Original Research Article

Empirical evidence on the validity of the Conditional Higher Moment CAPM in the Bombay Stock Exchange

Abstract

The present study investigated empirical validity of the conditional higher order moment Capital Asset Pricing Model (CAPM) in the context of the Indian stock market, specifically the Bombay Stock Exchange utilizing the data from sectoral indices for the period from April 2011 to March 2021. The study's findings reveals that the higher moment i.e. conditional coskewness and cokurtosis hold significant pricing implications and do have an impact on returns in the Indian stock market. The explanatory power of the model showed an increase as compared to the unconditional CAPM as the R-square value obtained was greater than the latter model. The results of the study were found to be mixed and inconclusive. Hence, the conditional higher moment CAPM cannot be validated in the Indian context.

Keywords: covariance, coskewness, cokurtosis, Indian stock market, Bombay Stock Exchange (BSE).

1. Introduction

Comment [MS1]: However, the research background failed to fully elucidate the issues at the research site. However, the novelty of this study is that it is insufficient.

Despite being widely used, the traditional Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965), and Mossin (1966) has failed to explain the cross-sectional variation in average returns over several decades of empirical testing by numerous researchers like Black et al. (1972), Douglas (1969), Fama & French (1992), Fama and MacBeth (1973) and Lintner (1965). Because of its theoretical appeal and easy to use, it has developed its place as the most popular model being used by US companies to estimate the cost of equity capital.

The attempts to improve the model by retaining its spirit led to two different categories i.e. conditional and unconditional version. The early studies conducted by Jagannathan and Wang (1966) suggested that the CAPM may hold but in a conditional way. The later studies like Lettau and Ludvigson (2001), Lewellen and Nagel (2006), Chung et al. (2006) and Vendrame et al. (2016) etc. presented mixed results related to conditional CAPM.

The efforts to find the better model that could explain the asset returns let to the development of extended versions of CAPM by incorporating skewness and kurtosis. Kraus and Litzenberger (1976) and Fang and Lai (1997) were the first to incorporate the third and fourth moment respectively. The positive skewness was favored by the investors whereas the investors expected a risk premium on the assets having positive kurtosis.

It was assumed that the systematic risk and the risk premia are constant i.e. they do not vary with respect to time which is unrealistic assumption. Hawawini (1980), Hanson (1994), Harvey and Siddiqui (1999), Javid and Ahmad (2008) and Adrian and Franzoni (2009) suggested that these measures changes with respect to time. Hence, to account for the conditional information, the unconditional higher moment was modified. An attempt has been in the present study to test the conditional higher moment CAPM with time varying betas and risk premia using the specification derived from Bollerslev et al. (1988) which is represented by the following equation:

$$E_{t-1}\,(R_{it}) = \alpha + \hat{\beta}^{c}{}_{imt}E_{t-1}(R_{mt}) + \hat{\gamma}^{c}{}_{imt}E_{t-1}(R_{mt})^{2} + \hat{\delta}^{c}{}_{imt}E_{t-1}(R_{mt})^{3} + \mu_{t} \qquad (1)$$

The $\hat{\beta}^{c}{}_{imt}$, $\hat{\gamma}^{c}{}_{imt}$ and $\hat{\delta}_{imt}$ denote the covariance, co-skewness risk and co-kurtosis risks respectively.

2. Literature review

A substantial portion of financial research emphasizes the need to expand the conventional CAPM model to account for skewness and kurtosis. Although the mean-variance methodology holds a dominant position in asset pricing, there exists a wealth of evidence supporting the inclusion of higher moments. Soon after the creation of the CAPM, researchers contended that rational risk-averse investors possess a preference for positive skewness (Adritti and Levy (1972)) and dislike for kurtosis (Scott and Horvath (1980)).

The first notable work to include skewness into the conventional CAPM framework was done by Kraus and Litzenberger (1976). A significant positive beta premium and significant negative coskewness premium were obtained, which was consistent with their expectations. Friend and Westerfield (1980), Lim (1989), and Harvey and Siddique (2000a, 2000b) investigated the conditional versions of the three-moment CAPM under certain conditions. The latter researchers illustrated that adding skewness to the standard CAPM led to an increase in the adjusted R-squared statistic. Fang and Lai (1997) and Athayde and Flores (1997) were the first to evaluate the four-moment CAPM.

Fang and Lai (1997) engaged in a triple-sorting process involving portfolios categorized by beta, coskewness, and cokurtosis and obtained significant improvement in the R-squared value for the four-moment CAPM model, accompanied by positive significant risk premium associated with cokurtosis. Conversely, Athayde and Flores (1997) reached that skewness outweighs kurtosis in terms of importance.

