Bandwagon Eccentric Domination Polynomial and its Energy in Graphs

ABSTRACT

Kathiresan In this article, bandwagon eccentric domination polynomial $BED(G,x)$ isintroduced. Theorems related to the characteristics of coefficients of $BED(S_n,x)$ of astar graph is discussed and the coefficients of $BED(S_n,x)$ are tabulated. Minimum bandwagoneccentric dominating energy$\mathbb{E}_{bed}(G)$ is also defined. $\mathbb{E}_{bed}(G)$ of families of graphsare computed and the properties of minimum bandwagon eccentric dominating eigenvaluesare obtained. Results related to upper and lower bounds for $\mathbb{E}_{bed}(G)$ of cocktail party,complete, crown and star graphs are discussed.

Keywords: Elected neighbour, Bandwagon distance, Bandwagon eccentricity, Bandwagoneccentric domination, Bandwagon eccentric domination polynomial, minimumbandwagon eccentric dominating energy.

1. INTRODUCTION

“The concept of domination in graphs was introduced by Ore and Berge”[12, 3]. There are different types of dominations. The two books ‘Towards theory of domination’ and ‘Fundamentals of domination’ by T.W.Haynes gives a comprehensive overview ofthe literature regarding domination in graphs. ’Distance in graphs’ by F. Buckley and F. Harary[6] is a textbook where some distances in graphs are discussed. There are many types of distances and distance based dominations. Some are metric and some are non-metric distance. J. Arocha et al[2] introduced domination polynomial, Saeid Alikhani et al[1] contributed majorly to the area. T.N. Janakiraman et al[11] introduced “eccentric domination in graphs”. A.M. Ismayil et al introduced “eccentric domination polynomial in graphs”. Ivan Gutman[8] introduced theenergy of graphs. Rajesh Kanna et al[13] introduced “minimum dominating energy of graphs”. Tejaskumar R, A Mohamed Ismayil and Ivan Gutman[15] introduced “minimum eccentric dominating energy of graphs”.

Bandwagon distance and bandwagon eccentric domination was introduced by Tejaskumar R et al[14]. In this paper, we introduce bandwagon eccentric dominating polynomial and bandwagon eccentric energy. Bandwagon eccentric dominating polynomials of star graph is studied. The minimum bandwagon eccentric dominating energy of cocktail party, crown and star graphs are found. Upper and lower bounds for the minimum bandwagon eccentric dominating energy are established.

2. PRELIMINARIES

Definition 2.1. [1]. Let $D(G,k)$ be the family of dominating sets of a graph G with cardinality k and let $d(G,k) = |D(G,k)|$. Then the domination polynomial $D(G,x)$ of G is defined as $D(G,x) = \sum_{k=\gamma(G)}^{|V(G)|} d(G,k)x^k$, where $\gamma(G)$ is the domination number of G.

Definition 2.2. [14]. A vertex u is said to be an elected neighbour of v if u is adjacent to v and has the maximum degree among all vertices adjacent to v. The walk between any two vertices where all the vertices are connected to al least one of its elected neighbour is called bandwagon walk. The shortest bandwagon walk between any two vertices v_i and v_j is known as bandwagon distance given by $Bd(v_i, v_j)$.

Definition 2.3. [14]. The bandwagon eccentricity $Be(v)$ of a vertex v is the bandwagon distance to a vertex farthest from v. Thus, $Be(e) = max\{Bd(u,v) : v,u \in V\}$. For a vertex v, each vertex at a distance $Be(v)$ from v is a bandwagon eccentric vertex. The bandwagon eccentric set of a vertex v is defined by $BE(v) = \{u \in V(G): d(u,v) = Be(e)\}$.

Definition 2.4. [14]. The bandwagon radius $Brad(G)$ is the minimum bandwagon eccentricity of the vertices. The bandwagon diameter $Bdiam(G)$ is the maximum bandwagon eccentricity. v is a bandwagon central vertex if $Be(v) = Brad(G)$. The bandwagon center $BC(G)$ is the set of all bandwagon central vertices. v is a bandwagon peripheral vertex if $Be(v) = Bdiam(G)$. The bandwagon periphery $BP(G)$ is the set of all bandwagon peripheral vertices. A graph G is said to be bandwagon self-centered if and only if $Brad(G) = Bdiam(G)$.

Definition 2.5. [14]. A dominating set $D \subseteq V(G)$ is a bandwagon eccentric dominating set (BED set) if for every vertex $v \in V - D$, there exists at least one bandwagon eccentric vertex of v in D. A BED set D is called a minimal BED set if no proper subset of D is a BED set. The BED-number $\gamma_{bed\,(G)}$of a graph G is the minimum cardinality among the minimal BED sets of G. The upper BED-number $\Gamma_{bed\,(G)}$ of a graph G is the maximum cardinality among the minimal BED sets of G.

Theorem 2.1. [14] For star graph S_n, where $n \geq 3$, $\gamma_{bed}\,(S_n) =\ 2$.

Definition 2.6. [16]. The adjacency matrix $A(G)$ of the graph G is a square matrix of order n, whose (i,j)-entry is equal to 1 if the vertices v_i and v_j are adjacent and is equal to zero otherwise.

Definition 2.7. [16]. The characteristic polynomial of the adjacency matrix, that is,$det(\lambda\, I_n - A(G))$, where I_n is the unit matrix of order n, is said to be the characteristicpolynomial of the graph G.

Definition 2.8. [16]. The eigenvalues of a graph G are defined as the eigenvalues of its adjacency matrix $A(G)$. Since $A(G)$ is symmetric, its eigenvalues are all real. Denote them by $\lambda_1, \lambda_2, \ldots, \lambda_n$ and as a whole, they are called the spectrum of G and denoted by $Spec(G)$.

Definition 2.9. [16]. If G is a graph on n vertices and $\lambda_1, \lambda_2, \ldots, \lambda_n$ are its eigenvalues, then the energy of G is$\mathbb{E}(G) = \sum_{i=1}^{n} |\lambda_i|$.

3. BANDWAGON ECCENTRIC DOMINATION POLYNOMIAL OF STAR

In this section, the bandwagon eccentric domination polynomial (BED-polynomial) is defined and BED-polynomial of star graphs $S_n \forall\, n \geq 3$ are discussed. The properties of BED-polynomial of star graphs are studied.

