A STUDY OF MULTI- SERVER QUEUING ANALYSIS TO MEASURE THE PERFORMANCES OF HEALTHCARE SYSTEMS IN HOSPITALS.

Abstract- Prolonged wait (queue) times in medical out-patients department is a growing concern in Nigerian hospitals/clinic, because of its numerous effects like overcrowding, patients leaving out of anger without been attended to, as well as been faced with stress for not staying too long in the system. The main motive of this paper is to research on the various technique or methods used to decrease prolonged waiting in queue. Patients waiting for some minutes, hours, days or months to receive medical services could result in waiting costs to them. The time wasted on the queue would have been adequately utilized elsewhere (opportunity cost of time spent in the queue). This paper tends to determine an optimal server level and at a minimum total system cost which include expected service costs and the expected waiting costs in a multi-server system in order to reduce patient's congestions in the hospital. Data for the study was collected in two forms. Secondary method was first used to select the most congested OPD out of numerous OPDs considered in the study. Then primary was considered for the performance measures costs calculations. The performance measures of the queuing system were calculated by using TORA optimization software. For the costs calculations and charts plots, MS excel was used. Based on the result of the analysis, one physician was suggested to be increased at the medical OPD of the hospital in order to reduce overcrowding of patients and their waiting times. Therefore, this call for refocusing so as to improve the overall patient care in our cultural context and meet the patients' needs in our society.

Keywords: Multi-server, utilization factor, reneging, waiting costs, service costsand servers.

1.0 Introduction

According to the definition by WHO, health refers to the overall state of a complete social, emotional, mental and physical well-being that considered as a resource for living a full life, and therefore, the access for the highest achievable level of health is the basic rights of every human

being without the distinction of social or economic conditions (WHO, 1948). Health is not as simple as the absence of disease, however, it is the ability to get well from ailment and/or other problems.

Healthcare on the other hand, is an act delivered by the health professionals. Is the maintenance of health through prevention, diagnosis and treatment from mental and physical impairments, Healthcare systems exists to ensure and help people, to maintain their optimal state.

An outpatient department is a department at the healthcare devoted to diagnose and consults an outpatients (American Heritage Dictionary, 2007; Zhu, Heng, & Teow, 2009). Outpatients are patients that come to see a physicians for treatment and go back home without been admitted in hospital.

In a country, like Nigeria, the number of trained physicians is much less than what the system demands. Thus, the physicians are subjected to overloading, attending to outpatients in multiple centers and being required to show up at the in-patients in multiple hospitals. These results to consultation hours to be too limited for only a short period of time, usually either of forenoons or afternoons. Moreover, even on top of this short period of service, regrettably, the service often becomes inactive due to late arrival of physicians and other interruptions (Babes & Sarma, 1991).

The study of multi-server queuing analysis to measure the performance of healthcare systems in hospitals is a topic of significant importance in healthcare management and operations research. Queuing theory is a mathematical approach used to model and analyze waiting lines, and it has been widely applied to healthcare settings to improve patient flow and reduce waiting times. The improvement of health industry in a country is one of the indicators for the economic development and prosperity, as it is directly associated with the value of human resources.

Around 1909, a Danish engineer named Erlang developed queuing theory as a result of his research work (Winston, 1991). He conducted an experiment and discovered that telephone traffic demand fluctuates. Following that, he released a report on the delays in automatic dialing equipment. Shortly after his published work and during the end of World War II. Erlang's early work was extended to more general challenges and business applications of waiting lines and to various service industries. Modeling a service industry as a queuing system has a number of benefits, including diagnosing problems and identifying restrictions in order to better understand real-world systems rather than making specific predictions about them. Queuing theory is the mathematical study of waiting lines, or the act of joining a line (queues). In queuing theory, a model is built so that queue lengths and waiting times can be calculated (Sundarapandian, 2009). The issue of queuing has been a subject of scientific debate for there is no known society that is not confronted with the problem of queuing. Queuing situations arise in all aspects of work and life and are typified by the process of queuing for services, i.e., a set of physical units (people or things) which wait in a queue or queues subject to certain rules of behavior before some services are performed on or for each unit in the queue one after the other (Burodo, Suleiman and Shaba, 2019). Wherever there is competition for limited resource, queuing is likely to occur (Koko, Burodo& Suleiman, 2018). Queues emerged when individuals requesting service, usually called customers, arrive at a service facility and cannot be served on time (Suleiman, Burodo& Ahmed, 2022).

