Observations on the Three-Plus-One Conjecture And aProposed Solution

Abstract

A block format was used in the study of the three-plus-one conjecture. The odd positive integer series has been divided into three families. The block format treats these families in relation to the three plus one conjecture. Statistical analyses of these blocks suggest the validity of the conjecture.

Key Words: Three plus one conjecture - **Collatz conjecture – Ulam conjecture - Kakutani's problem – Block format – Odd positive integer families – Base block – Root blook – Block extension – Lead-follower blocks - Block statistics**

Introduction

Since its introduction by Collatz and others more than eighty years ago, the three-plus-one conjecture has attractedvarying interest. Recently, there has been a revival of interest in the problem. Researchers tackled the problem from different angles. This mathematical problem has been known under various names, including the Ulam conjecture, Kakutani's problem, The Thwaites conjecture, Hasse's algorithm, and the Syracuse problem.

Watching a video on the three-plus-one conjecture created by Dr. Derek Muller in his Veritasium video series about a year ago triggeredmy interest in the subject. The footage includeddepressingand warning statements to those who delve into the topic.Thus, I decided to proceed with a fresh mind. The following treatment is rather basic but explores the behavior of the natural positive number series and the odd natural numbers series in many ways and relates them to the conjecture. A unique format called the block format helped in conducting this research and in formulating governing rules. Formulations, rules, and statistics of the behavior of these blocks proved very useful in the study and drew a proving picture of the three plus one conjecture (The conjecture).

After preparing the present work as shown herein, I did a quick search using the internet search sites and came across the clever work of Surendran and Babu [1]. I left my treatment as written. Common elements are evident; their work is cited as a general reference. Among these observations and formulations are their theorems 2.1 and 4.1 and some statistics and behavior formulas of odd numbers regarding the conjecture (The three-plus-one conjecture). The present formulations of the block format summarize some of these and other relationships and formalize them in a well-ordered way. Their observation about the statistics of the occurrence of a single division by two and the odd numbers divisible by three are defined here, too.In addition, other statistics of the occurrence of all other odd positive integers are presentedhere,along with general conclusions. Their article includes observations about the occurrence of the “Starters” (The odd numbers divisible by 3). Their work extends to other formulations and endeavors. The “Alternation” between the *F*1, *F*2, and *F*3 families of the positive odd integers along the convergence process or the reverse bottom-up generation process and the skipping of *F*3 odd numbers along the convergence process is described here. The credit goes to the original authors for any other formulations or concepts presented here that have appeared in previously published work.

This current study explores and defines patterns in which natural numbers behave with consideration of the three plus one conjecture. Natural odd numbers are divided into families and groups. Patterns and behaviors of these families and groupsare investigated through the light of the three-plus-one conjecture. A unique block format was used during the study, which helped developgeneral governing rules. Distinctive characteristics of these families and the block groups formed from them have been discovered. The study investigates these characteristics in the convergence (Conjecture) mode and the opposite generation (bottom-up, revers-conjecture) mode and suggests a solution proving the validity of the conjecture.

The convergencemode moves in one direction in one chain from the start to number 1 in a final converging step (If so happens). During this one-direction path, other chains starting from other positive integers join the path at specific points. This creates a look of coming down from many (infinite) branchesof a tree or a bush to more and more common ones to final trunks above ground, which are also infinite. These trunks join only underground at one root, the number one. On the other hand,in the generation mode(Reverse-conjecture rules),the movement starts at the root and spreads intoprimary and secondary branches. Different branchesare explored until the target number is reached. This procedure is tedious andexhausting if one specific number is the target. However, general rules applicable to the whole set of the natural positive integers can be obtained.The current study aims to develop relationships and formulas that prove that all natural numbers (Positive integers) can be generated by applying these rules.There has been work along the convergence mode in checking individual numbers to prove or disprove the final convergence to number one. Other than finding a non-converging number, this kind of work just adds to a stronger belief in the validity of the conjecture.