In the context of emerging market stocks, Hwang and Satchell (1999) tested an unconditional four-moment CAPM and found better explanation for the variability observed in average stock returns. Dittmar (2002) was the first to test the four moment CAPM and obtained a substantial reduction in pricing errors.

Kostakis, Muhammad, and Siganos (2012) performed the test using the data from UK stock market and obtained a negative risk premium associated to coskewness and positive risk premium associated with cokurtosis risk. Furthermore, the incorporation of coskewness and cokurtosis improved the explanatory power of the FF and momentum factors.

Lambert and Hubner (2013) expand the four-moment CAPM to encompass the Fama and French factors in the US market and found that cokurtosis risk holds greater relevance for portfolios with low book-to-market ratios, while small portfolios exhibited more sensitivity to coskewness risk. Also the inclusion of these higher moments consistently led increase of the R-squared value. Using the US daily data, Young et al. (2013) found that high exposure to market skewness corresponds to diminished returns, while elevated kurtosis exposure corresponds to higher returns. Moreno and Rodriguez (2009) reach a similar conclusion regarding coskewness.

Ajibola et al. (2015) examined the higher order CAPM (both conditional and unconditional form) in the Nigerian stock market. The unconditional version showed that only the co-skewness risk is priced, whereas the weak relationship was seen between the covariance and co-skewness with asset returns. The results of the conditional version showed that in the up market, none of the risk factors are priced whereas the covariance and coskewness explained the asset returns in the down market. The ability of the model was improved while considering the conditional version.

Cederburg and O'Doherty (2016) showed that conditional CAPM explains the risk premium that low beta portfolios earn higher unconditional alpha than high beta portfolios. Vendrame et al. (2016) showed that investors forego some returns (for the same variance) for positive skewness while for positive cokurtosis, the investors need to be compensated as it delivers large losses for expecting higher expected returns. Nishantha (2018) showed that coskewness was found to be significant while the covariance and cokurtosis were insignificant in explaining the returns of stocks in Colombo Stock Exchange.

3. Materials and Methods

Weekly data of 10 sectoral indices listed in S&P BSE for the period from April 2011 to March 2021 downloaded from Bombay Stock Exchange (BSE) website has been used in the study. The 91 days Treasury bill (T-Bill) rates, obtained from the website of Reserve Bank of India (RBI), used as the proxy for the risk free rate of interest.

The methodology proposed by Pettengill et al. (1995) has been used in the study. The weekly sector returns, market returns for each sector were computed similar to that of Asthana and Ahmed (2023). The betas (down market and up market) for each sector was obtained using equation 1 with the help of GARCH(1,1) specification given by Bollerslev et al. (1988). The rolling regression with 60 months rolling window and step size 1 was used to estimate the conditional rolling beta.

When the beta value obtained is greater than 1 ($\beta > 1$), it signifies that the Indices' price displays higher volatility compared to the market. Conversely, when the beta value obtained is less than 1 but greater than 0 ($\beta < 1$), it suggests that the Indices' price exhibits lower volatility than the market. If the beta value is less than zero ($\beta < 0$), it indicates an inverse relationship with the market. The Jarque-Bera test serves as a measure of goodness of fit, determining whether data attributes such as skewness and kurtosis align with those expected in a normal distribution. A higher Jarque-Bera value, accompanied by a p-value below 0.05, signifies that the data exhibits deviations from normality.

After the beta estimation (down market and up market) for each index, the portfolios were formed similar to unconditional CAPM (Asthana and Ahmed, 2023). The cross sectional regression equation of the model was used to examine the validity of conditional higher moment CAPM which is represented as:

$$R_{pt} = \hat{\eta}_0 + \hat{\eta}_1 \beta^c_{pmt-1} + \hat{\eta}_2 \gamma^c_{pmt-1} + \hat{\eta}_3 \delta^c_{pmt-1} + \mu_{pt} \qquad (2)$$

The following hypotheses must hold true to empirically validate the model theory:

$\hat{\eta}_0 = 0$ i.e. the intercept term is statically equal to zero and insignificant.

$\hat{\eta}_1 > 0$ i.e. the systematic risk should be positive and significant.

$\hat{\eta}_2 < 0$ i.e. the co-skewness risk should be negative and significant.

$\hat{\eta}_3 > 0$ i.e. the co-kurtosis risk should positive and significant.

