Research Article

Group Decision Method Without Consensus Threshold Based on Personalized Semantic Continuous Learning

Abstract
The primary concern in group decision-making lies in the objective reasonable creation of a decision outcome that is agreed upon by all decision-makers. To achieve this purpose, a dynamic adjustment of preferences is necessary, in which personalized semantic continuous learning of linguistic information initially provided by decision-makers is a key process. This study explores a way for guiding decision-makers to continuously learn from individual linguistic preferences and establish an adaptive consensus-reaching method. The continuous personalized semantic learning model is firstly designed to simulate individual preferences in a dynamic decision-making environment, addressing the issue of quantifying semantics for decision-makers. Secondly, an adaptive weight allocation method is proposed to capture the changing process of a decision-maker's weight while measuring its importance based on the current decision environment. Furthermore, we establish an adaptive consensus-reaching model without subjective parameters facilitates objective evaluation of decision-making. Finally, some experiments are conducted to examine the effectiveness of the proposed model.

Keywords: Group decision-making; Consensus reaching process; Personalized semantics; Continuous learning; Dynamic weights

INTRODUCTION

Group decision-making (GDM) refers to the collaborative process of reaching decisions collectively as a group, rather than relying on individual judgment. Within the realm of decision-making, the capacity to address intricate real-world challenges stands as a fundamental necessity [1]. It involves a collection of individuals discussing and analyzing different alternatives from various perspectives and ultimately reaching a consensus or making a decision. This process is accompanied by multiple participants with varying opinions in the decision-making problem. Linguistic terms are communication tool among group members, which help experts for expressing their viewpoints and exchanging information. Decision-makers express their preferences for evaluation objects or alternatives using natural language such as 'good', 'bad', or 'average'. As a result, GDM problems with linguistic variables as preference information arise in various settings, such as E-commerce, travel platforms and community management [2][3][4].

Zadeh introduced the paradigm of Computing with Words (CWW) [2], which seeks to bridge the gap between human cognition and computational systems. It allows users to express their preferences and uncertainties in natural language rather than precise numerical values. The word computing model [5] is the fundamental method of expressing and calculating linguistic preferences. This approach has been applied in processing linguistic assessment information for solving decision-making problems. However, considering the differences in cultural background, experience, and knowledge of decision-makers, linguistic preference has significant personalized semantic features.

Personalized semantics refers to the significant individual differences in the expression and understanding of linguistic preferences among different decision-makers [6][5]. On a hotel booking platform, for example, customers often look at other customers' reviews. Different consumers may interpret the term 'good' differently due to differences in travel aims, consumption patterns, and other aspects. While some customers may find the hotel's amenities to be extremely pleasant, others may find them to be only acceptable. Hence, considering the fact that both linguistic expression and semantic comprehension involve complexity and uniqueness, it is possible that implementing approaches that consider personalized semantics could improve the effectiveness of GDM.

In recent years, there has been a focus on developing a personalized numerical scale for linguistic labels in the GDM problem, leading to significant advancements in this field. For example, in order to reflect the differences in individual understanding of linguistic expression, the existing word computing models have proposed type-2 word computing models [8][9] and multi-granularity linguistic models [10][11]. Mendel et al. [8][9] proposed the type-2 linguistic model, which uses type-2 fuzzy sets to solve the problem of semantic complexity. Dong and Herrera-Viedma [12][13] further established a consistency-driven optimization model to solve the personalized interval numerical scales corresponding to individual linguistic labels. Tang et al. [14][15] studied the distributed linguistic decision-making method based on personalized semantics to solve the linguistic terminology GDM problem. Zhang and Li [16] proposed a consistency control and consensus optimization method based on personalized semantics in linguistic GDM problems to develop the consistency control and consensus reaching models. Liang et al. [17] applied the effect analysis and opinion dynamics in social networks to propose the personalized semantic-based linguistic opinions dynamics model in the framework of bounded confidence effects. Liu et al. [18] proposed a consensus feedback adjustment model by considering the personalized self-confidence and trust semantics in dynamic social network GDM scenarios.

When seeking consensus, it is common to suggest that decision-makers modify their preferences in order to conform to the viewpoints of the group, thereby achieving a higher level of consensus. However, when addressing practical GDM challenges, it is essential to take into account not only the cognitive variations among decision-makers but also the ongoing evolution of individual semantics. Therefore, it becomes necessary to incorporate the characteristics of personalized semantic continuous learning to better simulate decision-makers evolving and personalized semantics.

In the process of solving GDM problems, it is crucial to determine the weights of decision-makers, as they are influenced by various factors, including the subjectivity of their opinions and preferences within groups [20]. Geng et al. [21] proposed a hybrid approach combining clustering and entropy weight method to solve the problem of large scale group decision-making (LSGDM) considering double expert weight determination. Li et al. [22] applied the cumulative prospect theory with double reference points to solve the risk-based multi-attribute GDM problem, in which the decision-maker's weight information and attribute value are represented as an interval number. Zhao et al. [23] constructed a directed weighted social network for characterizing decision-makers' cooperation and preference networks to determine their relative weights.

Objective weights are challenging to come by because of differences in knowledge backgrounds, as well as the features of real-world GDM problems. Some weight distribution methods can come with a risk of ignoring important objective factors during real-world implementation. In addition, in the consensus-reaching process (CRP), the preference information of both decision-makers may adjust as the decision-making environment changes. Suppose a dynamic method is ignored to determine the weight, then the current decision-making situation will not be fully reflected, resulting in reduced accuracy and effectiveness of decision-making results.