The present treatment and many other articles aimed to find the patterns and formulate general rules governing these patterns so a conclusion can be made based on simple mathematical formulations. If we can move from number one to all the natural positive integer series by applying the generation (Reverse conjecture) rules, then we can move down in the opposite direction from any natural positive number to number one, which proves the conjecture. Along this line of thought, the possibilities of looping, ever-increasing, and plateau formation are tackled.

Method

Procedures and formulations here apply a format called the block format. In this format, the process, whether convergence or generation, divides the chain of positive natural numbers into blocks. Blocks are treated as individual entities. A block starts with an odd number, called the left hook (L), and ends with an odd number, called the right hook (R). Even numbers are sandwiched between these odd numbers and are called the middle-even numbers (E). The odd number at one end of one block is repeated at the other end of the following block.Each block is a convergence from one odd positive natural number to another. Odd numbers shall always be treated in these joint couples like the domino tiles when dealing withthe three plus one conjecture. Block format in the convergence mode is presented in square brackets as [L/E/R] and in the generation mode as [>R/E/L]. The > sign is placed ahead of the R in the generation mode for distinguishing purposes. Examples in the convergence mode are [27/1/41] and [41/2/31], and in the generation mode, they are [> 41/1/27] and [> 31/2/41].

Relationships between the left hooks, right hooks, and middle-even numbers are formulated and produce specific criteria for the blocks they belong to. Formulations includebasic formulas as well as parametric ones. The parametric formulaspresent the left hook (L) in the form $L = A + BK$, and the right hook in the form $R = C + DK$, where A and C are odd numbers, B and D are even numbers, and K is an index taking values: 0, 1, 2, … to infinity. Blocks are categorized primarily based on the number of the middle-even numbers (The length of the block), then according to the R family ($R1$ or $R2$)) and according to the L family (Called the L brand:$L1$, $L2$, $L3$). Each categorized group of blocks has specific characteristics in general and in relation to the conjecture rules. Each group of blocks extends vertically as the K index increases to infinity. These block groups are connected in chains, which results in the well-observed unique behavior and properties of the convergence or generation chains. Statistics of the occurrence and repetition of specific block groups have been studied. Formulas and statistics for individual block types and block couples in a Lead-Follower arrangement and clusters of specific block groups have also been studied.

Odd numbers sandwich even numbers in the block format (The middle-even numbers). Analyses of the odd numbers are thus sufficient to generalize the conclusions to the whole positive integer series. Any even number is divided by two consecutively until an odd number is reached, which is considered the left hook of a block. This left hook is multiplied into three, and one is added, which results in an even number. This even number is divided by two consecutively until an odd number is reached, which is considered the right hook of this block. The odd number in this right hook is used as the left hook of the next block, and the process is continued until the convergence is completed. Convergence occurs in a finite number of calculations, no matter how big the starting L is.

The series of positive natural odd numbersis divided into three main families. The series of each family continues infinitely. The sum of these three families together forms the infinite odd numbers series. These families are:

Family $F1$: 5, 11, 17, ..., ∞

Family $F2$: 1, 7, 11, ..., ∞

Family $F3$: 3, 9, 15, ..., ∞

Formulas:

Family $F1$: $O = 5 + 6K$

Family $F2$: $O = 1 + 6K$

Family $F3$: $O = 3 + 6K$

O: Odd number

$K = 1, 2, 3, \ldots$

Blocks start with a left hook belonging to any of the three main families, described as brands $L1$, $L2$, and $L3$. Blocks end with an R. R hook belonging to $F1$ are denoted as $R1$ family and $R2$ family for R belonging to $F2$. All blocks of the $R1$ family have an odd number of the middle-even numbers, and all blocks of the $R2$ family have an even number of the middle-even numbers. There is no $R3$ family, as shall be described later.

Formulations and Results

Formulations and results are included in Boxes. Each Box containsa data set in one or more tables and relateddefinitions and formulas. This facilitates reading the data and grasping the related concepts included. Boxes D1, D2, and D3 include details of the block formulations and definitions.