4. Analysis and Results

Descriptive statistics helps us to understand the characteristics of returns data employed for CAPM testing, as well as the coefficients of portfolios. Given that the CAPM testing involves grouping companies to create portfolios, it becomes crucial to comprehend the tendencies of individual index returns. The table below provides descriptive statistics for portfolio returns (down and up market). (Table 1)

Table 1.a: Descriptive Statistics: Index returns (Down Market)

	Index 1	**Index 2**	**Index 3**	**Index 4**	**Index 5**	**Index 6**	**Index 7**	**Index 8**	**Index 9**	**Index 10**
Mean	-0.022	-0.024	-0.026	-0.021	-0.030	-0.023	-0.024	-0.025	-0.034	-0.016
Std. Dev.	0.053	0.373	0.457	0.504	0.332	0.183	0.892	0.155	0.755	0.193
Skewness	-0.860	-1.150	-0.911	-1.825	-0.594	-0.020	-0.470	0.244	-0.834	-0.875

Kurtosis	5.116	6.027	4.547	10.967	6.720	5.904	7.393	5.431	4.251	4.472
Jarque-Bera	62.290	21.094	47.838	43.062	27.684	17.633	16.979	51.506	36.389	43.795
Probability	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

Table 1.b: Descriptive Statistics: Index returns (Up Market)

	Index 1	**Index 2**	**Index 3**	**Index 4**	**Index 5**	**Index 6**	**Index 7**	**Index 8**	**Index 9**	**Index 10**
Mean	0.023	0.025	0.025	0.024	0.028	0.020	0.020	0.020	0.032	0.019
Std. Dev.	0.464	0.593	0.565	0.395	0.461	0.890	0.915	0.551	0.231	0.708
Skewness	1.383	2.449	1.769	2.841	1.907	2.809	0.772	1.735	0.902	2.148
Kurtosis	7.385	5.490	9.014	3.772	9.514	6.905	6.759	8.743	7.601	5.064
Jarque-Bera	22.091	20.761	40.766	12.201	47.171	18.676	18.308	37.059	24.553	52.452
Probability	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

It can be seen from the above table that for the down market, the mean weekly returns ranges between 3.024 (P3) to 0.559 (P5) while for the up market, the mean weekly returns ranges between 1.614 (P1) to 0.445 (P4). The skewness values for all portfolios were obtained positive (up market) and negative for down market except P4. Negative skewness indicates that the data is skewed to the left, on the other hand, positive skewness signifies the data is skewed to the right. In finance, negative skewness corresponds to low returns (frequent small gains) or high risk (infrequent significant losses), while positive skewness relates to low risk (minor losses) or high returns (occasional substantial gains). The obtained kurtosis values are greater than 3, indicating a leptokurtic distribution of returns. In financial terms, a leptokurtic distribution signifies increased investment risk. The high value of Jarque-Bera statistic and p-value below 0.05 suggest that more than half of the portfolios have return distributions that deviate from normality.

Using the specification of Bollerslev (1988), in the first step, the conditional higher moment beta, lambda and gamma for each sector were estimated using 60months rolling regression applying the GARCH(1,1) as an estimation technique.

The beta values for six indices (down market) and for eight indices (up market) were found to be greater than 1 (β > 1), indicating that these indices' prices exhibit higher volatility compared to the broader market. Additionally, beta values for four indices (down market) and two indices (up market) were obtained to be less than 1 but greater than 0 (β < 1), signifying that the price movements of these indices are relatively less volatile than the overall market. A beta value of 1 (β = 1) implies that the indices' prices move in proportion to the market, both in terms of increase or decrease. (Table 2)

Table 2: Estimated beta values of sectoral indices

Indices	Beta (Down Market)	Beta (Up Market)
Index 1	0.981	1.424
Index 2	1.007	1.803
Index 3	0.341	1.477
Index 4	1.506	1.994
Index 5	1.681	1.330
Index 6	1.367	0.950
Index 7	1.660	0.691
Index 8	1.596	1.400
Index 9	0.296	1.482
Index 10	0.822	1.926

In the second step, after the estimation of unconditional betas, portfolios were formed by arranging the betas in ascending order such that the lowest betas were put in first portfolio and the highest betas were put in last portfolio. A total of five portfolios were made. Further, the portfolio returns were calculated using the weekly returns as average of the returns of individual sectors. Similarly, the portfolio betas were calculated using betas as average of the betas of the individual sectors for each portfolio.