After making adjustments based on the feedback mechanism, reaching a conclusion on the CRP poses another crucial challenge [25]. Two key parameters usually are employed to conclude CRP: the consensus threshold [26][27][28][29][30] and the maximum number of adjustment rounds [31][32]. The above parameters are subjectively determined according to the specific GDM problem. Consequently, certain investigations have been conducted on how to objectively determine these parameters [33][34]. Wu et al. [33] developed a sensitivity consistency evolution network using a consistency matrix as a reference to ascertain the consistency threshold. Tang et al. [34] investigated the CRP in the context of heterogeneous LSGDM. They developed a measure of consensus that is based on preference orderings and has an objective threshold. CRP without a threshold can effectively remove the reliance on subjective judgment in the decision-making process and more accurately align with the actual decision-making situation. This study will expore a termination condition that does not rely on a predetermined threshold, thereby seeking to establish a consensus.

The current research methods not adequately consider the differences in cognitive abilities among decision-makers and the continuous evolution of individual interpretations over time, while also neglecting the potential adaptation of decision makers to changing preference information during consensus building. Additionally, subjective factors pose challenges that are difficult to overcome in the decision-making process. Based on the problems above, this study focus on investigating personalized semantic learning models that addressing dynamic linguistic preference information in specific decision scenarios and proposes an adaptive personalized semantic consensus GDM model based on continuous learning in the linguistic decision-making environment, which has the following contributions:

1. Establishing the consensus-driven personalized numerical scales by considering the dynamic nature of personalized semantics to guide decision-makers to continuously learning.

2. An adaptive weight allocation method is designed to determine the dynamic weight of decision-makers as the decision-making process unfolds.

3. An objective termination condition for CRP is developed to minimize the dependence on subjective parameters, enhancing its reliability and effectiveness.

The study is organized as follows: Section presents a comprehensive review of personalized concepts. Section elaborates the GDM method based on personalized semantic continuous learning without a consensus threshold. The model's practicality is examined in Section 14 and Section 14 through some comparative analysis. Finally, Section 14 draws the conclusions and provides the future research directions.

PRELIMINARIES

This section expounds the related concepts of linguistic preference relation, which provides the theoretical support for the subsequent content.

Due to the complexity and ambiguity of human cognition, individuals often rely on natural language to articulate their preferences for objective matters when making decisions. For instance, the terms like 'very good', 'good', 'fair', 'bad', 'very bad', and similar linguistic expressions are used to convey decision-makers' preference information. Let $S = \{s_\tau \mid \tau = 0, 1, \cdots, g\}$ be the set of linguistic terms, where s_τ represents individual linguistic terms, and $g + 1$ denotes the cardinality of set S. Typically, g is chosen to be an even number. The terms s_0 and s_g correspond to the minimum and maximum values, respectively, in the linguistic term set S. For instance, the set S comprising five linguistic terms can be expressed as $S = \{s_0 = \text{very bad}, s_1 = \text{bad}, s_2 = \text{medium}, s_3 = \text{good}, s_4 = \text{very good}\}$. The property of linguistic terms is as follows.

Definition 1 *[35] Let $S = \{s_\tau | \tau = 0, 1, \cdots, g\}$ be a linguistic term set, define S existence orderliness: if $i > j$, then s_i is superior to s_j, where '>' denotes 'superior to' and s_τ represent the linguistic variable.*

A 2-tuple linguistic term refers to a linguistic expression consisting of two components: a fuzzy linguistic term and the corresponding deviation value is shown as follows.

Definition 2 *[35] Let $NS\left(s_\tau, \alpha\right)$ represent the 2-tuple linguistic terms,*

$$NS\left(s_\tau, \alpha\right) = \begin{cases} NS\left(s_\tau\right) + \alpha\left(NS(s_{\tau-1}) - NS\left(s_\tau\right)\right) & \alpha \geq 0 \\ NS\left(s_\tau\right) + \alpha\left(NS\left(s_\tau\right) - NS(s_{\tau-1})\right) & \alpha < 0 \end{cases}, \tag{1}$$

where $\alpha \in [-0.5, 0.5)$, and $NS(s_\tau) = a_\tau(a_\tau \in [0,1])$ represents the numerical scale for the linguistic term s_τ.

Definition 3 *[35] Let $NS^{-1}(a)$ represent the inverse function of the ordered numerical scale $NS(a)$,*

$$NS^{-1}(a) = \begin{cases} \left(s_\tau, \frac{a-NS(s_\tau)}{NS(s_{\tau+1})-NS(s_\tau)}\right), & NS\left(s_\tau\right) < a < \frac{NS(s_\tau)+NS(s_{\tau+1})}{2} \\ \left(s_\tau, \frac{a-NS(s_\tau)}{NS(s_\tau)-NS(s_{\tau-1})}\right), & \frac{NS(s_{\tau-1})+NS(s_\tau)}{2} \leq a \leq NS\left(s_\tau\right) \end{cases}. \tag{2}$$

GROUP DECISION METHOD WITHOUT CONSENSUS THRESHOLD BASED ON PERSONALIZED INDIVIDUALIZED SEMANTIC CONTINUOUS LEARNING

This section introduces the group decision method without a consensus threshold based on personalized semantic continuous learning. The flowchart of the designed method is shown in Fig. 1.