2.0 Literature

Ali *et al* (2021) conducted a study on the Application of Multi-Server Queuing Model to Analyze the Queuing System of OPD during Covod-19 Pandemic. This study is aimed at suggesting the optimum service level of queuing system of reception and the outpatient department. The data was collected from the reception and OPD of the ABC public hospital of Hyderabad. The arrival times, service times of patients and number of doctors and receptionist at the workplace, their salaries and waiting cost of patients were all included in the data collection. Input analysis of patients` arrivals and service was conducted by input analyzer of Rockwell Arena software.TORA optimization software was adopted for the calculation of performance measures. Various costs of queuing system were calculated in MS Excel and the required graphs were also plotted. Therefore, the study recommended that one receptionist and one doctor should be increased to bring optimality in the queuing system and patients` flow. Moreover, waiting cost of patients should be decreased to greater extent.

Kazemi *et al.* (2017) which aimed to evaluate the performance of emergency departments in two hospitals using simulation models. The study used queuing performance measures to compare the performance of single-server and multi-server systems and found that multi-server systems can reduce waiting times and improve patient satisfaction.

The investigative study of multi-server queuing analysis to measure the performances of healthcare systems of two hospitals can provide insights into the design and operation of healthcare systems to improve patient care and resource utilization. The study can be guided by queuing theory and operations management principles and can use various data collection and analysis methods to obtain meaningful results.

The multi-server queuing model is a mathematical model that represents a system with multiple servers and a single waiting line. This model is particularly useful for healthcare systems, as it

allows us to analyze the impact of different factors on patient waiting times, such as the number of servers, the arrival rate of patients, and the service time of each server.

To measure the performance of the healthcare systems in the two hospitals, we will use various performance metrics, such as the average waiting time, the average queue length, and the utilization rate of each server. These metrics will help us to identify the bottlenecks in the system and to optimize the allocation of resources to improve the overall efficiency of the healthcare system.

Segun (2020) conducted a research on the Performance Modelling of Health-care Service delivery in Adekunle Ajasin University, Akungba-Akoko, Nigeria Using Queuing Theory. The purpose of the study was to determine the waiting, arrival and service times of patients at AAUA Health- setting and to model a suitable queuing system by using simulation technique to validate the model.

The study employed analytical and simulation methods to develop a suitable model. A stopwatch was used to calculate the number of minutes spent by each patient from the reception section where patients arrive and collect their hospital cards or register to the last section (the consulting room section). Data on the arrival time, waiting time and service time of each patient was collected on Weekdays (Mondays through Fridays) for three (3) weeks. The data was calculated and analyzed using Microsoft Excel. Based on the analyzed data, the queuing system of the patient current situation was modelled and simulated using the PYTHON software. The result obtained from the simulation model showed that the mean arrival rate of patients on Friday week1 was lesser than the mean service rate of patients (i.e. 5.33> 5.625 ($\lambda > \mu$). What this means is that the waiting line would be formed which would increase indefinitely; the service facility would always be busy. The analysis of the entire system of the AAUA health center

revealed that queue length increases when the system is very busy. The study recommends the need for the AAUA Health-Centre to improve the quality of service offered to the patients visiting this health center.

Nor and Binti (2018*)* Applied Queuing Theory Model and Simulation to Patient Flow at the Outpatient Department. The objective of the study is to determine the waiting time, arrival time and service time of patients at the outpatient counter and to model suitable queuing system using simulation technique. The research was carried out at a Public Health Clinic in southern Malaysia. Descriptive and simulation method were employed to develop suitable model. The collection of data on waiting time for this study is based on the arrival rate and service rate of patients at the outpatient counter. The data calculated and analyzed using Microsoft Excel. Using the ARENA, the patient's existing queuing system was modelled and simulated based on the analyzed data. Descriptive analysis and observations study was used to determine the time taken of patients from the registration until seen by pharmacist at the outpatient clinic. The results obtained from the ARENA simulation stated that the average waiting time of patient have to wait before getting the treatment is 54.295 minutes whereas the maximum waiting time is 144.48 minutes. Then, the average service time for patient to get the treatment is 13.48 minutes whereas the maximum service time for several patients is 23.724 minutes. Therefore, the average total time spend by patients in outpatient department is 68.315 minutes and maximum total time in system is 156.718 minutes. Total average number of patients that arrived at outpatient counter is 327 patients per day. Thus, based on the result average total number of patient gives the utilization of server at outpatient department is 78.84%.