In the second stage, following the estimation of conditional betas, portfolios were constructed similar to unconditional CAPM (Asthana and Ahmed, 2023). To test the validity of conditional higher moment CAPM, the cross sectional regression (equation 2) was used and the associated hypothesis must hold true to validate the model theory.

Table 3: Result of Conditional Higher Moment CAPM (Down Market)

Portfolios	Coefficient		R-square	t-statistic	p-value
Full Sample	**α**	0.723	0.032	0.807	0.420
	β	0.583		0.804	0.421
	λ	1.267		19.035	<0.001
	γ	-1.175		-9.233	<0.001
P1	**A**	0.918	0.090	1.310	0.192
	β	-3.897		-0.137	0.891
P2	**α**	1.159	0.050	1.297	0.196
	β	2.347		0.332	0.740
P3	**α**	4.668	0.011	3.498	0.001
	β	1.870		1.552	0.122

P4	α	1.775	0.010	1.468	0.144
	β	1.202		0.153	0.878
P5	α	0.415	0.020	0.521	0.603
	β	-1.832		-0.227	0.821

Table 4: Result of Conditional Higher Moment CAPM (Up Market)

Portfolios	Coefficient		R-square	t-statistic	p-value
Full Sample	α	-1.066	0.027	-2.349	0.019
	β	-5.037		-2.340	0.020
	λ	0.761		15.900	<0.001
	γ	0.432		7.734	<0.001
P1	α	1.296	0.031	3.428	0.001
	β	-2.081		-2.561	0.011
P2	α	4.701	0.033	2.561	0.011
	β	-1.969		-1.941	0.054
P3	α	1.220	0.018	3.528	0.001
	β	-2.725		-2.631	0.009
P4	α	2.812	0.011	2.116	0.036
	β	-1.445		-1.548	0.123
P5	α	1.959	0.031	3.469	0.001
	β	-1.093		-2.544	0.012

To examine the validity of the conditional CAPM, the cross-sectional regression framework was utilized. Analysis of the above table showed that the intercept term for the entire sample is positive but insignificant (down market), and it is negative but statistically significant (up market). This observation supports the first hypothesis for down market while rejecting it for up market. In contrast, the risk premiums for the entire sample are positive but statistically insignificant (down market), and negative but statistically insignificant (up market), leading to the rejection of the second hypothesis.

As for the market risk premium with respect to conditional co-skewness, it is positive and holds statistical significance for both down and up market. However, when considering the market risk premium with respect to conditional co-kurtosis, it is negative and statistically significant (down market), and it is positive but statistically significant (up market). This leads to the rejection of the third and fourth hypotheses, with the exception of the fourth hypothesis which is supported for the up market.

Comment [MS2]: In the expanded discussion, the findings are further contextualized and compared with the theory employed.

Although the hypotheses related to the coskewness and cokurtosis risk have been rejected, they still hold significant pricing implications. The inclusion of these higher moments in the conventional model has increased the model's explanatory power, evident from the higher R-square value exhibited by the conditional higher moment CAPM (0.032 for down market) and (0.027 for up market) in comparison to the unconditional CAPM (0.00035).

The low R-square value indicates its inability to explain the variations in cross-sectional returns. Similar finding were obtained by Fletcher and Kihanda (2005), Javid (2009), Ergun (2010) and Akbar (2013) as they also found mixed and inconclusive findings.

5. Conclusion

The objective of the present study was to examine validity of the conditional higher moment CAPM and to check whether the higher moments are significantly priced in the Indian stock market. The results showed that the higher moments i.e. coskewness and cokurtosis are significantly priced. The estimations indicated that the intercept terms were positive but insignificant in most cases. However, there were instances where some intercept terms were statistically significant, thereby contradicting the hypotheses postulated by the CAPM. Further, the hypotheses related to systematic covariance, co-skewness, and co-kurtosis were rejected with the exception of the fourth hypothesis which was supported for the up market. Hence, the validity of the conditional model was found to be inconclusive in the Indian stock market.

Availability of data and materials: The data used in the study that support the findings of this study have downloaded from BSE (https://www.bseindia.com/indices/IndexArchiveData.html) and RBI (https://www.rbi.org.in/Scripts/BS_NSDPDisplay.aspx?param=4) websites which is publically accessible.