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Figure 1: The Flowchart of Group Decision Method Based on Personalized Individualized Semantic Continuous Learning

Personalized Individualized Semantic Model

The factors typically encompassed in addressing GDM problems include decision-makers, alternatives, and preference relations. Let $S = \{s_0, s_1, \cdots, s_g\}$ be the linguistic term set, $E = \{e_1, e_2, \cdots, e_m\}$ denote a collection of m decision-makers, $B = \{b_1, b_2, \cdots, b_n\}$ represent a set of n alternatives, and $L^{k,t} = (l_{ij}^{k,t})_{n\times n}$ represent the linguistic preference relations of decision-maker e_k at round t. Subsequently, a consensus-driven optimization model is designed to obtain personalized numerical scales, transforming linguistic preference relations into numerical values.

The calculation of the numerical scales can be realized as follows,

$$NS^{k,t}(s_\tau, \alpha) = \begin{cases} NS^{k,t}(s_\tau) + \alpha\left(NS^{k,t}(s_{\tau-1}) - NS^{k,t}(s_\tau)\right) & \alpha \geq 0 \\ NS^{k,t}(s_\tau) + \alpha\left(NS^{k,t}(s_\tau) - NS^{k,t}(s_{\tau-1})\right) & \alpha < 0 \end{cases}, \quad (3)$$

where $NS^{k,t}(s_\tau^{k,t}) = a_\tau^{k,t}$ represent the personalized numerical scales representation of decision-maker e_k with respect to linguistic terms at round t. The inverse function of the numerical scale $NS^{-1,k,t}(a^t)$ is calculated as shown in Eq.(2).

To address the optimal personalized numerical scales and achieve a maximal consensus level, an optimization model is given as follows,

$$\begin{aligned} &\max \sum_{k=1}^{m} w^{k,t} \cdot CL^{k,t} \\ &\text{s.t.} \begin{cases} a_\tau^{k,t} + a_{g-\tau}^{k,t} = 1 \\ a_\tau^{k,t} - a_{\tau-1}^{k,t} \geq \theta \\ 0 \leq a_0^{k,t} < a_1^{k,t} < \cdots < a_t^{k,t} < \cdots < a_g^{k,t} \leq 1 \\ k = 1, 2, \cdots, m \\ \tau = 0, 1, \cdots, g \end{cases} \end{aligned}, \quad (4)$$

where the parameter θ severs for regulating the minimum distance between personalized semantic scales, $CL^{k,t}$ represent the the collective consensus level at round t calculated under the result of $N^t(e_k)$, which is defined in Eq.(10). By solving Eq.(4), the linguistic values corresponding to the provided linguistic terms by decision-maker e_k are denoted as $N^t(e_k) = \{a_0^{k,t}, a_1^{k,t}, \cdots, a_g^{k,t}\}$.

Dynamic Weight Allocation Method

The weight of decision-makers is crucial in the decision-making process. It is important to update individual weights to ensure their reasonableness by considering the variations in decision environment. When decision-makers' preference that closely aligns with the collective preference, their support level of the group's preference would increase. Appropriate adjustments must be made to the weights assigned to them as a result.

The distance calculation between $p^{u,t} = (p_{ij}^{u,t})_{n\times n}$ and $p^{v,t} = (p_{ij}^{v,t})_{n\times n}$ is as follows,

$$d(p^{u,t}, p^{v,t}) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\sqrt{(p_{ij}^{u,t} - p_{ij}^{v,t})^2}}{n \times n}, \quad (5)$$

where $p^{u,t}$ and $p^{v,t}$ represent the numerical preference matrix obtained from the transformation of personalized numerical scale at round t.

Equal weights are initially assigned to each m decision-maker as $\omega^{k,0} = \frac{1}{m}(k = 1, 2, \cdots, m)$ at round $t = 0$. And taking into account the difference between decision-maker e_k's adjusted preference and the collective preference at round $t-1$, the weight calculation method is as follows [36],

$$\omega^{k,t} = Q\left(\frac{n^{k,t-1}}{m}\right) - Q\left(\frac{n^{k,t-1} - 1}{m}\right), \quad (6)$$

where $n^{k,t}$ denotes the sorted sequence of the distance between the preference information of decision-maker e_k and the collective preference in the induced sequence, m represent the number of decision-makers, $Q(r)$ represents the induced function, and the calculation method of $Q(r)$ is as follows,

$$Q(r)=\begin{cases} 0 & r<a \\ \frac{r-a}{b} & a\leq r\leq b \\ 1 & r>b \end{cases}, \tag{7}$$

where the values of parameters a and b depend on the quantization rules of fuzzy linguistic term and the relationship between parameter values [36].

The weighted average method [37] is used to aggregate the collective preference information $cp^t=(cp^t_{ij})_{n\times n}$ at round t. The calculation of the term cp^t_{ij} can be realized as follows,

$$cp^t_{ij}=\sum_{k=1}^{m}\omega^{k,t}\cdot p^{k,t}_{ij}. \tag{8}$$

Consensus Reaching Process

The consensus reaching process consists of two key steps: consensus measurement and feedback adjustment. The former is employed to gauge the consensus level among decision-makers, while the latter provides adjustment recommendations for decision-makers.

• **Consensus measurement process**

The consensus level is primarily assessed based on the divergence of preferences. Specifically, the consensus level $cl^{k,t}$ of decision-maker e_k following adjustments in round t can be determined as follows,

$$cl^{k,t}=1-d(p^{k,t},cp^t), \tag{9}$$

where the consensus level on the preference relation between alternatives b_i and b_j can be calculated as $cl^{k,t}_{ij}=1-|p^{k,t}_{ij}-cp^t_{ij}|$.

