Empirical evidence on the validity of the Unconditional Higher Moment CAPM in the Bombay Stock Exchange

Abstract

The traditional Capital Asset Pricing Model (CAPM) assumed a normal distribution of returns, which was criticized by various researchers who recognized the non-normal distribution of returns in the Sharpe-Lintner CAPM.The introduction ofunconditional higher moments, namely co-skewness and co-kurtosis, as additional measures of systematic risk may enhance the model's explanatory power, especially when the distribution function of stock returns is asymmetric.The present study empirically investigates the applicability of unconditional higher order moment CAPM and the impact of higher moments in the Indian stock market i.e. Bombay Stock Exchange using the data of sectoral indices for the period from April 2011 to March 2021. To test the four moment CAPM empirically, the specification given by Fang and Lai (1997) has been used in the study. The findings of the present study revealed that the higher moments (coskewness and cokurtosis) are significantly priced and have impact on the returns in the Indian stock market. The market risk premium for covariance was found to be insignificant. Further the hypothesis related to intercept term was accepted and market risk premiums were rejected. The results showed an increase in the explanatory power of the model as compared to the unconditional CAPM as the R-square value of the model was obtained better than the latter model. The mixed and inconclusive findings contradicted the model in the Indian context.The alternate models like the Fama-French three factor and five factor model should be exploited in the Indian context as those models have been rarely used in the Indian context.

Keywords: covariance, coskewness, cokurtosis, Indian stock market, Bombay Stock Exchange (BSE).

1. Introduction

The value of any given asset is determined by a set of activities known as asset pricing. The most widely used asset pricing model in the financial literature is CAPM as it is quite simple and easy to use. The relationship between the systematic risk and expected return was assumed to be independent of market conditions and variance to be the only measure of risk that an investor should consider while making investment decisions. The non normal distribution and non quadratic utility function of investors' wealth implied considering of higher order moments beside mean and variance. Researchers like Cheung and Wong (1992), Ostermark (1991) and Zarnowski and Rutkowska (2012) either rejected the study or found the theory empirically unverifiable. The unrealistic assumptions made by the CAPM highlighted the need for alternative approach to test and estimate the market risk premium.

The traditional Capital Asset Pricing Model (CAPM) assumed a normal distribution of returns, a notion criticized by researchers such as Rubinstein (1993), Harvey (2000), and Harvey and Siddiqui (1999, 2000), who recognized the non-normal distribution of returns under the traditional CAPM framework. The failure of the traditional CAPM is attributed to its assumption that stock returns follow a normal distribution, despite empirical evidence showing that stock returns exhibit non-normal distribution, with higher moments such as skewness and kurtosis influencing stock returns.

In response, an extension of the Sharpe-Lintner CAPM,the introduction of higher moments, namely co-skewness and co-kurtosis, as additional measures of systematic risk. Including these higher moments, namely co-skewness and co-kurtosis, in the pricing of stocks may enhance the model's explanatory power, especially when the distribution function of stock returns is asymmetric.

Rubinstein (1973) demonstrated that investors are concerned about all moments of returns when asset returns do not conform to an elliptical distribution and quadratic utility function. Kraus and Litzenberger (1976) were among the first to propose a model incorporating higher moments, specifically co-skewness, while Fang and Lai (1997) expanded this model to include the fourth moment, co-kurtosis, referred to as the four-moment CAPM.

Furthermore, Kraus and Litzenberger (1976) argued that "if investors exhibit non-quadratic utility functions with non-increasing absolute risk aversion, they would prefer negative skewness over positive skewness in the distribution of stock returns", a view supported by Kimball (1993) and Dittmar (2002).

The three moment CAPM of Kraus and Litzenberger (1976) and the four moment model of Hamaifar and Graddy (1988) are known models for asset pricing which incorporated the higher moments in the traditional model. Gibbons, Ross and Shanken (1989) found that these higher moments cannot be diversified by increasing the size of portfolio, hence, proved to be important in asset valuation.