Kumar *et al*. (2020) used a queuing model to analyze the patient flow in a hospital emergency department. Similarly, in a study by Brailsford et al. (2008) used a queuing model to analyze the impact of different scheduling policies on patient waiting times in a hospital outpatient department.

Queuing theory provides a mathematical framework for analyzing waiting lines or queues. It can be used to study the performance of service systems, such as healthcare facilities, by examining key performance indicators such as waiting times, queue lengths, and service rates. Queuing theory has been widely applied in healthcare research to evaluate system performance, identify bottlenecks, and improve patient flow (Wang et al., 2018).

Healthcare management theory focuses on the effective and efficient management of healthcare systems. It includes aspects such as patient safety, quality improvement, resource allocation, and strategic planning. Healthcare management theory can be used to analyze the performance of hospitals in terms of their ability to provide high-quality care while managing resources effectively (Fottler, Khatri, & Savage, 2010).

Operations research is a quantitative approach to decision-making that uses mathematical models to optimize system performance. It involves identifying and analyzing trade-offs between different performances measures, such as cost, quality, and throughput. Operations research can be used to design and evaluate healthcare systems, including hospitals, to improve efficiency and quality of care (Ozcan, 2012).

Khan *et al* (2021) conducted a research on improving the performance of reception and OPD by Using Multi-Server Queuing Model. The aim of the research work was to examine the performance of current queuing system and offer solutions on how to achieve its optimum service level. The present data was acquired in two steps. The first step was to collect the data

from reception. While the 2nd step was to gather the data from OPD. Data included variables like arrival rate, service rate and wage of the front desk staff and the variables consisted of service times of the different customers concerning seeing different physicians sitting at the OPD. Moreover, the salary of appointed medical doctors and waiting cost of the patients were additionally gathered.

The Rockwell Arena software was used to import the data and reveal its dispersion. When it was validated that arrival and service distribution of people followed the poison and exponential distribution respectively, then the ordinary arrival and service rate were put into TORA optimization software in addition to the variety of doctors. Performance measures were determined by the assistance of TORA optimization software application. The required performance measures were taken into MS succeed for estimation of cost and outlining graphs. Since there were already 10 doctors; they calculated performance measures and cost analysis were done and the results showed that 2 more doctors should be hired. As a result of the aforementioned decision, the system utilization decrease from 86.6% to 72.2%. The time spent by patients in the system reduced from 0.238 hours to 0.178 hours; and the probability of the system to remain idle increased from 1.2% to 1.6%. This research study was only carried out on the medical OPD; it can additionally be conducted in all the OPDs of hospital in order to improve the healthcare delivery at public healthcare. The seasonal analysis of queuing system of the OPDs can also be carried out.

Kembe*et al.* (2012) analyzed the queuing characteristics at the Riverside Specialist Clinic of the Federal Medical Centre, Makurdi using a Multi-server queuing Model and determined the waiting and service Costs with a view to determining the optimal service level. The results of the analysis showed that average queue length, waiting time of patients as well as overutilization of

doctors could be reduced when the service capacity level of doctors at the Clinic is increased from ten to twelve at a minimum total costs which include waiting and service costs. The most common objectives of the study have included the reduction of patient's time in the system (outpatient clinic), improvement on customer service, better resource utilization, and reduction of operating costs (Gorunescu, McClean and Millard, 2002) Analysis in such cases involves, in depth analysis of the patient's arrival and flow, structure of the system, manpower characteristics and the scheduling system. Appropriate queuing models are then developed and applied for process modifications, appropriate staffing, scheduling or facility changes. The M/M/s model therefore is the best queuing model to be used in this study based on the objectives.