According to the consensus level of decision-makers, the collective consensus level at round t is calculated in the following manner,

$$CL^t=\frac{\sum_{k=1}^{m}cl^{k,t}}{m}. \tag{10}$$

• **Feedback adjustment process**

An identification mechanism is established to determine the decision-maker e_h with the lowest $cl^{h,t}_{ij}$ and adjust their preference information if the consensus level $cl^{h,t}_{ij}$ less than $cl^{h,t}$. The identification mechanism can be computed as follows,

$$AD^t=\{(h,(i,j))|h=\arg\min\left\{cl^{1,t}_{ij},\ldots,cl^{m,t}_{ij}\right\},cl^{h,t}_{ij}<cl^{h,t}\}. \tag{11}$$

By utilizing the collectively modified preference at round t and leveraging individual semantics associated with decision-maker e_h as a reference to enhance consensus level, the adjustment mechanism of decision-maker e_h at round $t+1$ is constructed as follows,

$$p^{h,t+1}_{ij}=\beta^{h,t}p^{h,t}_{ij}+\left(1-\beta^{h,t}\right)\cdot\overline{cp}^t_{ij}, \tag{12}$$

$$\overline{cp}^t_{ij}=NS^{h,t}\left(NS^{h,t,-1}\left(cp^t_{ij}\right)\right), \tag{13}$$

where $\overline{cp}^t_{ij}$ represents the personalized semantic value of the collective preference corresponding with $NS^{h,t}\left(s_\tau\right)$, and $\beta^{h,t}$ represents the weight adjustment parameter. It is worth noting that the value of $\beta^{h,t}$ also changes dynamically. Therefore, we set $\beta^{h,t}=\omega^{h,t}$ in the presented method.

Termination conditions of CRP

In the context of CRP, it is observed that as the adjustment rounds increase, the adjustment cost associated with the adjustment process rises. Moreover, uncertainty exists regarding how much the consensus level will improve. To measure preference information adjustments and establish a stopping criterion for CRP based on considerations of the adjustment cost, we introduce a metric known as consensus enhancement rate (CER). The calculation of CER can be outlined as follows,

$$CER^t = \frac{\Delta CL^t}{\Delta CL^{t-1}} + \frac{1}{\ln t}(t \geq 2), \tag{14}$$

where $\Delta CL^t = CL^t - CL^{t-1}(t \geq 2)$ represents the amount of increase in the consensus level at round t, and $\frac{1}{\ln t}$ serves a role in controlling the number of adjustments and minimizing expenses.

The termination of CRP is recommended when the CER^l is lower than the average, indicating inefficiency in adjustment considering the adjustment cost. The termination condition is defined as follows,

$$CER^l \leq \frac{1}{l}\sum_{t=1}^{l} CER^t. \tag{15}$$

Upon meeting the termination condition, the matrix $cp^l = (cp^l_{ij})_{n\times n}$ represents the preference information that satisfies the consensus requirement, and alternatives are ranked based on cp^l, the score of the alternative b_i is given as below,

$$SQ^l_i = \sum_{j=1}^{n} cp^l_{ij}. \tag{16}$$

The alternatives are subsequently ranked based on their final preference scores, and the alternative with the highest preference score is selected. The procedure for the group decision method without a consensus threshold based on personalized semantic continuous learning is presented in Algorithm 1.

Algorithm 1: Group Decision Method without Consensus Threshold Based on Personalized Semantic Continuous Learning

Input: The initial evaluation information $L^{k,0} = (l^{k,0}_{ij})_{n\times n}$, the initial weight of decision-makers $\omega^{k,0} = \frac{1}{m}(k = 1, 2, \cdots, m)$, the parameter θ, $t = 0$

Output: The final preference score SQ^t_i

Apply personalized semantic model in Section to obtain personalized numerical scales.
Aggregate the collective preference information $cp^t = (cp^t_{ij})_{n\times n}$ by using Eq.(8).
Calculate the global consensus level CI^t.
Calculate CER^t by using Eq.(14).
if $t < 2$ *or* $CER^l \leq \frac{1}{l}\sum_{t=1}^{l} CER^t$ **then**
    $t = t + 1$.
    Reallocate weights for decision-makers using the dynamic weight allocation method in Section .
    Update the preference information for decision-makers via Eq.(11) and Eq.(12).
    Back to line 2.
end
else
    Calculate the final preference score SQ^t_i for the alternative b_i.
end
return SQ^t_i

NUMERICAL EXPERIMENTATION

In this section, we illustrate the practicability and effectiveness of the above approach by using a venture capital problem as an example. There are four possible alternatives, denoted as $B = \{b_1, b_2, b_3, b_4\}$, a dedicated decision-making team is formed comprising six decision-makers from different departments (represented as $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$). To enhance the feasibility of the optimal selection process, it is assumed that each decision-maker can utilize linguistic information for evaluating different alternatives. The relevant set of linguistic terms is provided below,

$$S = \left\{ \begin{array}{c} s_0 = \text{extremely bad}, s_1 = \text{very bad}, s_2 = \text{bad}, s_3 = \text{fair}, \\ s_4 = \text{good}, s_5 = \text{very good}, s_6 = \text{extremely good} \end{array} \right\}.$$

At the initial round $t = 0$, the preference information denoted as $L^0 = \{L^{1,0}, L^{2,0}, L^{3,0}, L^{4,0}, L^{5,0}, L^{6,0}\}$ is provided by six decision-makers.