This study empirically investigates the significance of these higher moments in determining investors' required rate of return within the context of the Indian stock market, specifically the Bombay Stock Exchange (BSE).To analyze the impact of coskewness and cokurtosis on the stock returns in the Indian context, the present study incorporated the coskewness and cokurtosis. An attempt has been made in the present study to know whether the higher order moments have its impact on stock returns in the Indian context or not.

2. LITERATURE REVIEW

“The traditional Sharpe-Lintner CAPM assumed the stocks returns are normally distributed and the variation is explained by the first two moments. The traditional CAPM assumed the utility function of an investor to be quadratic and the only factor that determined the price of stock is the co-movement of stock returns with the market return”. [29] Further studies like Carhart (1997), Fama-French (1992, 1993) etc contradicted “the validity of the model which led to be development of alternative models which can determine the stock prices”. Mandelbrot (1963), Arditti (1967) and Fama (1965) found “the stock returns were not normally distributed. The explanatory power of the traditional model may increase if the higher moments were included”. Kraus and Litzenberger (1976) suggested that “the investors will prefer positive skewness to negative skewness if the utility function is non quadratic and express non-increasing absolute risk aversion”.

Kimball (1993) and Dittmar (2002) stated that “if the utility function showed decreasing absolute risk aversion, the kurtosis in the distribution of stock returns will be disliked by investors”. Rubinstein (1973) proved that “the investors do care about higher moments hence, skewness and kurtosis must be included in the determination of stock prices. The investors must be compensated in the form of higher returns as the negative skewness and excess kurtosis are disliked by the investors. On the other hand, the investors may forego some returns for the same variance as they prefer for positive skewness and low excess kurtosis. Hence, the higher order co-moments i.e. coskewness and cokurtosis should be included in pricing of stocks in the traditional CAPM if the distribution of stock returns is asymmetric”.

Since, the late 1990s, the testing on four moment CAPM has been done by various researchers across the globe. The impact of coskewness and cokurtosis was examined by Fang and Lai (1997), Dittmar (2002), Doan et al. (2010) and Conrad et al. (2013) in the US stock market. Similarly the importance of coskewness and cokurtosis was examined by Hung et al. (2004) and Fletcher and Kihanda (2005) in the UK stock market. The importance of higher moments including the volatility was examined by Galagedera et al. (2003), Doan et al. (2010) and Doan et al. (2014) in the Australian stock market.

The importance of coskewness and cokurtosis on stock returns for Brazilian and Russian stock market was examined by Silva (2005) and Teplova and Shutova (2011) respectively. Chunhachinda et al. (1997), Bekaert et al. (1998), Eftekari and Satchell (1999), Hwang and Satchell, 1999; Mitra and Low (1998), Ghysels et al. (2011) etc. analyzed the importance of higher moments in pricing of risky securities for emerging economies like India. Vishnani (2013) performed the study to analyze the impact of coskewness only in the Indian stock market.

Andersen et al. (2001), Linden (2001), Choi and Nam (2008) and Buckle et al. (2016) suggested that the stock returns were observed to be asymmetric and leptokurtic, therefore it obvious for higher order moments to explain the stock returns. Hasan and Kamil (2014) found that the stock returns were explained by including the higher order moments. Kostakis et al. (2012) found that higher expected returns were because of higher cokurtosis.

"In this article, co-skewness and co-kurtosis are incorporated in the traditional CAPM model of Sharpe-Lintner. Thus, this article analyses the impact of higher moments on returns of sectoral indices listed in BSE in the Indian context. There are limited number of studies which have analyzed both the impact of co-skewness and co-kurtosis on stock returns. The empirical literature shows that testing on four-moment CAPM model has been done more recently (since the late 1990s)".[29] Dittmar (2002), Fang and Lai (1997), Conrad et al. (2013) and Doan et al. (2010) have examined" the impact of co-skewness and co-kurtosis on stock returns for the US market". Fletcher and Kihanda (2005) and Hung et al. (2004) have examined "the importance of co-skewness and co-kurtosis in explaining the variation in the stock returns for the UK stock market". Galagedera et al. (2003), Doan et al. (2010) and Doan et al. (2014) examine "the importance of higher moments (volatility, skewness and kurtosis) for the Australian stock market". Chiao et al. (2003) tested "the four-moment CAPM model for Taiwan stock market". Teplova and Shutova (2011) and Silva (2005) analyse "the impact of co-skewness and co-kurtosis on stock returns for the Russian stock market and Brazilian stock market, respectively".