Mittal and Sharma, (2020) Studied the Probabilistic Model for the Assessment of Queuing Time of Corona Virus Disease (Covid-19) Patients Using Queuing Model. To determine patient waiting time in hospitals for the confirmation of disease, the queuing theory was applied for the multi-server system. In order to estimate the time, it will take to detect and identify diseases under severe loading conditions, this study provides a sequential queuing model. The objective is to propose a simplified probabilistic model to determine the general behavior to predict how long the treatment cycle takes to diagnose and categorize the infected individuals. The law of the isolated logarithm is proved for this type of method, demonstrating that the general process of recognition is consistent right of iterated logarithm. The numerous measurement criteria are shown graphical representations in some cases. The findings of the modelling revealed that the waiting time for patients in the course of studies, detections, detecting or treatment of corona viruses will increase according to the logarithm rule.

3.0Methods

Data for this study were collected from the federal Medical Centre, keffi, Nasarawa State. The methods employed during data collection were secondary data for the selection of study area and also primary data (Direct observations, personal interview and questionnaire) by the researcher.

The following assumptions were made for the queuing system at the Federal Medical Centre, keffi which is in accordance with the queuing theory. They are:

i. Length of the line is not restricted
ii. The size of the calling population is infinite. This assumption implies that the input source is unlimited; the assumption is also warrant in case where the number of customers is finite but after being served customer rejoins the input source.
iii. The arrival rate distribution is approximately by a Poisson distribution**.**
iv. There is no balking
v. There is no reneging. This assumption implies that customers stay in line until served**.**
vi. The queue discipline is first-come first-served (FIFO)**.**
vii. The service time distribution is approximated by an exponential distribution.

2.1 The M/M/S Model

The model adopted in this work is the (M/M/s: FCFS/∞)- multi server queuing model. For this queuing system, it is assumed that the arrivals follows a Poisson probability distribution at an average of ג customers (Patients) per unit of time. It is also assumed that they are served on a first come, first serve basis by the any of the servers (in this case the physicians). The service time are distributed exponentially with an average of μ customers (patients) per unit of time and the number of servers S. If there are n customers in the queuing system at any point in time, then the following two cases may arise:

1. If n<S, (number of physicians in the system is less than the number of servers), then there will be no queue. However, (S-n) number of servers will not be busy, the combined service rate will then be $\mu_n = \mu n$; n<S.
2. If n≥S, (number of physicians in the system is more than or equal to the number of servers), then all servers will be busy and the maximum number of physicians in the queue will be (n-S). The combined service rate will be $\mu_n = S\mu$; n≥S

Therefore,

$$\boldsymbol{p_0} = \left[\left[\sum_{\mathbf{n=0}}^{\mathbf{s-1}} \frac{(\lambda/\mu)^{\mathbf{n}}}{\mathbf{n!}}\right] + \frac{(\lambda/\mu)^{\mathbf{s}}}{\mathbf{s!}}\left(\frac{\mathbf{s\mu}}{\mathbf{s\mu-\lambda}}\right)\right]^{-\mathbf{1}} \tag{3.0}$$

Now the other properties of the multi-channel system can be found out.

The expected (average) number of customers in the system denoted by $L_s$will be,

$$\boldsymbol{L_s} = \frac{\boldsymbol{\lambda.\mu}\left(\frac{\boldsymbol{\lambda}}{\boldsymbol{\mu}}\right)^{\boldsymbol{s}}}{\boldsymbol{(s-1)!(s-2)^2}}\boldsymbol{p_0} + \frac{\boldsymbol{\lambda}}{\boldsymbol{\mu}} \tag{3.1}$$

while the expected (average) number of customers waiting in the queue L_q is,

$$\mathbf{L_q} = \frac{\lambda.\mu\left(\frac{\lambda}{\mu}\right)^{s}}{(s-1)!(s\mu-\lambda)^2} * \mathbf{p_0} \tag{3.2}$$

In order to check the arrival of patients, the necessary parameter, is the average time a customer spends in the system defined as,

$$\boldsymbol{W_S} = \frac{L_S}{\lambda} = \frac{\mu\left(\frac{\lambda}{\mu}\right)^{s}}{(s-1)!(s\mu-\lambda)^2} * \boldsymbol{p_0} + \frac{1}{\mu} \tag{3.3}$$

Before a patient is served, the patient is expected to wait in the queue defined as,

$$\boldsymbol{W_q} = \frac{L_q}{\lambda} = \frac{\mu\left(\frac{\lambda}{\mu}\right)^{s}}{(s-1)!(s\mu-\lambda)^2} \cdot \boldsymbol{p_0} \tag{3.4}$$