References

1. Adrian, T., & Franzoni, F. Learning about beta: Time-varying factor loadings, expected returns, and the conditional CAPM. Journal of Empirical Finance, 16(4), (2009), 537–556.
2. Ajibola, A., Kunle, O. A., & Prince, N. C. Empirical proof of the CAPM with higher order co-moments in Nigerian stock market: the conditional and unconditional based tests. Journal of Applied Finance and Banking, 5(1), (2015), 145.
3. Akbar, M., & Nguyen, T. T. The explanatory power of higher moment capital asset pricing model in the Karachi stock exchange. Research in International Business and Finance, 36, 241-253.
4. Arditti, F., & Levy, H. (1972). Distribution moments and equilibrium: A comment.Journal of Financial and Quantitative Analysis, 7, (2016), 1429–1433.
5. Asthana, A., & Ahmed, S. S. Empirical Testing of Capital Asset Pricing Model using Generalized Method of Moments in Context of Bombay Stock Exchange. Indian Journal of Natural Sciences, 14(77), (2023).
6. Athayde, G., & Flores, R. A CAPM with higher moments: Theory and econometrics.Ensaios

Economicos, (1997), 317, Rio de Janeiro.

7. Black, F., Jensen, M., & Scholes, M. The capital asset pricing model: Some empirical tests. Studies in the theory of capital markets. New York: Praeger Publishers, (1972).
8. Cederburg, S., & O'Doherty, M. S. Does it pay to bet against beta? On the conditional performance of beta anomaly. Journal of Finance, 71, (2016), 737–774.
9. Dittmar, R. Nonlinear pricing kernels, kurtosis preference and cross-section ofequity returns. Journal of Finance, 57,(2002), 369–403.
10. Douglas, G. Risk in the equity markets: An empirical appraisal of market efficiency. Yale Economic Essays, 9, (1969), 3–45.
11. Fama, E., & French, K. The cross-section of expected stock returns. Journal of Finance, 47, (1992), 427–465.
12. Fama, E., & MacBeth, J. Risk, return and equilibrium: Some empirical tests. Journal of Political Economy, 81, (1973), 607–636.
13. Fang, H., & Lai, T. Co-kurtosis and capital asset pricing. The Financial Review, 32, (1997),293–307.
14. Friend, I., & Westerfield, R. Co-skewness and capital asset pricing. Journal ofFinance, 35,(1980), 897–914.
15. Harvey, C., & Siddique, A. Conditional skewness in asset pricing tests. Journal ofFinance, 55,(2000a), 1263–1295.
16. Hwang, S., & Satchell, S. Modelling emerging market risk premia using highermoments. International Journal of Finance and Economics, 4,(1999), 271–296.
17. Jagannathan, R., & Wang, Z. The conditional CAPM and the cross-section of expected returns. Journal of Finance, 51, (1996), 3–54.
18. Kostakis, A., Muhammad, K., & Siganos, A. Higher co-moments and asset pricingon London Stock Exchange. Journal of Banking and Finance, 36,(2012), 913–922.
19. Kraus, A., & Litzenberger, R. Skewness preference and the valuation of riskassets. Journal of Finance, 31,(1976), 1085–1100.
20. Lambert, M., & Hubner, G. Comoment risk and stock returns. Journal of EmpiricalFinance, 23,(2013), 191–205.
21. Lettau, M., & Ludvigson, S. Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying. Journal of Political Economy, 109, (2001), 1238–1287.
22. Lewellen, J., & Nagel, S. The conditional CAPM does not explain asset-pricing anomalies. Journal of Financial Economics, 82, (2006), 289–314.
23. Lim, K. A new test of the three-moment capital asset pricing model. Journal ofFinancial and Quantitative Analysis, 24,(1989), 205–216.
24. Moreno, D., & Rodriguez, R. The value of coskewness in mutual fundperformance evaluation. Journal of Banking and Finance, 33,(2009), 1664–1676.

25. Nishantha, J. M. Testing the Validity of Conditional Four Moment Capital Asset Pricing Model: Empirical Evidence from the Colombo Stock Exchange. Staff Studies, 48(1), (2018), 99-129.
26. Pettengill, G., Sundaram, S., & Mathur, I. The conditional relation between beta and returns. The Journal of Financial and Quantitative Analysis, 30, (1995), 101–116.
27. Scott, R., & Horvath, P. On the direction of preference for moments of higherorder than the variance. Journal of Finance. 35,(1980), 915–919.
28. Vendrame, V., Tucker, J., & Guermat, C. Some extensions of the CAPM forindividual assets. International Review of Financial Analysis, 44,(2016), 78–85.
29. Young, B., Christoffersen, P., & Jacobs, K. Market skewness risk and the cross-section of stock returns. Journal of Financial Economics, 107(1),(2013), 46–68.