Box B1 includes blocks with fixed middle-even numbers (Fixed *E*) in the convergence mode. Box B2 shows the first occurrence and the repetitions of different block lengths (Block groups) along the odd positive integers' series. Box B3 includes a summary of the generated *L* values versus *E* and *K*.Box B3-B contains observations and calculation formulas for Data in Box B3. Box B4 summarizes statistics of the occurrence of different block lengths. Box B4-B includes morestatistics for *R* and *L* from thedata presented in Box B4.

Box F1 includes tables of two consecutive blocks with both the lead and follower blocks having an odd number of *E*. Box F2 tabulates similar data for odd *E* in the lead block and even *E* in the follower block. Box F3 includes tables of two consecutive blocks where the lead block has an even number of *E* and the follower block has an odd number of *E*. Box F4 tabulates similar data for even *E* in both the lead and the follower blocks. Data in these four Boxes is arranged in the generation mode. Boxes F5 and F6 summarize the frequency of occurrence of these combinations and conclude statistics about their *R* and *L*.

Box C1 includes tables of Consecutive Single Middle-even number Blocks (CSMB) starting with *L*3, while Box C2 includes similar tables for blocks starting with *L*2. Box C3 includes formulas governing data in Boxes C1 and C2. Box C4 includes a table and formulas of one clustertype of the Consecutive Double Middle-even numbers Blocks (CDMB) starting with *L*3. Box C5 includes formulas governing data presented in Box C4. Box C6 includes examples of CSMB and CDMB havingthree blocks (3*M*) and starting with *L*3. These examples are shown in the parametric form.

Box EX1 includes examples of the *L*3 skipping in the convergence mode and compares it to the generation mode. Box EX2includesthe parametric form of the convergence of number 27 in a one-lead, one-follower combination. Box EX3containsan example of CSMB clusters along with a figure. Box EX4includesdata for 20*M* CSMB (A cluster of 20 Consecutive Single Middle-even numbers Blocks) and the complete convergence of this chain to number one.

Box EX5 includes data for two types of CDMB, starting with *L*1. Box EX6containsan example of the CDMB cluster having twenty blocks (20*M*) starting with *L*3 and completing the chain to final convergence to 1.

Box EX7includes examples of stretches of similar sequences of varyingblock lengths. Box EX8 contains the application of the proposed stretch formulas to selected sequences of the convergence of number 27.

Discussion

According to the three plus one conjecture (The conjecture), even positive numbers are divided by two until an odd number is reached. Then, the odd number is multiplied into three, and one is added. The resulting even number is divided by two, and the process is repeated consecutively. The final convergence should reach number one. Even numbers descend smoothly to an odd number. Therefore, studying the positive odd numbers is sufficient to prove or disprove the conjecture. The Definition Boxes D1, D2, and D3detail thisprocedure and state important properties and formulas for the blocks.

The general formula relating R to L in the convergence mode is:

$$R = (3/2^E)L + (1/2^E) \quad (1)$$

Only one single E value (Number of the middle-even numbers) produces a specific wholeodd$R$-value for any odd whole L value. That means each block is unique, starting with the selected L value producing a unique R-value.E is determined by the group L belongs to. BOX B1 shows this relationship. Blocks are tabulated in groups of block lengths (Fixed E). General governing formulas are also contained in the box.Each odd number as an L in the block brings its specific R according to the group it belongs to.

The general formula for the generation mode is:

$$L = (2^E/3)R - (1/3) \quad (2)$$

Series of E values satisfy the formula for whole odd L and R values. These Rseries produce a corresponding series of L values. The L values in these series obey specific rules and can be predicted. Reference is made to BOX B3 for that property. The L series is unique to each R-value. One specific L can not be generated from two different R values.

The base block for the $R1$ family has E = 1, and the $R2$ family has E = 2. There is no limit for E, as shown from the E series in the generation mode. For one R-value, infinite E values produce infinite odd L values in extended blocks having E = 3, 5, 7, … for $R1$ Blocks and E = 4, 6, 8, … for $R2$ family.