Definition 3.1 Let $bed(G,k)$ be the set of all bandwagon eccentric dominating sets of a graph G with cardinality k then the bandwagon eccentric dominating polynomial (BED-polynomial) $BED(G,x)$ of G is defined by $BED(G,x) = \sum_{k=\gamma_{bed}\,(G)}^{n} |bed(G,k)| x^k$, where $|bed(G,k)|$ is the number of distinct bandwagon eccentric dominating set with cardinality k and $\gamma_{bed}\,(G)$ is the BED-number of G.

Example 3.1 Consider the claw graph given in Figure-1

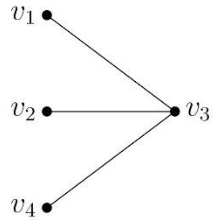

Fig. 1.Claw graph

Here $\{v_1, v_2, v_3, v_4\}$ is the only bandwagon eccentric dominating set with cardinality four. There are four bandwagon eccentric dominating sets$\{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4\}$ with cardinality three. There are three bandwagon eccentric dominating sets $\{v_1, v_3\}, \{v_2, v_3\}, \{v_3, v_4\}$ with cardinality two. There is no bandwagon eccentric dominating set with cardinality one.
Therefore the BED-polynomial is given by $BED(G, x) = x^4 + 4x^3 + 3x^2$.

Theorem 3.1 For a star graph S_n, $bed(S_n, k) = \emptyset$ if $k \geq n$ or $k \leq 2$.
Proof: The cardinality of the minimum bandwagon eccentric dominating set of a star graph S_n is given by $\gamma_{bed}(S_n) = 2$. Hence there is no proper subset of S_n whose cardinality is less than 2. $V(S_n)$ forms the largest bandwagon eccentric dominating set and $|V(S_n)| = n$ therefore there exists no bandwagon eccentric dominating set whose cardinality is greater than n. Hence $bed(S_n, k) = \emptyset$ if $k \geq n$ or $k \leq 2$.

Observation 3.1
1. $|bed(S_1, 1)| = 0$, (from Definition-2.5)
2. $|bed(S_2, 1)| = 2$, since $\{v_1\}$ and $\{v_2\}$ both form BED sets.
3. $|bed(S_2, 2)| = 1$, since $V(S_2)$ is a BED set.

Theorem 3.2 There exist no bandwagon eccentric dominating set of cardinality one for a star graph S_n i.e, $|bed(S_n, 1)| = 0$ for $n > 2$.
Proof: Let S_n be a star graph with vertices $\{v_1, v_2, \ldots, v_n\} = V$. Let us assume v_1 is the central vertex of star graph S_n then the central vertex dominates all the other vertices in the star graph, $\gamma(S_n) = 1$. But the bandwagon eccentric vertex of a central vertex is $V - \{v_1\}$ and the bandwagon eccentric vertices of the pendant vertices are all the vertices in the graph excluding the central vertex and itself. Hence the central vertex alone forms the dominating set which is not an bandwagon eccentric dominating set. Therefore the bandwagon eccentric dominating set contains atleast two vertices. Hence $|bed(S_n, 1)| = 0, \forall\, n > 2$.

Theorem 3.3 For a star graph S_n, $|bed(S_n, k)| = {}^{n-1}C_{k-1} + {}^{n-1}C_k$ for $n = 3,4$ and $k \neq 1$.
Proof: Let $\{v_1, v_2, v_3\}$ be the vertices of the star graph S_3, from Theorem-3.2, we know that there exists no bandwagon eccentric dominating set of cardinality one. Therefore $|bed(S_3, 1)| = 0$ and for all vertices of $n - 1$ cardinality we have n combinations. It is obvious to have one bandwagon eccentric dominating set of cardinality n. Similar proof follows for $n = 4$.

Theorem 3.4 For a star graph S_n where $n > 4$,

$$|bed(S_n, k)| = \begin{cases} |bed(S_{n-1}, k)| + 1, & k = 2 \\ |bed(S_{n-1}, k-1)| + |bed(S_{n-1}, k)| - 1, & k = n-2 \\ |bed(S_{n-1}, k-1)| + |bed(S_{n-1}, k)|, & otherwise \end{cases}$$

Proof: Case(i): If $k = 2$, let $\{v_1, v_2, v_3, v_4, \ldots, v_n\}$ be the vertex set of the star graph S_n. Let v_1 be the central vertex of star graph and all others are pendant vertices. Then $BE(v_1) =$

$\{v_2, v_3, v_4, \dots, v_n\}$ is a bandwagon eccentric set of vertices of v_1 and $Be(v_1) = 1$. Therefore $\{v_1\}$ is a dominating set but not a bandwagon eccentric dominating set. The possible number of bandwagon eccentric dominating sets of cardinality two in S_5 is $|bed(S_5, 2)| = |bed(S_4, 2)| + 1 = 4$. The possible number of bandwagon eccentric dominating sets of cardinality two in S_6 is $|bed(S_6, 2)| = |bed(S_5, 2) + 1 = 5$. Proceeding like this the possible number of bandwagon eccentric dominating set of cardinality two in S_{n-1} is $|bed(S_{n-1}, 2)| = |bed(S_{n-2}, 2) + 1 = n - 2$ and the possible number of bandwagon eccentric dominating set of cardinality two in S_n is $|bed(S_n, 2)| = |bed(S_{n-1}, 2)| + 1 = n - 1$.

Case(ii): If $k = n - 2$, for a star S_5, the possible number of bandwagon eccentric dominating set of cardinality three in S_5 is $|bed(S_5, 3)| = |bed(S_4, 2)| + |bed(S_4, 3) - 1 = 3 + 4 - 1 = 6$. The possible number of bandwagon eccentric dominating set of cardinality four in S_6 is $|bed(S_6, 4)| = |bed(S_5, 3)| + |bed(S_5, 4) - 1 = 6 + 5 - 1 = 10$. Similarly, for a star S_{n-1}, the possible number of bandwagon eccentric dominating set of cardinality $n - 3$ in S_{n-1} is $|bed(S_{n-1}, n-3)| = |bed(S_{n-2}, n-2)| + |bed(S_{n-2}, n-3)| - 1$. Proceeding like this the possible number of bandwagon eccentric dominating set of cardinality $n - 2$ in S_n is $|bed(S_n, n-2)| = |bed(S_{n-1}, n-1)| + |bed(S_{n-1}, n-2)| - 1$. Since $k = n - 2$, we obtain $|bed(S_n, k)| = |bed(S_{n-1}, k-1)| + |bed(S_{n-1}, k)| - 1$.