"There are a few studies in the context of emerging economies like India that have analysed the importance of higher moments in pricing risky assets. However, most of these studies in the context of emerging economies have concentrated more on individual stocks rather than on sectoral indices. There are hardly any studies in the context of emerging economies like India that have analysed the impact of higher moments on the sectoral indices returns and this has motivated us to write this article".[29] Vishnani (2013) has analysed "the impact of only co-skewness on the returns of the Indian stocks. To the best of our knowledge, there are no studies in the Indian context that have analysed the impact of both co-skewness and co-kurtosis on the returns of sectoral indices. This study is an attempt to fill this gap. The results of this article will help us to know whether the impact of higher moments on expected sectoral index return in the Indian context is the same as it is found in the developed countries like the USA".

3. MATERIALS AND METHODS

The study was conducted using secondary data. The data of 10 sectoral indices from Bombay Stock Exchange listed in S&P BSE 500 index for a period of 10 years ranging from April 2011 to March 2021 was used. The 91 days Treasury bill yield was used as proxy for the risk free rate of return was downloaded from *Reserve Bank of India (RBI)* website.

The methodology proposed by Fama and Macbeth (1973) has been used in the study. The weekly sector returns, market returns, and beta for each sector were obtained similar to Asthana and Ahmed (2021). To test the four moment CAPM empirically, the specification given by Fang and Lai (1997) were used which is represented as:

$$R_{it} = \alpha + \hat{\beta}_{imt}\,(R_{mt}) + \hat{\gamma}_{imt}\,(R_{mt})^2 + \hat{\delta}_{imt}\,(R_{mt})^3 + \mu_t \qquad (1)$$

The $\hat{\gamma}_{imt}$ (lambda) and $\hat{\delta}_{imt}$ (delta) denotes the co-skewness and co-kurtosis risk respectively.

The inclusion of higher moments adds complexity to the model, making it more difficult to interpret and apply compared to the traditional CAPM. This complexity may discourageresearchers from adopting the model. While the traditional CAPM assumes a normal distribution of returns, the unconditional higher moment CAPM relaxes this assumption by considering higher moments. However, this may not fully capture the complexities of real-world return distributions, leading to potential inaccuracies in risk assessment and pricing. Estimating higher moments such as co-skewness and co-kurtosis can be challenging, particularly with limited data or in markets with low liquidity. Inaccurate estimation of these moments can lead to unreliable risk assessments and pricing estimates.

The beta for each sector was obtained using equation 1 using Generalized Method of Moments (GMM) as an estimation technique. The rolling regression was employed with 60months (5 years) rolling window with step size 1 and was used to estimate the rolling beta. The excess market return and lagged excess market returns were used as an instrumental variable.

The beta value greater than 1 ($\beta>1$) indicates that the Indices' price is more volatile than the market. The beta value for seven Indices was obtained less than 1 but greater than 0 ($\beta<1$) which indicates that the Indices' price is less volatile than the market. The beta value for three indices was obtained less than zero ($\beta<0$) which indicates an inverse relation to the market. The Jarque-Bera test is a goodness of fit test which shows whether the data characteristics like skewness and kurtosis matches to that of the normal distribution. The higher value of Jarque-Bera having p-value less than 0.05 shows that the data deviates from normality.

After the estimation of rolling betas for each sector, the portfolios were formed in order of ascending betas such that the lowest betas were contained in the first portfolio and the highest betas were contained in the last portfolio. The cross sectional regression equation of the model was used to examine the validity of unconditional higher moment CAPM which is represented as:

$$R_{pt} = \hat{\eta}_0 + \hat{\eta}_1\,\beta_{pmt-1} + \hat{\eta}_2\,\gamma_{pmt-1} + \hat{\eta}_3\,\delta_{pmt-1} + \varepsilon_{pt} \qquad (2)$$

The coefficients representing the slope in the cubic CAPM model, as illustrated in the equation (1), serve as explanatory variables in equation (2) of the cross-section equation to estimate the associated risk premium. Given investors' preference for high skewness, negative market skewness is deemed a risk and is anticipated to be compensated with a positive skewness premium.