With the chances of having to wait given by the proportion defined in form of

Probability as; $p(n \geq s) = \frac{\mu.\left(\frac{\lambda}{\mu}\right)^{s}}{(s-1)!(s\mu-\lambda)} \cdot p_0$ (3.19)

The utilization factor (ρ).

$$\rho = \frac{\lambda}{\mu s} \tag{3.20}$$

There are very slim chances that a patient arrives and finds no queue. This happens when the service rate μ is faster than the arrival rateλ. The interpretation of this in the physical situation is that idle, thus will have a cost impact to the facility. The chances of a customer or a patient to enter the service without waiting is given by $1 - p(n \geq s)$

The analysis of parameters used to check the minimum number of servers necessary to meet the requirements of the patients without idle servers is obtained from the average number of idle servers given by s.

The utilization rate of the servers is defined by $\rho = \frac{\lambda}{s\mu}$ and thus the efficiency of M/M/s model is obtained from the ratio,

$$= \frac{Average\ number\ of\ customers\ served}{total\ number\ of\ customers}$$

3.2 Introducing Costs into the Model

Three cost were incorporated in the study that is service cost, waiting cost and total system cost.

1. Expected service cost: Salaries of doctors paid by the hospital for rendering their services.
2. Expected opportunity cost: The cost of patients while being at the hospital because of doing nothing to earn.
3. Expected total system cost: This cost is the addition of expected service cost and expected waiting cost.

Economic analysis of the cost will assist the hospitals administration to line up the balance between the waiting cost, the service cost that is increase of service cost by providing better service and decreasing waiting cost of the patients that incurs for waiting in the hospitals.

$$E(Sc) = SCs \qquad (3.5)$$

Where S= number of servers, Cs= service cost of each server.

Cost paid by the customers/patients due to waiting in the system

$$E(Wc) = (\lambda\ Ws)*Cw \qquad (3.6)$$

Where, ג= number of arrivals, Ws= Average time an arrival spends in the system.

4.0 Analysis and Results

In this paper, analysis regarding selection of the study area, profession and associated waiting costs of patents, service costs, system cost, etc. were done in order to determine the optimum cost. The results will informed the management on the minimum number of servers required in order to reduce/minimize the waiting costs of patients and at the same time, provides services at a minimum service costs in the OPD's of the case study hospitals.

4.1 Selection of the study Area

The researcher considered eleven OPD's in the case study hospital for the selection of the study area. Data on the number of arrival of patients at various out-patients Department was collected from the research and Statistics department for one month (September, 2023). Frequency distribution of the collected data revealed that the highestcongested OPD among the eleven OPD's was the medical out-patients department (See fig. 1), which was selected for the study to be conducted. On the basis of maximum arrival rate of patients, medical out-patients (OPD) was considered as the study area.

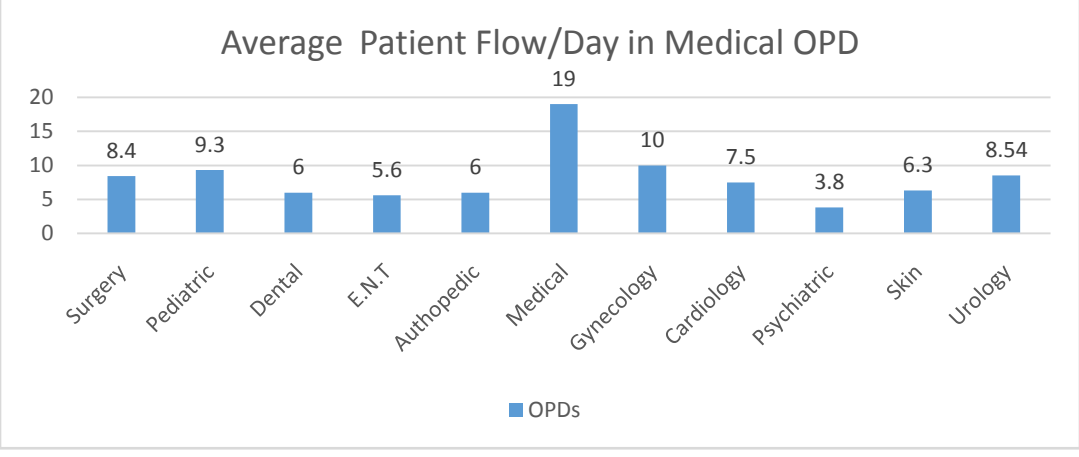

Fig. 1 Average per day of patients' arrival at Medical OPD

Based on the maximum number of arrivals, medical out-patients Department of the case hospital that required the research attention and optimization because as at the time the OPD was confronted, the researcher saw the problem of long queues and also long waiting time. Since the waiting time of patients was associated with the opportunity cost, it was required to study and optimize the queuing system of medical out-patient Department.