Case(iii): If $k = 3, 4, \dots, n-3, n-1, n$, here we have$\binom{n-2}{k-2}$ bandwagon eccentric dominating sets of cardinality k. Therefore $|bed(S_n, k)| = \binom{n-2}{k-2}$. $|bed(S_{n-1}, k-1)| = \binom{n-3}{k-3}$. Similarly $|bed(S_{n-1}, k)| = \binom{n-3}{k-2}$. Then we have $\binom{n-2}{k-2} = \binom{n-3}{k-3} + \binom{n-3}{k-2}$. Therefore $|bed(S_n, k)| = |bed(S_{n-1}, k-1)| + |bed(S_{n-1}, k)|$.

Theorem 3.5 For a star graph $n \geq 4$,
$BED(S_n, x) = x\, BED(S_{n-1}, x) + BED(S_{n-1}, x) - x^{n-2} + x^2$.

Proof: We prove the following theorem by taking the summation of bandwagon eccentric dominating sets of every possible cardinality.

When $k = 2$, $|bed(S_n, 2)| = |bed(S_{n-1}, 2)| + 1$

$\Rightarrow x^2\, |bed(S_n, 2)| = x^2\, |bed(S_{n-1}, 2)| + x^2$.

By the Theorem-3.4 we have $|bed(S_n, k)| = |bed(S_{n-1}, k-1)| + |bed(S_{n-1}, k)|$.

When $k = 3$, $|bed(S_n, 3)| = |bed(S_{n-1}, 2)| + |bed(S_{n-1}, 3)|$

$\Rightarrow x^3\, |bed(S_n, 3)| = x^3\, |bed(S_{n-1}, 2)| + x^3\, |bed(S_{n-1}, 3)|$

When $k = n - 2$, by Theorem-3.4,

$|bed(S_n, n)| = |bed(S_{n-1}, k-1)| + |bed(S_{n-1}, k)| - 1$.

$\Rightarrow |bed(S_n, n-2)| = |bed(S_{n-1}, n-3)| + |bed(S_{n-1}, n-2)| - 1$.

$\Rightarrow x^{n-2}\, |bed(S_n, n-2)| = x^{n-2}\, |bed(S_{n-1}, n-3)| + x^{n-2}\, |bed(S_{n-1}, n-2)| - x^{n-2}$.

When $k = n - 1$, $|bed(S_n, n-1)| = |bed(S_{n-1}, n-2)| + |bed(S_{n-1}, n-1)|$.

$\Rightarrow x^{n-1}\, |bed(S_n, n-1)| = x^{n-1}\, |bed(S_{n-1}, n-2)| + x^{n-1}\, |bed(S_{n-1}, n-1)|$.

When $k = n$, $|bed(S_n, n) = |bed(S_n, n-1)| + |bed(S_n, n)|$.

$\Rightarrow x^n\, |bed(S_n, n)| = x^n\, |bed(S_n, n-1)| + x^n\, |bed(S_n, n)|$.

$x^2\, |bed(S_n, 2)| + x^3\, |bed(S_n, 3)| + \cdots + x^{n-2}\, |bed(S_n, n-2)| + x^{n-1}\, |bed(S_n, n-1)| + x^n\, |bed(S_n, n)| =$
$x^2\, |bed(S_{n-1}, 2)| + x^2 + x^3\, |bed(S_{n-1}, 2)| + x^3\, |bed(S_{n-1}, 3)| + x^4\, |bed(S_{n-1}, 3)| + x^4\, |bed(S_{n-1}, 4)| + \cdots + x^{n-2}\, |bed(S_{n-1}, n-3)| + x^{n-2}\, |bed(S_{n-1}, n-2)| - x^{n-2} + x^{n-1}\, |bed(S_{n-1}, n-2)| + x^{n-1}\, |bed(S_{n-1}, n-1)| + x^n\, |bed(S_n, n-1)| + x^n\, |bed(S_n, n)| \rightarrow 1$

By rearranging the terms of equation(1),

$x^2\, |bed(S_n, 2)| + x^3\, |bed(S_n, 3)| + \cdots + x^{n-2}\, |bed(S_n, n-2)| + x^{n-1}\, |bed(S_n, n-1)| + x^n\, |bed(S_n, n)| = x[x^2\, |bed(S_{n-1}, 2)| + x^3\, |bed(S_{n-1}, 3)| + \cdots + x^{n-2}\, |bed(S_{n-1}, n-2)| + x^{n-1}\, |bed(S_{n-1}, n-1)| + x^n\, |bed(S_{n-1}, n)|] + [x^2\, |bed(S_{n-1}, 2)| + x^3\, |bed(S_{n-1}, 3)| + \cdots + x^{n-2}\, |bed(S_{n-1}, n-2)| + x^{n-1}\, |bed(S_{n-1}, n-1)| + x^n\, |bed(S_{n-1}, n)|] + x^2 - x^{n-2}$.

Since, $|bed(S_{n-1}, 1)| = |bed(S_{n-1}, n)| = 0$,

we get $\sum_{k=2}^{n} |bed(S_n, k)| x^k = x \sum_{k=2}^{n} |bed(S_{n-1}, k)| x^k + \sum_{k=2}^{n} |bed(S_{n-1}, k)| x^k - x^{n-2} + x^2$.

$\Rightarrow BED(S_n, x) = x\,BED(S_{n-1}, x) + BED(S_{n-1}, x) - x^{n-2} + x^2.$

Using the Theorem-3.2, Theorem-3.3 and Theorem-3.4 we get $BED(S_n, x)$ for $3 \le n \le 12$ as shown in the table below.

n \ k	1	2	3	4	5	6	7	8	9	10	11	12
1	-											
2	2	1										
3	0	3	1									
4	0	3	4	1								
5	0	4	6	5	1							
6	0	5	10	10	6	1						
7	0	6	15	20	15	7	1					
8	0	7	21	35	35	21	8	1				
9	0	8	28	56	70	56	28	9	1			
10	0	9	36	84	126	126	84	36	10	1		
11	0	10	45	120	210	252	210	120	45	11	1	
12	0	11	55	165	330	462	462	330	165	55	12	1

Table 1. Construction of coefficients of bandwagon eccentric domination polynomial of S_n

Theorem 3.6 Thefollowing properties for the coefficients of $BED(S_n, x)$ hold.