If L and R are known,E can be calculated by the formula:

$$E = (\mathrm{Log}(3L + 1) - \mathrm{Log}(R))/\mathrm{Log}(2) \quad (3)$$

The $F3$ family of positive integers has unique properties. Numbers of family $F3$ can start a chain in the convergence mode, where they are used as the first left hooks ($L3_0$) in the first block. $F3$ numbers can not occur as an R of a block and consequently do not occur in the middle of a converging chain. This is a consequence of the convergence rules.

In the generation mode, members of the $F3$ family can be generated as L, but only once, and the generation process stops there. This terminates the procedure of generating more blocks along this chain. In this mode, $L3$ can be skipped by extending the block lengthby adding more to the middle-even numbers (Two more even numbers at a time or multiplying the middle-even number by 2^2), as shown in Box EX1.

In the generation mode, it is possible to move from the $R1$ or $R2$ valuesto reach all the members of the$L3$ (3, 9, 15, …). Thus, moving back to the convergence mode starting from the $L3$ brands (Left hooks belonging to the $F3$ family) is sufficient for studying the conjecture, and there is no need to study $L1$ and $L2$ separately in this mode. After the starting block, L always belongs to $F1$ or $F2$ ($L1$ or $L2$), and R also belongs to $F1$ or $F2$ ($R1$ or $R2$).

The following Table (Table 1) shows the six elementary blocks starting with *L*3.

Table 1: *R*1 and *R*2 produced from *L3* – Convergence Mode

*L*3	*E*	*R*1 &*R*2	Series
3 + 12*K*	1	5 + 18*K*	5, 23, 41, …
117 + 192*K*	5	11 + 18*K*	11, 29, 47, …
45 + 48*K*	3	17 + 18*K*	17, 35, 53, …
21 + 384*K*	6	1 + 18*K*	1, 19, 37, …
9 + 24*K*	2	7 + 18*K*	7, 25, 43, …
69 + 96*K*	4	13 + 18*K*	13, 31, 49, …

The six formulas in the *R*1 &*R*2 columns cover all members of the *F*1 and *F*2 families.

The parametric form of these six formulas can be written in a block format [*A*-*BK*/*E*/*C*-*DK*] as follows:

[3+12*K*)/1/5+18*K*]

[117+192*K*/5/11+18*K*]

[45+48*K*/3/17+18*K*]

[21+384*K*/6/1+18*K*]

[9+24K/2/7+18K]

[69+96*K*/4/13+18*K*]

The general parametric forms for*L* and *R* are presented as follows:

$L = A + BK$ (4)

$R = C + DK$ (5)

Table 4 shows the parametric forms of *L* and *R* as *E*steps up (Increases by two). Odd *E* is for *R*1, and even *E* is for *R*2.

Table 2: *L* and *R* Parametric Forms

E	*L* Formula	*R* Formula
1	3 + 4*K*	5 + 6*K*
3	13 + 16*K*	5 + 6*K*
5	53 + 64*K*	5 + 6*K*
7	213 + 256*K*	5 + 6*K*
9	853 + 1024*K*	5 + 6*K*
11	3413 + 4096*K*	5 + 6*K*
2	1 + 8*K*	1 + 6*K*
4	5 + 32*K*	1 + 6*K*
6	21 + 128*K*	1 + 6*K*
8	85 + 512*K*	1 + 6*K*
10	341 + 2048*K*	1 + 6*K*
12	1365 + 8192*K*	1 + 6*K*

The index *K*takes values along the 1, 2, 3, … series. Thus, the tabulated *L*-*R* in the boxes can extend infinitely as the *K* value increases infinitely.Specific formulas govern the *L*-*R* relationship within each group (Block length). These, as well as group-to-group relationships, are presented in Box B1. The general formulas for *L, R, E*, and *K* are written below.