4.1.2 Profession distribution at case hospital

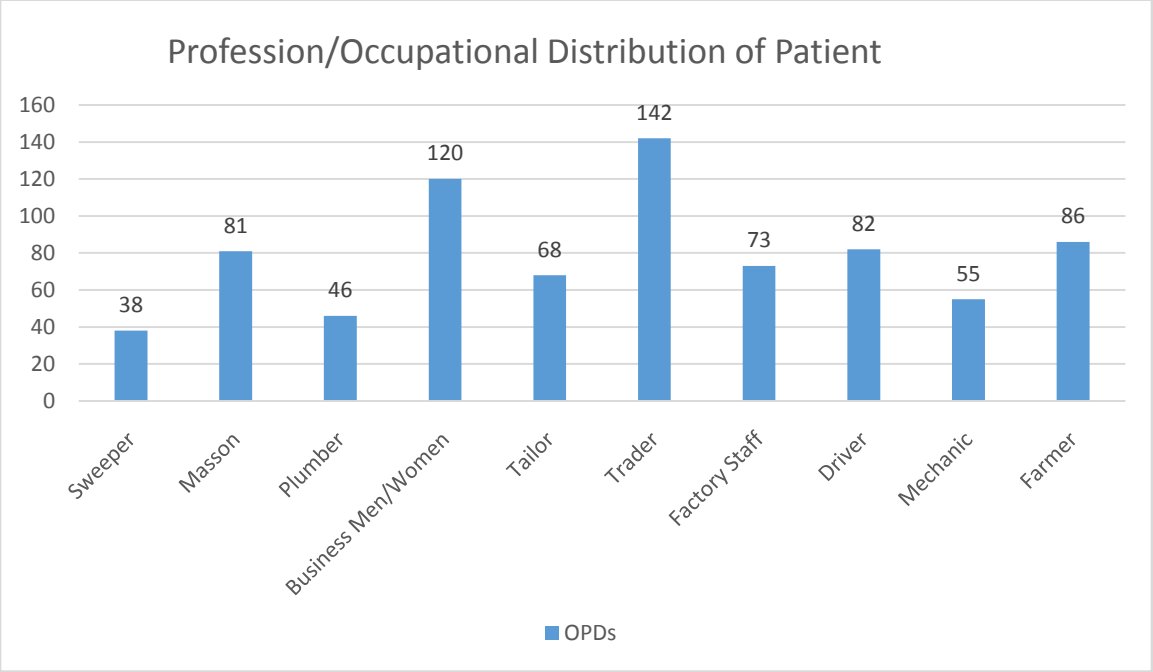

Fig. 2Profession distribution of the patients arriving at the OPD

Data on the patient's professions was also collected. Based on the data collected, patients who arrived at the OPD belonged to ten (10) different professions (see Fig 2). It can be seen from fig. 2 that the maximum number of patients were trader i.e 142.

The average waiting cost of patients was computed to be #119.34 (see fig 3). The highest waiting cost among all the costs i.e #214.3 was of the trader because of remaining idle for the whole day.