1. $|bed(S_n, n)| = 1, \forall\, n \ge 2.$
2. $|bed(S_n, n-1)| = n, \forall\, n \ge 2.$
3. $|bed(S_n, 1)| = 0, \forall\, n > 2.$
4. $|bed(S_n, n-3)| = \frac{(n-1)(n-2)(n-3)}{6}, \forall\, n \ge 5.$
5. $|bed(S_n, n-4)| = \frac{(n-1)(n-2)(n-3)(n-4)}{24}, \forall\, n \ge 6.$
6. $\sum_{k=2}^{n} |bed(S_n, k)| = 2[\sum_{k=2}^{n-1} |bed(S_{n-1}, k)|], \forall\, n \ge 4.$
7. Total number of bandwagon eccentric dominating sets in $S_n$is $2^{n-1} \forall\, n \ge 3$.

Proof:

1. The whole vertex set of a graph G is an bandwagon eccentric dominating set. Therefore $|bed(S_n, n)| = 1, \forall\, n \ge 2$.
2. Every set of cardinality $n-1$ has a singleton set in its complement. The bandwagon eccentric vertex of the singleton vertex lies in the set of cardinality $n-1$. Therefore it must be an bandwagon eccentric dominating set and there are n combinations of bandwagon eccentric dominating sets with cardinality $k = |n-1|$. Therefore $|bed(S_n, n-1)| = n,\ \forall\, n \ge 2$.
3. The proof follows from the Theorem 3.2.
4. By induction on n. The result is true for $n = 5$. Since $|bed(S_5, 2)| = 4$. Assume the result is true for all natural numbers less than n. Now we prove it for n. By the Theorem 3.4,

$$|bed(S_n, n-3)| = |bed(S_{n-1}, n-4)| + |bed(S_{n-1}, n-3)|$$

$$|bed(S_n, n-3)| = \frac{(n-2)(n-3)(n-4)}{6} + \frac{(n-2)(n-3)}{2}$$

$$|bed(S_n, n-3)| = (n-2)(n-3)(n-4) + \frac{3(n-2)(n-3)}{6}$$

$$|bed(S_n, n-3)| = \frac{(n-2)(n-3)[n-4+3]}{6}$$
$$|bed(S_n, n-3)| = \frac{(n-1)(n-2)(n-3)}{6}$$

The result is true for all n.

5. By induction on n. The result is true for $n = 6$, since $|bed(S_6, 2)| = 5$. Assume that the result is true for all natural numbers less than n. Now we prove it for n. By the Theorem 3.4
$$|bed(S_n, n-4)| = |bed(S_{n-1}, n-5)| + |bed(S_{n-1}, n-4)|$$
$$|bed(S_n, n-4)| = \frac{(n-2)(n-3)(n-4)(n-5)}{24} + \frac{(n-2)(n-3)(n-4)}{6}$$
$$|bed(S_n, n-4)| = \frac{(n-2)(n-3)(n-4)[(n-5)-4]}{24}$$
$$|bed(S_n, n-4)| = \frac{(n-1)(n-2)(n-3)(n-4)}{24}$$
The result is true for all n.
6. From Theorem 3.4, we have
$$|bed(S_n, n)| = |bed(S_{n-1}, n-1)| + |bed(S_{n-1}, n)|$$
$$\sum_{k=2}^{n} |bed(S_n, k)| = \sum_{k=2}^{n} |bed(S_{n-1}, k-1)| + \sum_{k=2}^{n-1} |bed(S_{n-1}, k)|$$
$$\sum_{k=2}^{n} |bed(S_n, k)| = \sum_{k=2}^{n-1} |bed(S_{n-1}, k)| + \sum_{k=2}^{n-1} |bed(S_{n-1}, k)|$$
$$\sum_{k=2}^{n} |bed(S_n, k)| = 2\left(\sum_{k=2}^{n-1} |bed(S_{n-1}, k)|\right)$$
7. By induction on n. When $n = 3, |bed(S_3, k)| = 2^{3-1} = 2^2 = 4$. Therefore this is true for $n = 3$. Let us assume, this result is true for all natural numbers less than n. Similarly, when $n = n-1, |bed(S_{n-1}, k)| = 2^{n-1-1} = 2^{n-2}$. Proceeding like this for n, we get $|bed(S_n, k)| = 2^{n-1}$. Therefore total number of bandwagon eccentric dominating sets in S_n is $2^{n-1} \forall\, n \geq 3$.

Theorem 3.7 The bandwagon eccentric dominating polynomial of a star graph is given by $BED(S_n, x) = x(1+x)^{n-1} + x^{n-1} - x, \forall\, n \geq 3$.
Proof: The theorem is a direct consequence of Theorem 3.2, Theorem 3.3, Theorem 3.4 and Theorem 3.5.

Example 3.2 Let S_n be the star graph then $BED(S_n, x) = x(x+1)^{n-1} + x^{n-1} - x$.
For $n = 7$,
$$BED(S_7, x) = x(x+1)^{7-1} + x^{7-1} - x$$
$$BED(S_7, x) = x(x+1)^6 + x^6 - x$$
$$BED(S_7, x) = x(x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1) + x^6 - x$$
$$BED(S_7, x) = x^7 + 6x^6 + 15x^5 + 20x^4 + 15x^3 + 6x^2 + x + x^6 - x$$
$$BED(S_7, x) = x^7 + 7x^6 + 15x^5 + 20x^4 + 15x^3 + 6x^2$$
Refer the coefficients of $BED(S_7, x)$ in the Table 1.

4. MINIMUM BANDWAGON ECCENTRIC DOMINATING ENERGY

Definition 4.1: For $G = (V, E)$ be a simple graph where $V(G) = \{v_1, v_2, \dots, v_n\}$ where $n \in \mathbb{N}$ is the set of vertices and E is the set of edges. Let D be a minimum bandwagon eccentric dominating set of G then the minimum BED matrix of G is a $n \times n$ matrix defined by $\mathbb{A}_{bed}(G) = (e_{ij})$, where

$$(e_{ij}) = \begin{cases} 1, & if\ v_j \in BE(v_{ij})\ or\ v_i \in BE(v_j), \\ 1, & if\ i = j\ and\ v_i \in D, \\ 0, & otherwise \end{cases}$$

Definition 4.2: The characteristic polynomial of the minimum BED matrix $\mathbb{A}_{bed}(G)$ is defined by $\mathcal{G}_n(G,\beta) = \det(\mathbb{A}_{bed}(G) - \beta I)$, where I is the identity matrix.