For odd *E*, $R_0 = 5$

$$L_K = (5/3)*2^E - (1/3) + 2^{E+1}K \quad (6)$$

$$R = + 6K \quad (7)$$

For even *E*, $R_0 = 1$

$$L_K = (1/3)*2^E - (1/3) + 2^{E+1}K \quad (8)$$

$$R = 1 + 6K \quad (9)$$

$$R_0 = R \text{ at } K = 0 \quad (10)$$

Box B1 also shows that each *L* brand repeats sequentially when *E*or*K* increases. This adds more of the characteristics of the families of odd numbers. The *L* brands can be categorized together at jumping *E* (*E* increases by six). Categorization of the same *L* brands along the *K* is done by increasing it by three at a time.

Box B2 categorizes the first occurrence and subsequent repetition(Reoccurrence) of block groups (Blocks having the same number of middle-even numbers *E)* per the *L* brand (*L*3, *L*2, *L*1). When the three L brands are merged, the move along the series of the odd numbers is the stepwise addition of two (1, 3, 5, …). 1*E* blocks occur every two steps (Addition of four) starting from number three along the odd number series. The 2*E* blocks reoccur every four steps (Addition of eight) starting from 1.The 3*E* blocks reoccur every addition of sixteen,beginning at number thirteen, and so on. Each linear increase of the *E* by one decreases the occurrence of the block by doubling the *B* value (A multiplier of two on the *B*), as shown in Box B1.

The place of the odd number in the odd number series (Its value) very well determinesits Lbrand, the block length they have, and the R family they belong to. Theresulting *R*, used as *L* in the subsequent block, has specific block lengths, R, and so on.

Each block has predetermined R, which becomes *L*1 or *L*2 in a subsequent block in the convergence mode. The block of this new L has its specific length and frequency of occurrence, as determined in Box B1.

Box B4 summarizes the statistics of all block lengths for the *R*1 and *R*2 families in the generation mode. The sum of the *L* series in the *R*1 family equals one-third of the positive integer series, and the sum of the L in the R2 family equals one-sixth of the positive integer series. The total of these *L* values equals one-half of the positive integer series or the whole series of the odd positive integers. Thus, no number along the odd positive number series can be missing in allpossible generation chains.

Box B3 shows *L* values as *E* increases horizontally and as *K* increases vertically for *R*2 and *R*1 in the generation mode. Both *E* and *K* can increase infinitely.

The general formulas presented in Box B3 are repeated here.

For even *E:*

$$L_{E,0} = 4^{n} + 4^{n-1} + 4^{n-2} + \ldots + 4^{n-n} \quad (11)$$

$$n = (E-2)/2 \quad (12)$$

For odd *E:*

$$L_{E,0} = 3*4^{n} + 4^{n-1} + 4^{n-2} + \ldots + 4^{n-n} \quad (13)$$

$$n = (E-1)/2 \quad (14)$$

For even and odd *E*:

$$L_{E,K} = L_{E,0} + 2^{E+1}K \quad (15)$$

These formulas generate one single *L* for each *E*, and *K*. *R* values in the far-left column are themselves generated *L* in the body of the table except for number one, which is the root number for all generations. One can move from number 1 and generate any number in the odd positive numbers series by successive generations and connect the generated *L* in one block as *R* in the next block. Then, moving back to the convergence mode can be traced on a reversed path.

A block can be extended to a huge number of middle-even numbers.The first occurrence of long blocks gets bigger, too, and its reoccurrence frequency diminishes, but its effect on its *L* (The *R* over *L* ratio) is bigger, too. This trend is illustrated in Box B2. The first block occurrence of a block having *E* = 30starting at *L*3 and having *R* = 1 is *L*3 =357,913,941. This block statement is: [357,913,941/30/1]

The second occurrence of such a block with *E* = 30 is after adding2,147,483,648 to the first *L*. The*L* brand of this block is *L*2, and it has *R* = 7. The *L*3 hook brand repeats occurrence after adding6,442,450,944tothe first *L*3. This block statement is [6,800,364,885/30/19].