$$L^{1,0} = \left\{ \begin{array}{cccc} s_3 & s_1 & s_4 & s_2 \\ s_5 & s_3 & s_5 & s_1 \\ s_2 & s_1 & s_3 & s_4 \\ s_4 & s_5 & s_2 & s_3 \end{array} \right\}, \quad L^{2,0} = \left\{ \begin{array}{cccc} s_3 & s_2 & s_1 & s_3 \\ s_4 & s_3 & s_4 & s_5 \\ s_5 & s_2 & s_3 & s_6 \\ s_3 & s_1 & s_0 & s_3 \end{array} \right\}, \quad L^{3,0} = \left\{ \begin{array}{cccc} s_3 & s_4 & s_5 & s_6 \\ s_2 & s_3 & s_2 & s_1 \\ s_1 & s_4 & s_3 & s_5 \\ s_0 & s_5 & s_1 & s_3 \end{array} \right\},$$

$$L^{4,0} = \left\{ \begin{array}{cccc} s_3 & s_5 & s_4 & s_1 \\ s_1 & s_3 & s_4 & s_2 \\ s_2 & s_2 & s_3 & s_3 \\ s_5 & s_4 & s_3 & s_3 \end{array} \right\}, \quad L^{5,0} = \left\{ \begin{array}{cccc} s_3 & s_1 & s_2 & s_3 \\ s_5 & s_3 & s_4 & s_2 \\ s_4 & s_2 & s_3 & s_5 \\ s_3 & s_4 & s_1 & s_3 \end{array} \right\}, \quad L^{6,0} = \left\{ \begin{array}{cccc} s_3 & s_1 & s_2 & s_4 \\ s_5 & s_3 & s_5 & s_4 \\ s_4 & s_1 & s_3 & s_5 \\ s_2 & s_2 & s_1 & s_3 \end{array} \right\}.$$

Initially, equal weights are assigned to each decision-maker as $\omega^{k,0} = \frac{1}{6}$ $(k = 1, 2, \cdots, 6)$, and the value of θ is set to 0.02 in Eq. (4). The resulting personalized numerical scales for each decision-maker are as follows,

$$N^0(e_1) = \{0.09, 0.28, 0.37, 0.50, 0.63, 0.72, 0.91\},$$

$$N^0(e_2) = \{0.13, 0.31, 0.39, 0.50, 0.61, 0.69, 0.87\},$$

$$N^0(e_3) = \{0.15, 0.30, 0.42, 0.50, 0.58, 0.70, 0.85\},$$

$$N^0(e_4) = \{0.14, 0.30, 0.38, 0.50, 0.62, 0.70, 0.86\},$$

$$N^0(e_5) = \{0.09, 0.26, 0.35, 0.50, 0.65, 0.74, 0.91\},$$

$$N^0(e_6) = \{0.07, 0.23, 0.41, 0.50, 0.59, 0.77, 0.93\}.$$

Figure 2: Numerical scales at round $t = 0$

The preference relation matrix is derived based on decision-makers' personalized semantics and the collective preference relation matrix cp^0 at round $t = 0$ is calculated by using Eq.(8).

$$cp^0 = \left\{ \begin{array}{cccc} 0.50 & 0.41 & 0.50 & 0.52 \\ 0.59 & 0.50 & 0.64 & 0.55 \\ 0.50 & 0.36 & 0.50 & 0.70 \\ 0.48 & 0.45 & 0.30 & 0.50 \end{array} \right\}.$$

The weights of decision-makers are reallocated based on the distribution of personalized semantic preference relations using the dynamic weight allocation method (described in Section) with $a = 0.3$ and $b = 0.8$, they are

$\omega^{1,1} = 0.34$, $\omega^{2,1} = 0$, $\omega^{3,1} = 0.25$, $\omega^{4,1} = 0.20$, $\omega^{5,1} = 0.21$, and $\omega^{6,1} = 0.04$. The collective consensus level is obtained from Eq.(10), resulting in a value of $CL^0 = 0.708$.

The decision-maker requiring modification of individual preferences and the positional information in AD^1 can be identified based on Eq.(11). By using Eq.(12), we can calculate the adjusted preference information with a value of $\beta^{3,1} = 0.25$ for decision-maker e_3 as follows,

$$L^{3,1} = \left\{ \begin{array}{llll} s_3 & \{s_4, -0.18\} & s_5 & \{s_5, -0.37\} \\ \{s_2, 0.18\} & s_3 & \{s_2, 0.32\} & \{s_1, 0.46\} \\ s_1 & \{s_4, -0.32\} & s_3 & s_5 \\ \{s_1, 0.37\} & \{s_5, -0.46\} & s_1 & s_3 \end{array} \right\}.$$

According to the updated preference information, the personalized numerical scales for each decision-maker are obtained as follows,

$$N^1(e_1) = \{0.12, 0.26, 0.34, 0.50, 0.66, 0.74, 0.88\},$$

$$N^1(e_2) = \{0.09, 0.30, 0.36, 0.50, 0.64, 0.70, 0.91\},$$

$$N^1(e_3) = \{0.19, 0.33, 0.45, 0.50, 0.55, 0.67, 0.81\},$$

$$N^1(e_4) = \{0.11, 0.28, 0.29, 0.50, 0.71, 0.72, 0.89\},$$

$$N^1(e_5) = \{0.12, 0.26, 0.34, 0.50, 0.66, 0.74, 0.88\},$$

$$N^1(e_6) = \{0.08, 0.19, 0.40, 0.50, 0.60, 0.81, 0.92\}.$$

Figure 3: Numerical scales at round $t = 1$

The collective preference relation matrix at round $t = 1$ can be obtained by using Eq.(8) as below,

$$cp^1 = \left\{ \begin{array}{cccc} 0.50 & 0.37 & 0.49 & 0.57 \\ 0.63 & 0.50 & 0.59 & 0.33 \\ 0.51 & 0.41 & 0.50 & 0.49 \\ 0.43 & 0.67 & 0.31 & 0.50 \end{array} \right\}.$$

The dynamic weight allocation method in Section is employed to reallocate weights for decision-makers based on the distribution of personalized semantic preference relations, resulting in $\omega^{1,2} = 0.34, \omega^{2,2} = 0.04, \omega^{3,2} = 0.20, \omega^{4,2} = 0.21, \omega^{5,2} = 0.25$, and $\omega^{6,2} = 0$. By applying Eq.(10), the collective consensus level is calculated to be $CL^1 = 0.797$.