Definition 4.3: The eigenvalues of $\mathbb{A}_{bed}(G)$ are called minimum BED eigenvalues of G. Since $\mathbb{A}_{bed}(G)$ is symmetric, the eigenvalues of $\mathbb{A}_{bed}(G)$ are real. We label the eigenvalues in non-increasing order $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_n$.

Definition 4.4: The minimum BED energy of G is defined by $\mathbb{E}_{bed}(G) = \sum_{i=1}^{n} |\beta_i|$.

Remark 4.1: The trace of $\mathbb{A}_{bed}(G)$= Bandwagon eccentric domination number.

Example 4.1: Consider the kite graph given in Fig. 2.

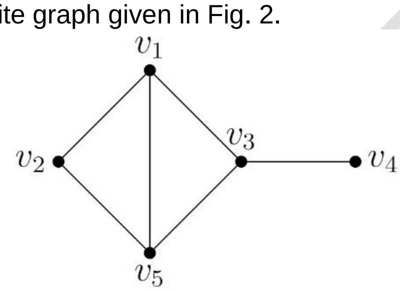

Fig. 2. Kite graph

Table 2. From the Fig. 2, we tabulate bandwagon eccentricity $Be(v)$ and bandwagon eccentric vertex $BE(v)$ of $v \in V$.

Vertex $v \in V(G)$	Bandwagon eccentricity $Be(v)$	Bandwagon eccentric vertex $BE(v)$
v_1	2	$\{v_2, v_4\}$
v_2	3	$\{v_4\}$
v_3	3	$\{v_4\}$
v_4	3	$\{v_2, v_3\}$
v_5	2	$\{v_2, v_4\}$

$Brad(G) = 2, Bdiam(G) = 3, BC(G) = \{v_1, v_5\}\ and\ BP(G) = \{v_2, v_3, v_4\}$.

The minimum bandwagon eccentric dominating sets of kite graph are $D_1 = \{v_1, v_4\}$, $D_2 = \{v_2, v_3\}$, $D_3 = \{v_2, v_4\}$ and $D_5 = \{v_4, v_5\}$.

1. $D_1 = \{v_1, v_4\}$,

$$\mathbb{A}_{bed}(G) = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial $\mathcal{G}_n(G,\beta) = -\beta^5 + 2\beta^4 + 5\beta^3 - 2\beta^2 - 3\beta + 1$.

Minimum bandwagon eccentric dominating eigenvalues are $\beta_1 \approx 3.2661, \beta_2 \approx 0.7162, \beta_3 \approx 0.3266, \beta_4 \approx -1, \beta_5 \approx -1.3089$.
Minimum bandwagon eccentric dominating energy $\mathbb{E}_{bed}(G) \approx 6.6178$.

2. $D_2 = \{v_2, v_3\}$,

$$\mathbb{A}_{bed}(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial $\mathcal{G}_n(G, \beta) = -\beta^5 + 2\beta^4 + 5\beta^3 - 4\beta^2 - 4\beta$.
Minimum bandwagon eccentric dominating eigenvalues are $\beta_1 \approx 3.0664, \beta_2 \approx 1.2222, \beta_3 \approx 0, \beta_4 \approx -0.6522, \beta_5 \approx -1.6364$.
Minimum bandwagon eccentric dominating energy $\mathbb{E}_{bed}(G) \approx 6.5772$.

3. $D_3 = \{v_2, v_4\}$,

$$\mathbb{A}_{bed}(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial $\mathcal{G}_n(G, \beta) = -\beta^5 + 2\beta^4 + 5\beta^3 - \beta^2 - 2\beta$.
Minimum bandwagon eccentric dominating eigenvalues are $\beta_1 \approx 3.3502, \beta_2 \approx 0.6735, \beta_3 \approx 0, \beta_4 \approx -0.641, \beta_5 \approx -1.3827$.
Minimum bandwagon eccentric dominating energy $\mathbb{E}_{bed}(G) \approx 6.0474$.

4. $D_4 = \{v_4, v_5\}$,

$$\mathbb{A}_{bed}(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{G}_n(G, \beta) = -\beta^5 + 2\beta^4 + 5\beta^3 - 2\beta^2 - 3\beta + 1$.
Minimum bandwagon eccentric dominating eigenvalues are $\beta_1 \approx 3.2661, \beta_2 \approx 0.7162, \beta_3 \approx 0.3266, \beta_4 \approx -1, \beta_5 \approx -1.3089$.
Minimum bandwagon eccentric dominating energy $\mathbb{E}_{bed}(G) \approx 6.6178$.

Observation 4.1: The minimum bandwagon eccentric dominating energy $\mathbb{E}_{bed}(G)$ of kite graph given in Fig. 2 varies for different minimum bandwagon eccentric dominating sets.
For the set $D_1, D_4, \mathbb{E}_{bed}(G) \approx 6.6178$,
For the set $D_2, \mathbb{E}_{bed}(G) \approx 6.5772$,
For the set $D_3, \mathbb{E}_{bed}(G) \approx 6.0474$.

Remark 4.2: The value of minimum bandwagon eccentric dominating energy $\mathbb{E}_{bed}(G)$depends on the bandwagon eccentric dominating set.

Theorem 4.1: The minimum bandwagon eccentric dominating energy of the cocktail partygraph $K_{2\times n}$ is$\mathbb{E}_{bed}(K_{2\times n}) = \left[\left|\frac{1+\sqrt{5}}{2}\right| + \left|\frac{1-\sqrt{5}}{2}\right|\right] n$.