Statistically, this block length occurs $1.863*10^{-9}$ times less frequently than blocks with 1*E*. In other words, blocks having 1*E* occur $5.37*10^{8}$ more times than blocks having 30*E*.

The effect of a block(*f*) in a converging series is the ratio of its *R* over its *L* (The *R* of the previous block). The impact (*i*) of the block is an approximation by neglecting the absolute term from the *R* definition.

$$f = R/L = (3L/2^{E} + 1/2^{E})/L \quad (16)$$

$$f = (3/2^{E}) + 1/2^{E})/L \quad (17)$$

$$i = 3/2^{E} \quad (18)$$

The impact (*i*) of the 30*E* block = $3/2^{30} = 2.79*10^{-9}$

The impact (*i*) of 1*E* block = $3/2^{1} = 1.5$ (19)

Converging chains include all sorts of skewing from the statistics.

Number 21 converges to number one in one block, and six middle even numbers, as follows:

[21/6/1]

Number 19 needs six blocks and fourteen middle even numbers for the final convergence to number one as follows:

[19/1/29] [29/3/11] [11/1/17] [17/2/13] [13/3/5] [5/4/1]

Number 27 requires 41 blocks and 70 middle-even numbers to converge to number one.

Number 21 belongs to the six middle even numbers group, while number 19 belongs to the 1E group (One single middle even number)

A strain of block would have the same R and jumping number of E (The Middle even numbers increase by six)

Examples are:

[3/1/5], [213/7/5], [13653/13/5]

All the L values 3, 213, 13653, 873813, … require two blocks only to converge to number one. The first blocks in the chains are extended, though, with increasing middle-even numbers.

Allowing more blocks puts these as skewed parts (stretches) within the complete converging chain. This is evident in the examples of clusters as shall be discussed.

The block study is further extended to the sequence of couples of blocks as contained in Boxes F1 through F6. From these boxes, it can be seen that in the generation mode, the sum of the occurrence of follower blocks of all lengths equals the odd natural numbers series.Moving through all generated branches generates the whole odd numbers series. Generation branches are infinite. This asserts what was found in Box B4 for the single-E block study.

In the convergence mode, Each *R* follows from the previous *L*. No *R* can be produced from two or more *L* values in two or more branches.After two **branches** join in a single *R* (A node), the formed trunksmove along the convergence rules and may join other branches and trunks along the convergence path, but they do not split into new branches again.

One possibility of not reaching the final 4, 2, 1 convergence includes infinite oscillation and not reaching the number one. This may happen when the *R*-value oscillates around a plateauor many plateaus. Another possibility isa continuous increaseof the *R*-value along the convergence of a chain without finally converging to number one.

One case of *R*-value ever increasing in a convergence mode chain is repeated single middle even number blocks in sequence.

Convergence to number one means that this sequence of ever-increasing *R*-value should stop at some point and not continue endlessly. This trend is seen in Boxes C1 through C4. Formulas for the CSMB cluster occurrence and the rule for starting *L* in such clusters are proposed and included in these boxes.

Proposed formulas conclude that the number of such blocks in those clusters is finite. After that, the chain continues in the typical observed up-and-down pattern until it finally converges into one. In these CSMB *R*/*L* reaches infinity only when the starting *L* is infinite. Clusters of different ranks (Number of the CSMB) occur along the convergence path of a chain and follow the governing formulas. The chain continues, including different block lengths and types of clusters, until it finally converges to 1.

This is shown in Box EX5, which includes a chain with 20 CSMB and the continuation to final convergence to number one in 164 blocks. With this relatively high number of blocks, longer (Extended) blocks (3*E*, 4*E*, …) are more probable, and their dampening effect on the *L*-value is more pronounced.The *R* goes high after one or more CSMB and then goes down in one or more following longer blocks (Having two or more *E*). Then, it goes up and down again in the commonly found zigzagging pattern. According to the statistics, the Single Middle, even number blocks (SMB) occur half the time of all block lengths. The sum of the dampening effect of the longer blocks on the *L*-value outweighs the shooting effect of the SMB, and a final convergence to number one occurs. It is only a matter of more blocks. The number of the blocks reaches infinity only if the starting *L* of a CSMB cluster is also infinite or if these SMB and CSMB reappear infinitely. Statistics prohibit this occurrence.