The decision-maker e_2 and the positional information in AD^2 are identified based on the mechanism described in Eq.(11). By employing the adjustment mechanism presented in Eq.(12) with $\beta^{2,2} = 0.04$, we can calculate the adjusted preference relations for decision-maker e_2 as follows,

$$L^{2,2} = \left\{ \begin{array}{llll} s_3 & s_2 & \{s_1, 0.18\} & s_3 \\ s_4 & s_3 & \{s_4, 0.07\} & \{s_6, -0.12\} \\ \{s_5, -0.18\} & \{s_2, -0.07\} & s_3 & \{s_6, -0.23\} \\ s_3 & \{s_0, 0.12\} & \{s_0, 0.23\} & s_3 \end{array} \right\}.$$

According to the updated preference information, the personalized numerical scales for each decision-maker are as follows,

$$N^2(e_1) = \{0.13, 0.36, 0.38, 0.50, 0.62, 0.64, 0.87\},$$

$$N^2(e_2) = \{0.09, 0.26, 0.43, 0.50, 0.57, 0.74, 0.91\},$$

$$N^2(e_3) = \{0.28, 0.35, 0.47, 0.50, 0.53, 0.65, 0.72\},$$
$$N^2(e_4) = \{0.14, 0.29, 0.38, 0.50, 0.62, 0.71, 0.86\},$$
$$N^2(e_5) = \{0.04, 0.16, 0.31, 0.50, 0.69, 0.84, 0.96\},$$
$$N^2(e_6) = \{0.06, 0.28, 0.46, 0.50, 0.54, 0.72, 0.94\}.$$

Figure 4: Numerical scales at round $t = 2$

By using Eq.(8) the collective preference relation matrix is calculated at round $t = 2$ as follows,

$$cp^2 = \left\{ \begin{array}{cccc} 0.50 & 0.39 & 0.51 & 0.36 \\ 0.61 & 0.50 & 0.58 & 0.33 \\ 0.49 & 0.42 & 0.50 & 0.57 \\ 0.64 & 0.67 & 0.43 & 0.50 \end{array} \right\}.$$

The weights are reallocated for decision-makers based on the distribution of personalized semantic preference relations using the dynamic weight allocation method described in Section , the corresponding results are $\omega^{1,3} = 0.25, \omega^{2,3} = 0, \omega^{3,3} = 0.34, \omega^{4,3} = 0.04, \omega^{5,3} = 0.21$, and $\omega^{6,3} = 0.20$. The collective consensus level is calculated using Eq.(10), resulting in a value of $CL^2 = 0.839$.

The identification of the decision-maker e_6 and the positional information in AD^3 is based on the mechanism described in Eq.(11). The adjustment mechanism presented in Eq.(12) is utilized with $\beta^{6,3} = 0.20$, to calculate the adjusted preference relations for decision-maker e_6 as follows,

$$L^{6,3} = \left\{ \begin{array}{llll} s_3 & s_1 & \{s_3, -0.18\} & \{s_4, -0.49\} \\ s_5 & s_3 & s_5 & s_4 \\ \{s_3, 0.18\} & s_1 & s_3 & \{s_4, 0.47\} \\ \{s_2, 0.49\} & s_2 & \{s_2, -0.47\} & s_3 \end{array} \right\}.$$

According to the updated preference information, the personalized numerical scales for each decision-maker are determined as follows,

$$N^3(e_1) = \{0.03, 0.23, 0.42, 0.50, 0.58, 0.77, 0.97\},$$
$$N^3(e_2) = \{0.39, 0.44, 0.45, 0.50, 0.55, 0.56, 0.61\},$$
$$N^3(e_3) = \{0.30, 0.36, 0.45, 0.50, 0.55, 0.64, 0.70\},$$
$$N^3(e_4) = \{0.11, 0.25, 0.44, 0.50, 0.56, 0.75, 0.89\},$$
$$N^3(e_5) = \{0.03, 0.27, 0.43, 0.50, 0.57, 0.73, 0.97\},$$
$$N^3(e_6) = \{0.13, 0.26, 0.41, 0.50, 0.59, 0.74, 0.87\}.$$

Figure 5: Numerical scales at round $t = 3$

The collective preference relation matrix is computed in round $t = 3$ using the weighted average aggregation operator, as depicted below according to Eq.(8),

$$cp^3 = \left\{ \begin{array}{cccc} 0.50 & 0.51 & 0.55 & 0.54 \\ 0.49 & 0.50 & 0.48 & 0.43 \\ 0.45 & 0.52 & 0.50 & 0.65 \\ 0.46 & 0.57 & 0.35 & 0.50 \end{array} \right\}.$$

The weights are reassigned to decision-makers based on the distribution of preference relations using the method described in Section , with $\omega^{1,4} = 0.34, \omega^{2,4} = 0, \omega^{3,4} = 0.20, \omega^{4,4} = 0.25, \omega^{5,4} = 0.21$, and $\omega^{6,4} = 0.04$. The collective consensus level is calculated as $CL^3 = 0.839$ using Eq.(10).