Consequently, in our model as given in equation (2), $\hat{\eta}_2$assumes a positive value when the market exhibits negative skewness and takes on a negative value in the presence of positive skewness in the market. Similarly, for kurtosis, the rationale mirrors that of the second moment; higher kurtosis, indicative of fat tails, represents a negative investment incentive, and accordingly, the corresponding risk premium, $\hat{\eta}_3$, is anticipated to be positive in our model.

The Following hypotheses must hold true to validate the model theory.

$\hat{\eta}_0 = 0$ i.e. the intercept term is statically equal to zero and insignificant.

$\hat{\eta}_1 > 0$ i.e. the systematic risk should be positive and significant.

$\hat{\eta}_2 < 0$ i.e. the co-skewness risk should be negative and significant.

$\hat{\eta}_3 > 0$ i.e. the co-kurtosis risk should be positive and significant.

4. RESULTS ANDDISCUSSION

The descriptive statistics i.e. mean, standard deviation, skewness and kurtosis have been computed and Jarque-Bera test value has been used to test the normality of the returns of the sectoral indices. (Table 1)

Table 1: Descriptive Statistics: Unconditional Higher Moment CAPM (Index returns)

	Mean	Std. dev	Skewness	Kurtosis	Jarque-Bera	p-value
Index 1	0.0003	0.719	2.423	9.967	37	<0.001
Index 2	0.0008	0.395	2.424	5.98	48	<0.001
Index 3	-0.0004	0.178	1.422	4.943	21	<0.001
Index 4	0.0013	0.99	-1.424	3.998	33	<0.001
Index 5	-0.0014	0.859	1.402	6.366	29	<0.001
Index 6	-0.0013	0.525	-1.424	5.981	56	<0.001
Index 7	-0.0017	0.492	1.42	7.878	39	<0.001
Index 8	-0.0024	0.951	2.421	4.913	41	<0.001
Index 9	-0.0006	0.338	-1.422	3.928	55	<0.001
Index 10	0.0012	0.688	1.418	6.833	49	<0.001

It can be seen from the above table that the mean weekly returns ranges between -0.0024 (Index 8) to 0.0013 (Index 4). The low value of standard deviation obtained for all indices indicates more stability and less volatility. The negative skewness (Index 4, 6 and 9) indicates that the data is skewed left (left tail is relatively longer than right tail) while the positive skewness (Index 1, 2, 3, 5, 7, 8 and 10) indicates that the data is skewed right (right tail is relatively longer than left tail). The negative skewness in finance refers to low returns (frequent small wins) or high risk (few big loss) while the positive skewness refers to low risk (minor loss) or high returns (few large gains). The kurtosis value for all portfolios obtained are >3 indicating leptokurtic distribution of the returns, means the investment is considered to be risky. The higher value of Jarque-Bera statistic and p-value <0.05 suggests that all the portfolios have return distributions that are non-normal (Table 1).

Using the specification model of Fang and Lai (1997), in the first step, the unconditional higher moment beta, lambda and gamma for each sector were estimated using 60months rolling regression applying the GMM as an estimation technique. The range of beta obtained is 1.179 with the minimum value -0.652 and maximum value 0.527. The beta value for seven Indices was obtained less than 1 but greater than 0 (β<1) which indicates that the Indices' price is less volatile than the market. The beta value for three indices was obtained less than zero (β<0) which indicates an inverse relation to the market. (Table 2)

Table 2: Estimated beta values of sectoral indices

Indices	**Beta (average)**
Index 1	0.452
Index 2	0.527
Index 3	0.429
Index 4	-0.652
Index 5	0.132
Index 6	-0.349
Index 7	0.210
Index 8	0.276
Index 9	-0.292
Index 10	0.129