Proof: Let $K_{2\times n}$ be a cocktail party graph with vertex set $V = \cup_{i=1}^{n}\{u_i, v_i\}$. Let D be the minimum bandwagon eccentric dominating set and $|D| = n$. Then $D = \{u_1, u_2, \dots u_n\}$ or $\{v_1, v_2, \dots v_n\}$. Then the minimum bandwagon eccentric dominating matrix is

$$\mathbb{A}_{bed}(K_{2\times n}) = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & \dots & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \end{pmatrix}_{n\times n}$$

Characteristic polynomial is $\mathcal{G}_n(K_{2\times n}, \beta) = \det(\mathbb{A}_{bed}(K_{2\times n}) - \beta I)$.

$$= \begin{vmatrix} 1-\beta & 0 & 0 & 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & 1-\beta & 0 & 0 & \dots & 0 & 0 & 0 & 1 \\ 0 & 0 & 1-\beta & 0 & \dots & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\beta & \dots & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & 0 & \dots & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & -\beta & 0 & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 & 0 & -\beta & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & -\beta \end{vmatrix}$$

Resulting in $\mathcal{G}_n(K_{2\times n}, \beta) = (\beta^2 - \beta - 1)^n$

The minimum bandwagon eccentric dominating eigenvalues are

$$\beta = \frac{1+\sqrt{5}}{2}(n\ \ times),$$

$$\beta = \frac{1-\sqrt{5}}{2}(n\ \ times).$$

The minimum bandwagon eccentric dominating energy of $K_{2\times n}$ is given by

$\mathbb{E}_{bed}(K_{2\times n}) = \left[\left|\frac{1+\sqrt{5}}{2}\right| + \left|\frac{1-\sqrt{5}}{2}\right|\right] n$.

Theorem 4.2: For the complete graph K_n where $n > 2$ the minimum bandwagon eccentric dominating energy is $\mathbb{E}_{bed}(K_n) = (n-2) + \left|\frac{(n-1)+\sqrt{n^2-2n+5}}{2}\right| + \left|\frac{(n-1)-\sqrt{n^2-2n+5}}{2}\right|$.

Proof: Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \dots v_n\}$. The minimum bandwagon eccentric dominating set is $D = \{v_1\}$ then

$$\mathbb{A}_{bed}(K_n) = \begin{pmatrix} 1 & 1 & 1 & & 1 & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 0 & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & 0 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 & 1 \\ 1 & 1 & 1 & & 1 & 1 & 0 \end{pmatrix}_{n\times n}$$

Characteristic polynomial is $\mathcal{G}_n(K_n, \beta) = \det(\mathbb{A}_{bed}(K_n) - \beta I)$.

$$= \begin{vmatrix} 1-\beta & 1 & 1 & & 1 & 1 & 1 \\ 1 & -\beta & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & -\beta & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & -\beta & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & -\beta & 1 \\ 1 & 1 & 1 & & 1 & 1 & -\beta \end{vmatrix}$$

From which it follows $\mathcal{G}_n(K_n, \beta) = (\beta+1)^{n-2}[\beta^2 - (n-1)\beta - 1]$.

The minimum bandwagon eccentric dominating eigenvalues are

$$\beta = -1\ ((n-2)\ times),$$
$$\beta = \frac{(n-1)+\sqrt{n^2-2n+5}}{2},$$
$$\beta = \frac{(n-1)-\sqrt{n^2-2n+5}}{2}.$$

The minimum bandwagon eccentric dominating energy of the complete graph K_n is given by

$\mathbb{E}_{bed}(K_n) = |(-1)|(n-2) + \left|\frac{(n-1)+\sqrt{n^2-2n+5}}{2}\right| + \left|\frac{(n-1)-\sqrt{n^2-2n+5}}{2}\right|.$

$\mathbb{E}_{bed}(K_n) = (n-2) + \left|\frac{(n-1)+\sqrt{n^2-2n+5}}{2}\right| + \left|\frac{(n-1)-\sqrt{n^2-2n+5}}{2}\right|.$

Theorem 4.3: The minimum bandwagon eccentric dominating energy of the crown graph H_n is$\mathbb{E}_{bed}(H_n) = \left[\left|\frac{1+\sqrt{5}}{2}\right| + \left|\frac{1-\sqrt{5}}{2}\right|\right]\frac{n}{2}$.

Proof: Let H_n be a crowngraph with vertex set $V = \cup_{i=1}^{n/2}\{u_i, v_i\}$. Let D be the minimum bandwagon eccentric dominating set and $|D| = n/2$. Then $D = \{u_1, u_2, \dots u_{n/2}\}$ or $\{v_1, v_2, \dots v_{n/2}\}$. Then the minimum bandwagon eccentric dominating matrix is

$$\mathbb{A}_{bed}(H_n) = \begin{pmatrix} 1 & 0 & 0 & & 1 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 1 & & 0 & 0 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 & 0 \end{pmatrix}_{n\times n}$$

Characteristic polynomial is $\mathcal{G}_n(H_n, \beta) = \det(\mathbb{A}_{bed}(H_n) - \beta I)$.

$$= \begin{vmatrix} 1-\beta & 0 & 0 & & 1 & 0 & 0 \\ 0 & 1-\beta & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 1-\beta & & 0 & 0 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 0 & 0 & & -\beta & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & -\beta & 0 \\ 0 & 0 & 1 & & 0 & 0 & -\beta \end{vmatrix}$$

From which we calculate $\mathcal{G}_n(H_n, \beta) = (\beta^2 - \beta - 1)^{n/2}$

The minimum bandwagon eccentric dominating eigenvalues are

$$\beta = \frac{1+\sqrt{5}}{2}(n/2\ times),$$
$$\beta = \frac{1-\sqrt{5}}{2}(n/2\ times).$$

The minimum bandwagon eccentric dominating energy of crowngraph H_n is given by

$\mathbb{E}_{bed}(H_n) = \left[\left|\frac{1+\sqrt{5}}{2}\right| + \left|\frac{1-\sqrt{5}}{2}\right|\right]\frac{n}{2}.$

Theorem 4.4: For a star graph S_n where $n \geq 3$ and wheel graph W_n, $n \geq 5$, the minimum bandwagon eccentric dominating energy is

$\mathbb{E}_{bed}(S_n) = |(-1)|(n-3) + \left|\frac{(n-1)+\sqrt{(n-1)^2+8}}{2}\right| + \left|\frac{(n-1)-\sqrt{(n-1)^2+8}}{2}\right|.$