The AD^4 is identified based on the identification mechanism in Eq.(11), and the adjusted preference relations for decision-maker e_6 are calculated by using the adjustment mechanism described in Eq.(12) with $\beta^{6,4} = 0.04$,

$$L^{6,4} = \left\{ \begin{array}{llll} s_3 & \{s_1, 0.17\} & \{s_3, -0.18\} & \{s_3, 0.47\} \\ \{s_5, -0.17\} & s_3 & s_5 & s_4 \\ \{s_3, 0.18\} & s_1 & s_3 & \{s_4, 0.47\} \\ \{s_3, 0.47\} & s_2 & \{s_2, -0.47\} & s_3 \end{array} \right\}.$$

The personalized numerical scales for each decision-maker have been updated according to the preference information as below,

$$N^4(e_1) = \{0.02, 0.23, 0.42, 0.50, 0.58, 0.77, 0.98\},$$
$$N^4(e_2) = \{0.29, 0.45, 0.46, 0.50, 0.55, 0.54, 0.71\},$$
$$N^4(e_3) = \{0.25, 0.34, 0.45, 0.50, 0.55, 0.66, 0.75\},$$
$$N^4(e_4) = \{0.11, 0.32, 0.44, 0.50, 0.56, 0.68, 0.89\},$$
$$N^4(e_5) = \{0.03, 0.24, 0.42, 0.50, 0.58, 0.76, 0.97\},$$
$$N^4(e_6) = \{0.13, 0.24, 0.41, 0.50, 0.59, 0.76, 0.87\}.$$

Figure 6: Numerical scales at round $t = 4$

The collective preference relation matrix is calculated based on Eq.(8),

$$cp^4 = \left\{ \begin{array}{cccc} 0.50 & 0.50 & 0.57 & 0.57 \\ 0.50 & 0.50 & 0.49 & 0.41 \\ 0.43 & 0.51 & 0.50 & 0.67 \\ 0.43 & 0.59 & 0.34 & 0.50 \end{array} \right\}.$$

Finally, the weights of decision-makers are reallocated by employing the dynamic weight allocation method in Section , we obtain $\omega^{1,5} = 0.25, \omega^{2,5} = 0.04, \omega^{3,5} = 0.20, \omega^{4,5} = 0.21, \omega^{5,5} = 0$, and $\omega^{6,5} = 0.34$. The collective consensus level is calculated using Eq.(10) as $CL^4 = 0.846$. Furthermore, the consensus termination condition of CPR at $t = 4$ is determined to be $CER^4 = 0.186$, which satisfies the termination condition criterion $0.186 < 0.187$. The scores for each alternative are calculated based on the preference relations using Eq.(16). The resulting scores are as $SQ_1^4 = 2.14$, $SQ_2^4 = 1.90$, $SQ_3^4 = 2.11$, and $SQ_4^4 = 1.86$. Finally, the alternatives are ranked based on their scores and the optimal alternative is identified as b_1.

COMPARISON ANALYSIS

In this section, we examine the efficacy of the personalized semantic approach based on continuous learning and the adaptive weight allocation method of the developed model through three sets of experimental analyses.

Comparison analysis on parameter β

This section utilizes a synthetic dataset, which is generated based on the data presented in the numerical experiment, incorporating linguistic preferences of 10 decision-makers and exploring various values for β, thereby highlighting the significance of selecting an appropriate parameter β. A larger value assigned to the parameter β signifies a reduced inclination of the decision-maker to modify preference information, resulting in a diminished reliance on collective preference during adjustment. Conversely, when β is assigned a lower value, it implies a heightened dependence on reference collective preference during adjustment.

CR.png

Figure 7: The result of comparison analysis on parameter β

As depicted in Fig. 7, an increased inclination on the part of the decision-maker to modify preference information accelerates the achievement of consensus. However, a low value for β does not always imply superiority. If β is excessively low, there exists a risk of over-modifying the decision-maker's preferences, thereby hindering the achievement of higher levels of consensus under the termination condition. In contrast, this study sets different values for the developed method depending on the changing decision environment. For instance, experimental results corresponding to ω in Fig. 7 can achieve greater consensus levels within shorter times.

Comparison analysis on continuous learning and adaptive weight allocation

This section employs a synthetic dataset, generated based on the information provided in the numerical experiment and incorporating linguistic preference information from 10 decision-makers, to empirically validate the effectiveness of the proposed continuous learning approach and adaptive weight allocation method. A comparative analysis is conducted with methodologies employed in [38] and [16]. Model 1 incorporates a personalized numerical scale of continuous learning while assigning a fixed weight for the method used in [16]. The personalized semantic model described in Algorithm 1 is replaced by the personalized semantic model employed in [38], referred to as Model 3. Subsequently, within the framework of Model 3, a continuous learning approach is considered to construct Model 2.