After the estimation of unconditional betas, next step was to construct the portfolios by arranging the betas in ascending order such that the lowest betas were allocated in the first portfolio and the highest betas were allocated in the last portfolio. The cross-sectional regression specification (equation 2) was employed to test the validity of Unconditional Higher Moment CAPM and the stated hypotheses must hold to empirically validate the CAPM theory. (Table 3)

Table 3: Result of Unconditional Higher Moment CAPM

Portfolios	**Coefficient**	**R-square**	**t-statistic**	**p-value**

Full Sample	**α**	-0.324	0.00468	-0.172	0.863
	β	0. 042		0.464	0.643
	λ	4.018		3.037	<0.001
	γ	-3.658		-2.873	<0.001
P1	**α**	0.486	0.00493	0.990	0.322
	β	1.902		0.476	0.633
P2	**α**	-0.110	0.00507	-0.987	0.323
	β	-1.518		-0.483	0.629
P3	**α**	-0.108	0.00509	-0.993	0.320
	β	-1.447		-0.484	0.628
P4	**α**	0.241	0.00488	0.994	0.320
	β	1.365		0.474	0.635
P5	**α**	-0.081	0.00485	-0.954	0.340
	β	-1.869		-0.472	0.636

It is observed from the above table that the intercept term for the full sample is negative but significant supporting the first hypothesis while the risk premiums for the full sample is positive but insignificant, rejecting the second hypothesis. The market risk premium for the unconditional co-skewness is positive and significant while the market risk premium for unconditional co-kurtosis is negative and significant, rejecting the third and fourth hypotheses.

Though the hypotheses related to coskewness and cokurtosis risk are rejected, but they are significantly priced and by including these higher moments in the traditional model, the explanatory power of the model has increased as the R-square value of the unconditional higher moment CAPM (0.00468) is greater than the unconditional CAPM (0.00035).[Asthana and Ahmed (2023)].

The values for intercept term for the portfolios obtained are significant at 5% significance level. The estimated risk premiums obtained for all the portfolios are statistically insignificant at 5% significance level. Thus, the systematic risk (i.e. unconditional beta) cannot be the appropriate measurement of risk in BSE. The low value of R-square suggests that the model was unable to explain the cross sectional returns. Hence, the validity of Unconditional Higher Moment CAPM is not justified in the Indian context. Similar finding were obtained by Sanchez-Torres and Sentana (1998), Wolfle and Fuss (2010) and Hung et al. (2004) which is consistent with our study of as they also found mixed and inconclusive findings.The results may vary if the higher frequency data is used.

The impact of higher order moments on stock returns in the Indian context is not the same as in the developed countries like USA. The main reason for the dissimilarity with the US stock market might be size of market, as the Indian stock market is much smaller than the US stock market.

The CAPM assumed that the higher (lower) risk stocks are associated with higher (lower) level of return. The portfolio average returns and portfolio average betas were ranked. To test the given assumption, the Karl Pearson correlation coefficient was calculated. The result of the given assumption was not supported as some of the portfolios with higher returns had lower beta and lower returns had higher beta. The positive correlation coefficients 0.487 indicated weak positive relationship.

5. CONCLUSION

The main purpose of the study was to examine the validity of the higher moment CAPM and to assess the explanatory power of coskewness and cokurtosis in the Indian market. The Fama-Macbeth two step approach was used in the study. The results showed that the coskewness and cokurtosis are significantly priced in the Indian market and the explanatory power of the model has increased. The cross-sectional results showed that the risk premium for covariance was insignificant and the hypothesis related to the risk premiums (covariance, coskewness and cokurtosis) was rejected. Hence, the validity of the unconditional higher moment cannot be justified in the Indian stock market.

Availability of data and materials: The data used in the study that support the findings of this study have downloaded from BSE (https://www.bseindia.com/indices/IndexArchiveData.html) and RBI (https://www.rbi.org.in/Scripts/BS_NSDPDisplay.aspx?param=4) websites which is publically accessible.

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