Proof: Let G be the star or wheel graph with the vertex set $V = \{v_1, v_2, \dots v_n\}$. The minimum bandwagon eccentric dominating set is $D = \{v_1, v_2\}$ then

$$\mathbb{A}_{bed}(G)=\begin{pmatrix}1&1&1&&1&1&1\\1&1&1&\cdots&1&1&1\\1&1&0&&1&1&1\\&\vdots&&\ddots&&\vdots&\\1&1&1&&0&1&1\\1&1&1&\cdots&1&0&1\\1&1&1&&1&1&0\end{pmatrix}_{n\times n}$$

Characteristic polynomial is $\mathcal{G}_n(G,\beta)=\det(\mathbb{A}_{bed}(G)-\beta I)$.

$$=\begin{vmatrix}1-\beta&1&1&&1&1&1\\1&1-\beta&1&\cdots&1&1&1\\1&1&-\beta&&1&1&1\\&\vdots&&\ddots&&\vdots&\\1&1&1&&-\beta&1&1\\1&1&1&\cdots&1&-\beta&1\\1&1&1&&1&1&-\beta\end{vmatrix}$$

From which it follows $\mathcal{G}_n(G,\beta)=(-1)^n(\beta)(\beta+1)^{n-3}(\beta^2-(n-1)\beta-2)$.

The minimum bandwagon eccentric dominating eigenvalues are

$$\beta=0\ (1\ time),$$
$$\beta=-1\ ((n-3)\ \ times),$$
$$\beta=\frac{(n-1)+\sqrt{(n-1)^2+8}}{2},$$
$$\beta=\frac{(n-1)-\sqrt{(n-1)^2+8}}{2}.$$

and thus the minimum bandwagon eccentric dominating energy of G is given by

$$\mathbb{E}_{bed}(S_n)=|(-1)|(n-3)+\left|\frac{(n-1)+\sqrt{(n-1)^2+8}}{2}\right|+\left|\frac{(n-1)-\sqrt{(n-1)^2+8}}{2}\right|.$$

5. PROPERTIES OF MINIMUM BANDWAGON ECCENTRIC DOMINATING EIGENVALUES

In this section, the properties of eigenvalues of $\mathbb{A}_{bed}(G)$ for crown, complete, star and coctail party graphs are discussed. Bounds for minimum bandwagon eccentric dominating energy of some standard graphs are obtained.

Theorem 5.1: Let D be a minimum bandwagon eccentric dominating set and $\beta_1,\beta_2,\ldots,\beta_n$ are the eigenvalues of minimum bandwagon eccentric dominating matrix $\mathbb{A}_{bed}(G)$, if G is

1. Any graph then $\sum_{i=1}^{n}\beta_i=|D|$,
2. A star, crown and complete graph then $\sum_{i=1}^{n}\beta_i^2=|D|+n(n-1)$,
3. A cocktail party graph $\sum_{i=1}^{n}\beta_i^2=|D|+n$,

Proof:

1. The trace of $\mathbb{A}_{bed}(G)$ is the sum of eigenvalues of $\mathbb{A}_{bed}(G)$.
$\sum_{i=1}^{n}\beta_i=\sum_{i=1}^{n}e_{ii}=|D|$.
2. For a star, crown and complete graph G sum of the squares of eigenvalues of $\mathbb{A}_{bed}(G)$is trace of $[\mathbb{A}_{bed}(G)]^2$

$$\sum_{i=1}^{n}\beta_i^2=\sum_{i=1}^{n}\sum_{j=1}^{n}e_{ij}e_{ij}=\sum_{i=1}^{n}(e_{ii})^2+\sum_{i\neq j}e_{ij}e_{ij}=\sum_{i=1}^{n}(e_{ii})^2+2\sum_{i<j}\left(e_{ij}\right)^2$$
$$\sum_{i=1}^{n}\beta_i^2=|D|+n(n-1)$$

Since for a star, crown and complete graph $2\sum_{i<j}\left(e_{ij}\right)^2=n(n-1)$.

3. For a cocktail party graph G sum of square of eigenvalues of $\mathbb{A}_{bed}(G)$ is trace of $[\mathbb{A}_{bed}(G)]^2$.

$$\sum_{i=1}^{n}\beta_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} e_{ij}e_{ij} = \sum_{i=1}^{n}(e_{ii})^2 + \sum_{i\neq j} e_{ij}e_{ij} = \sum_{i=1}^{n}(e_{ii})^2 + 2\sum_{i<j}\left(e_{ij}\right)^2$$

$$\sum_{i=1}^{n}\beta_i^2 = |D| + n$$

Since for a cocktail party graph G, $2\sum_{i<j}\left(e_{ij}\right)^2 = n$.

Theorem 5.2: For a star graph $S_n \forall n > 3$, crown $H_n\ \forall\ n \geq 6$ and complete graph K_n, if D be the minimum bandwagon eccentric dominating set and $W = |\det \mathbb{A}_{bed}(G)|$ then

$$\sqrt{|D| + n(n-1) + n(n-1)W^{\frac{2}{n}}} \leq \mathbb{E}_{bed}(G) \leq \sqrt{n(n(n-1) + |D|)}$$

Proof: By Cauchy schwarz inequality $(\sum_{i=1}^{n} g_i h_i)^2 \leq (\sum_{i=1}^{n} g_i^2)(\sum_{i=1}^{n} h_i^2)$. If $g_i = 1$ and $h_i = \wp_i$ then

$$\left(\sum_{i=1}^{n}|\beta_i|\right)^2 \leq \left(\sum_{i=1}^{n}1\right)\left(\sum_{i=1}^{n}\beta_i^2\right)$$

$$(\mathbb{E}_{bed}(G))^2 \leq n(|D| + n(n-1))$$

$$\Rightarrow \mathbb{E}_{bed}(G) \leq \sqrt{n(|D| + n(n-1))}$$

Since the arithmetic mean is not smaller than geometric mean we have

$$\frac{1}{n(n-1)}\sum_{i\neq j}|\beta_i|\,|\beta_j| \geq \left[\prod_{i\neq j}|\beta_i||\beta_j|\right]^{\frac{1}{n(n-1)}} = \left[\prod_{i=1}^{n}|\beta_i|^{2(n-1)}\right]^{\frac{1}{n(n-1)}} = \left[\prod_{i=1}^{n}|\beta_i|\right]^{\frac{2}{n}} = \left[\prod_{i=1}^{n}\beta_i\right]^{\frac{2}{n}}$$