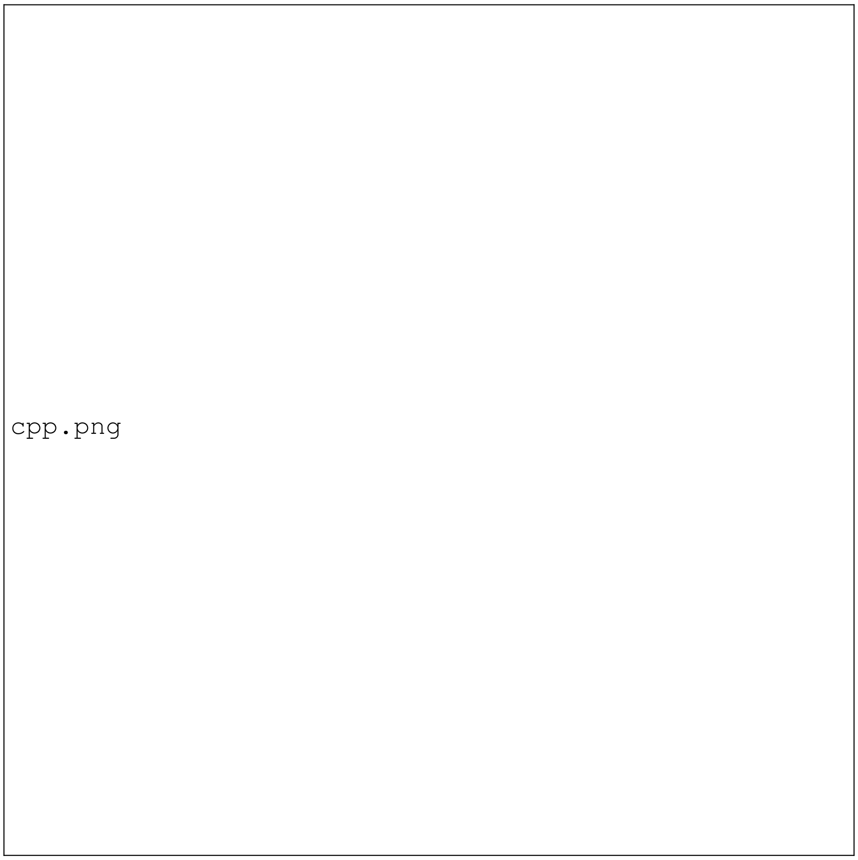

Figure 8: Comparison of continuous learning with adaptive weight allocation

The first part of the comparative experiment is conducted using the experimental data included in [16]. As shown in Fig. 8, Model 1 demonstrates a lower level of consensus in decision results compared to the developed model in this study. The adaptive weight allocation method can effectively adjust to changes in decision-makers' preferences and decision-making environments, thereby promoting consensus among them.

We can observe a significant disparity in the achieved consensus levels between Model 2 and Model 3, as shown in Fig. 8. The limited consensus observed in Model 3 can be attributed to the omission of continuous learning, which may result in adverse effects such as excessive adjustments leading to a decrease in consensus level due to the absence of gradual updating information and dynamic adaptive adjustment processes. In contrast, Model 2 incorporates continuous learning into its framework, effectively enabling it to dynamically capture constantly evolving information through personalized numerical scale. This allows for flexible adaptation to environmental changes while maintaining a balance between stability and adaptability. As a result, the possible negative impacts arising from excessive adjustments are mitigated. These comparative experimental results illustrate the practicality and efficacy of integrating personalized semantic continuous learning into decision-making processes.

Comparison analysis on termination condition

This part compares the impact of employing different termination conditions on CRP, and the designed termination conditions are contrasted and analyzed with conventional methods used for determining consensus threshold and maximum adjustment times [26][27][29][30][31][32], as depicted in Fig. 9.

reference.png

Figure 9: The result of comparison analysis on termination condition

We continue conduct the numerical experiment on the data that employed in Section 14. Setting a consensus threshold as $\delta = 0.85$ and limiting the maximum number of adjustments to 4 rounds for Method II and Method III. In Fig. 9, Method I represents the CER proposed in this study as a termination condition, Method III illustrates the approach employing a consensus threshold, and Method II demonstrates the technique utilizing a predefined consensus threshold while imposing an upper limit on adjustments. Method III allows for adjustment of the preset consensus threshold. Although the consensus level improves slightly compared to Method I and Method II after multiple adjustments, the growth range of the consensus level becomes significantly limited as the adjustment progresses. In such cases, it is crucial to consider the efficiency of adjustment and avoid making futile modifications when the growth range of the consensus level is minimal. While Method II limits the maximum number of adjustments based on Method III, setting this limit according to objective conditions is challenging and leading to low adjustment efficiency. Besides, Method I evaluates whether the termination condition is met based on the current decision-making environment.

CONCLUSIONS

This study develops a continuous learning consensus decision-making model based on personalized semantics, which incorporates the influence of historical and current preference information on personalized semantics. Firstly, a continuous learning model is designed to provide semantic quantification for decision-makers and better simulate the individual preference environment in dynamic decision-making. Secondly, using the adaptive weight allocation method to evaluate the importance of decision-makers in the current decision-making pattern can avoid setting objective parameters and improve the rationality of decision-making. Thirdly, CPR termination condition is introduced, enhancing objectivity and reasonability in decision outcomes. Finally, numerical and simulation analyses illustrate the model's effectiveness, elucidating the influence of dynamic adjustments in preference parameters on language-based decision-making and consensus formation.

However, in the face of increasingly complex decision-making environments brought about by rapid societal development, decision-makers encounter challenges when applying the methods proposed in this paper to address decision-making problems that necessitate extensive participant involvement. Therefore, our future research should prioritize investigating personalized semantic learning models within large-scale groups to effectively handle dynamic and incomplete language preference data in expansive decision-making scenarios.

AUTHOR CONTRIBUTIONS

Conceptualization, X.C. ; methodology, X.C.; software,X.C.; validation, X.C.; formal analysis, X.C.; investigation, X.C.; resources, X.C.; data curation, X.C.; writing—original draft preparation, X.C.; writing—review and editing, X.C.; visualization, X.C.; supervision, X.C.; project administration, X.C. All authors have read and agreed to the published version of the manuscript.

FUNDING STATEMENT

This research received no external funding.

DATA AVAILABILITY

Dataset available on request from the author.

CONFLICTS OF INTEREST

The author declares no conflicts of interest.

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