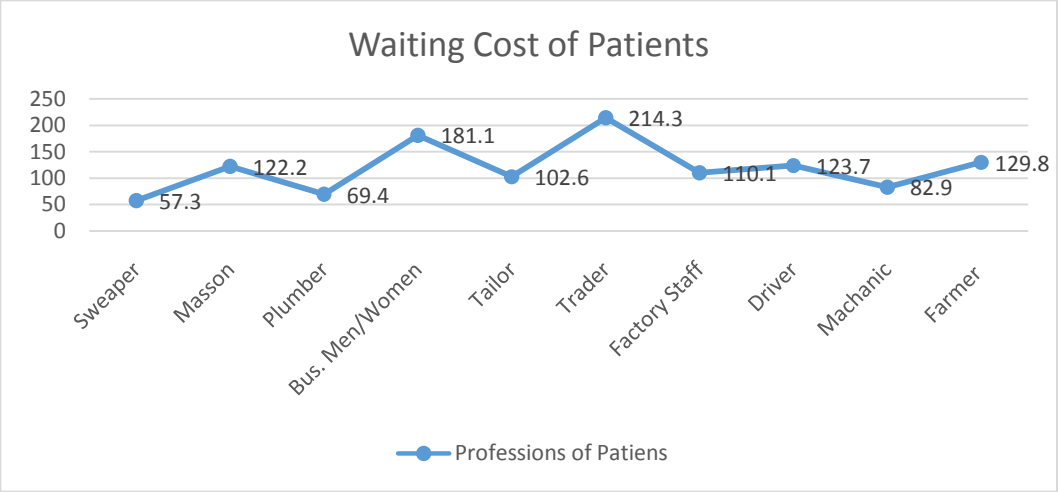

Fig. 3. Waiting cost of patients arriving at the OPD

1.5 Service cost/salary of physicians at medical OPD

In this research, service cost was termed as the cost which could be affected after the suggestion of the optimal scenario and it was only the salaries of physicians. Thus, the salaries of available physicians at medical out-Patient department of the hospital were considered as expected cost. There were five (5) physicians at the medical OPD and the corresponding salaries are presented in table 1.

Table: 1 physician's salaries at the case hospital

No of Physicians	Salaries of Physician Per month (₦)	Salaries of Physician Per Day (₦)	Salaries of Physician Per Hour (₦)
1	700,000.00	23,233.33	3,888.88
2	584,000.00	19,466.66	3,244.44
3	356,000.00	11,866.66	1,977.77
4	473,000.00	15,766.66	2,627.77
5	441,000.00	14,700	2,450.00

Average	510,800	17,006.66	2,837.77

Average of the monthly salaries of physician's turns out to be #510,800 at medical OPD.

2.1 Medical OPD of case study hospital

At the medical out-patients department (OPD) of case hospital, there were queue outside the physician hall waiting to be attended to. Multi-server queuing model was applied on the queues so that queuing system could be optimized

4.2.1 Performance measures of the medical OPD of the hospital

The medical OPD of the hospital was ranked as the busiest or the most congested in terms of arrival of patients to see the physicians for consultation than any other OPD.

The performance measures of the queuing system of medical OPD of case hospital is presented in table 2.

The calculation of performance measures was based on five (5) scenarios, so that the optimum number of physicians can be calculated in order to optimize the waiting time of patient at the medical OPD. In each scenario, one physician was assumed to be increased with same arrival and service rate of patients

The average arrival rate of patients per hour was calculated to be 28.8, 30.4, 24.3, 27.9 and 26.1 having five (5) physicians. And 57.9, 67.3, 52.5, 57.6 and 53.7 patients per hour were the average service rate of patients. On calculating the utilization factor, it came out to be 95%, 50%, 34%, 25% and 20% respectively, this means that, if one physician is increased, the congestion will be reduced by 50% against the 95% and so on.

Table 2: Performance measures of the queuing system of medical OPD of case hospital

Scenarios	1	2	3	4	5
Physicians	1	**2**	3	4	5
Arrival rate (arrived)	577	**608**	486	558	521
Service rate (served)	563	**597**	477	552	512
Total arrival time (hr)	32	**32**	32	32	32
Total service time (aims)	2363	**2149**	2226	2310	2289
Expected service cost/hr	3,888.88	**3,244.44**	1,977.77	2,627.77	2,450.00
Expected waiting cost/hr	870.64	**294.30**	199.50	200.35	4.95
Expected total system cost/hr	4759.52	**3,538.74**	2,177.27	2,828.12	2,454.95
L'daeff	577.0	**608.0**	486.0	558.0	521.0
Utilization factor (Rho)	95%	**50.9%**	34%	25%	20%
Probability (P_0)	0.35884	**0.36116**	0.36100	0.3639	0.3647
Ls (Number of patients in the system/hr)	24.02487	**9.01843**	7.01887	6.01087	5.01758
Lq (Number of patients in the queue/hr)	20.4211	**4.23016**	1.02106	0.31013	0.06001
Ws (Time spent by patients in the system/hr)	0.301781	**0.09682**	0.082101.	0.071810	0.00190
Wq (Time spent by patients in the queue/hr	0.28200	**0.03000**	0.01000	0.00000	0.00000