$$\frac{1}{n(n-1)}\sum_{i\neq j}|\beta_i|\,|\beta_j| = |\det \mathbb{A}_{bed}(G)|^{\frac{2}{n}} = W^{\frac{2}{n}}$$

$$\sum_{i\neq j}|\beta_i|\,|\beta_j| \geq n(n-1)W^{\frac{2}{n}}$$

Now consider

$$(\mathbb{E}_{bed}(G))^2 = \left(\sum_{i=1}^{n}|\beta_i|\right)^2 = \left(\sum_{i=1}^{n}\beta_i\right)^2 + \sum_{i\neq j}|\beta_i||\beta_j|$$

$$(\mathbb{E}_{bed}(G))^2 = (|D| + n(n-1)) + n(n-1)W^{\frac{2}{n}}$$

$$\mathbb{E}_{bed}(G) \geq \sqrt{(|D| + n(n-1)) + n(n-1)W^{\frac{2}{n}}}$$

Theorem 5.3: For a cocktail graph G, if D be the minimum bandwagon eccentric dominating set and $W = |\det \mathbb{A}_{bed}(G)|$ then

$$\sqrt{|D| + n + n(n-1)W^{\frac{2}{n}}} \leq \mathbb{E}_{bed}(G) \leq \sqrt{n(n + |D|)}$$

Proof: The proof follows on the similar lines to Theorem5.2.

Theorem 5.4: If $\beta_1(G)$ is the largest minimum bandwagon eccentric dominating eigenvalue of $\mathbb{A}_{bed}(G)$ then for a star, crown and complete graph G,

$$\beta_1(G) \geq \frac{|D| + n(n-1)}{n},$$

for a cocktail party graph G,

$$\beta_1(G) \geq \frac{|D| + n}{n}.$$

Proof: Let Y be a non-zero vector, then by applying Rayleigh-Ritz theorem [4],

$$\beta_1\big(\mathbb{A}_{bed}(G)\big) = \max_{Y \neq 0} \frac{Y^T \mathbb{A}_{bed}(G)\ Y}{Y^T\ Y}$$

$$\beta_1\big(\mathbb{A}_{bed}(G)\big) \geq \frac{U^T \mathbb{A}_{bed}(G)\ U}{U^T\ U} = \frac{|D| + n(n-1)}{n}$$

where U is the unit matrix.
Analogously,

$$\beta_1\big(\mathbb{A}_{bed}(G)\big) \geq \frac{U^T \mathbb{A}_{bed}(G)\ U}{U^T\ U} = \frac{|D| + n}{n}.$$

4. CONCLUSION

In this paper bandwagon eccentric domination polynomial $BED(G, x)$ is studied. Theorems related to the characteristics of coefficients of $BED(S_n, x)$ of a star graph is discussed. $BED(S_n, x)$ is found. The coefficients of $BED(S_n, x)$ are tabulated. Minimum bandwagon eccentric dominating energy$\mathbb{E}_{bed}(G)$is defined. $\mathbb{E}_{bed}(G)$of families of graphs are computed. Properties of minimum bandwagoneccentric dominating eigenvalues are obtained. Results related to upper and lower boundsfor $\mathbb{E}_{bed}(G)$of cocktail party, complete, crown and star graphs are discussed.

REFERENCES

1. S. Alikhani, Y.-H. Peng, et al. Dominating sets and domination polynomials of paths.International journal of Mathematics and mathematical sciences, Hindawi, 2009.
2. J. Arocha and B. Llano. Mean value for the matching and dominating polynomial. Discussiones Mathematicae Graph Theory, vol. 20, no. 1, pg. 57–69, Uniwersytet Zielonog´orski. Wydzia l Matematyki, Informatyki i Ekonometrii, 2000.
3. C. Berge. The theory of graphs and its applications. Methuen & Co, Ltd., London, 1962.
4. R. B. Bapat. Graphs and matrices, volume 27. Springer, 2010.
5. J. A. Bondy, U. S. R. Murty, et al. Graph theory with applications, volume 290. Macmillan London, 1976.
6. F. Buckley and F. Harary. Distance in graphs. Addison-Wesley Publishing Company, 1990.
7. E. J. Cockayne and S. T. Hedetniemi. Towards a theory of domination in graphs. Networks, volume 7, number 3, pages 247–261, Wiley Online Library, 1977.
8. I. Gutman. The energy of a graph. Ber. Math-Statist. Sekt. Forschungsz. Graz, volume 103, pages 1–22, 1978.
9. F. Harary. Graph Theory. Addison-Weslay Publ. Comp. Reading Massachusets, 1969.
10. A. M. Ismayil and R. Tejaskumar. Eccentric domination polynomial of graphs. Advances in Mathematics: Scientific Journal, vol. 9, no. 4, pg. 1729–1739, 2020.
11. T. Janakiraman, M. Bhanumathi, and S. Muthammai. Eccentric domination in graphs. International Journal of Engineering Science, Advanced Computing and Bio Technology, volume 1, number 2, pages 55–70, 2010.

12. O. Ore. Theory of Graphs. Amer. Math. Soc. Colloq. Publ. 38, American Mathematical Society, Providence, RI, 1962.
13. M. R. Kanna, B. Dharmendra, and G. Sridhara. Minimum dominating energy of a graph. International Journal of Pure and Applied Mathematics, volume 85, number 4, pages 707–718, 2013.
14. R. Tejaskumar and A. M. Ismayil. Bandwagon Distance and Bandwagon Eccentric Domination in Graphs. Asian Research Journal of Mathematics, volume 19, Issue 12, pages 109-119, 2023.
15. R. Tejaskumar, A. M. Ismayil, and I. Gutman. Minimum Eccentric Dominating Energy of Graphs. International Journal of Mathematics Trends and Technology, volume 69, number 6, pages 31–38, Seventh Sense Research Group, 2023.
16. X. Li, Y. Shi, and I. Gutman. Graph Energy. Springer, New York, 2012.
17. Riyaz Ur Rehman A and A. M. Ismayil. Minimum Injective Eccentric Dominating Energy of Graphs. Asian Research Journal of Mathematics, volume 20, Issue 1, pages 49-61, 2024.
18. R. Tejaskumar and A. M. Ismayil. The Minimum Superior Dominating Energy of Graphs. Asian Research Journal of Mathematics, volume 20, Issue 1, pages 62-69, 2024.