. Galato. “Allergic diseases in subjects under 18 years living with HIV.” *Allergy Asthma ClinImmunol*. 2014; 10(1):35. doi:10.1186/1710-1492-10-35
23. K. Marth, E. Wollmann, D. GalleranoPersistence of IgE-associated allergy and allergen-specific IgE despite CD4+ T cell loss in AIDS. *PLoS One*. 2014, 9(6) doi:10.1371/journal.