Suppose on duty, two physicians reported, the utilization factor (Rho) will be reduced from 95% to 50.9%. Which indicates that the server (physician) will be 50.9% less busy in scenario 2 as regards to scenario 1 (see table 2). And also the server (physician) will be 34% less busy in scenario 3 and so on. Furthermore, 20.42 patients will be left waiting in line and 24.02 patients in the system (See table 2). This means that when the server (physician) is increased by 2, patients that will be waiting in line is reduced by 9.018. The expected service cost/hr, expected waiting cost/hr and the total system cost/hr were calculated to be #3,244.44, #294.30 and #3,538.74 respectively (see fig 4). The performance measures calculated for the second scenario were consider to be optimum because the expected total system cost was seen to be minimum (see fig 4).

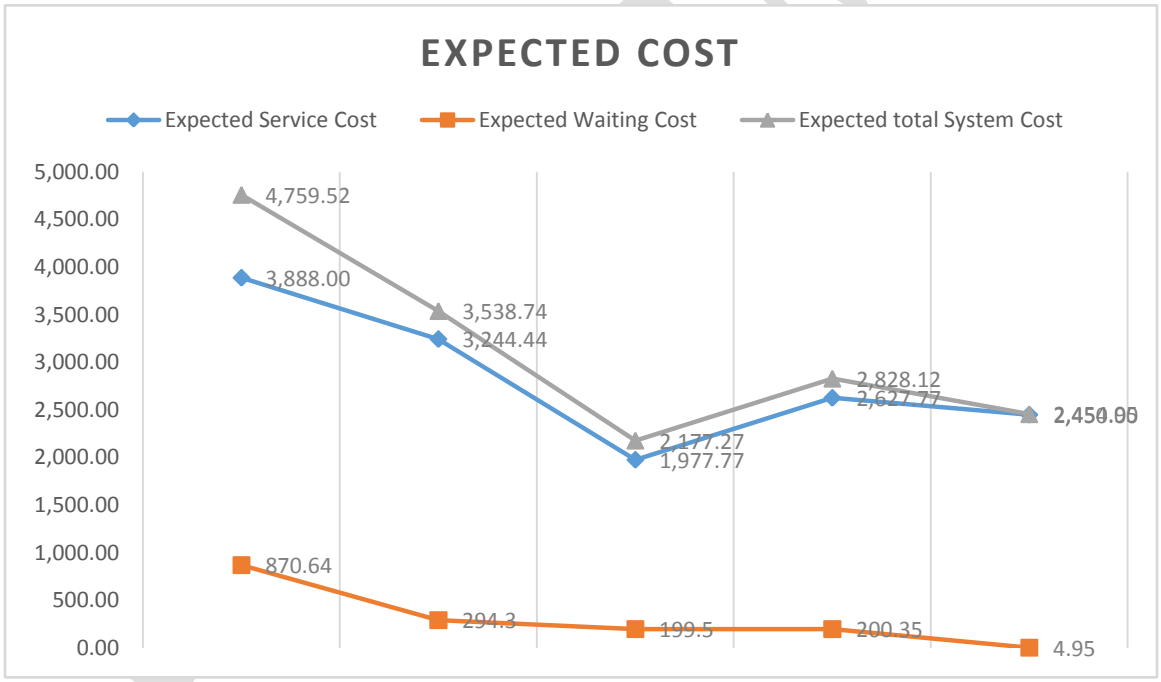

Fig. 4. Cost calculations of the medical OPD case hospital

4.3 Discussion of Results

The queuing characteristics at the Federal Medical Centre, keffi in Nasarawa State was analyzed using a multi- server queuing model and the waiting and service costs determined with a view to determining the optimal service level. The results of the analysis shows that the average queue length, waiting time of patients as well as overutilization of physicians could be minimized when the service capacity level of physicians at the hospital is increased and at minimum total costs which include service costs and waiting costs. Since they will have to wait for less time as compare to the existing scenarios, their waiting costs would certainly be decreased which will be good point on the side of the healthcare providers and the system at large.

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