Box C3 includes tables and formulas of one type of Consecutive DoubleMiddle-even number Blocks (CDMB) cluster.

The effect of this type of block is:

$$f = R/L = (3L/2^E + 1/2^E)/L \quad (20)$$

$$f = (3/2^2) + 1/2^2)/L \quad (21)$$

$$i = 3/4 \quad (22)$$

The effect and the impact of the CDMB is a moderate decrease in the *L*-value. As for the CSMB, the number of the CDMB increases to infinity as the starting L increases to infinity. Reference is made to Box EX7 to see the effect of a CDMB cluster having 20*M*and the complete convergence to number one, which shows the occurrence of SMB, CSMB, and other block lengths along the convergence path.

There are many other cluster patterns. Formulating the governing formula is similar in principle to those mentioned in the C1, C2, C3, and C4 examples.

BoxEX5 shows examples of groups of CDMB clusters. Box EX6 includesa 20M CDMB cluster workout andthe final convergence to number one.

The root blocks are the blocks having $R_0 = 1$. Their block formula is:

[*L*/*E*/1] on the convergence mode or

[> 1/*E*/*L*] in the generation mode

There are three possibilities:

1) Blocks that have *L*3 following the form:

[> 1/*E*/*L*3] where *E* = 6, 12, 18, … and *L*3 = 21, 1365, 87381, …

2) Blocks that have *L*2 following the form:

[> 1/*E*/*L*2] where *E* = 2, 8, 14, …*L*2 = 1, 85, 5461, …

3) Blocks that have $L1$ following the form:

[> 1/E/$L1$] where E = 4, 10, 16, … and $L1$ = 5, 341, 21845, …

Each $L1$ and $L2$ shall be the R for the next block in the generation process, and the process continues. This process creates branches along the positive integers' series but includes a selected set of numbers. Branches diverge and continue infinitely.

The *L* brand changes sequentially as the *E* steps up (Increases by two). An Example of the alternation from $L2$ to $L1$ to $L3$ for the same *K* value is shown in Table 3below:

Table 3: Example of *L*Brand Alternation as E Increases

>R	***E***	***L* Value**	***L* type**
5	1	**3**	***L3***
5	3	**13**	***L2***
5	5	**53**	***L1***
5	7	**213**	***L3***
5	9	**853**	***L2***
5	11	**3413**	***L1***

L changes from $L3$ to $L2$ to $L1$, and the sequence repeats every addition of six to *E*.

The generation (Reverse-conjecture) formulasare:

$$L_E = (2^E/3)R - (1/3) \qquad (23)$$

$$L_{E+2} = (2^{E+2}/3)R - (1/3) \qquad (24)$$

Which leads to the formula:

$$L_{E+2} = 4L_E + 1 \qquad (25)$$

Moving through the same *L* brand and jumping *E* (E increases by six at a time) follows the formula:

$$L_{E+6} = 64L_E + 21 \qquad (26)$$

The block [> 1/2/1] is the only root and base block.

Since its *R* is $R2$, then its E = 2

All other root blocks are extensions of this root block. All other base blocks occur along thegenerated chains. Block extension can continue infinitely.

The statistics in Boxes F5 and F6 show the frequency of different follower block lengths for R1 and R2. The occurrence frequency of each block length is well defined by its length (Group or number of the middle-even numbers). These frequencies addto the unity of the odd positive integer'sseries. This adds weight to what was calculated in Box B4 for single blocks of different lengths.

Table 4 below shows the sequence of block length and consequent *R*-value if we move along the positive odd number as *L* in the convergence mode. There is a very determined pattern, which is evident from the rearrangement shown in Box B1.

Table 4: *R-L* values

L	*E*	*R*
1	2	**1**
3	1	**5**
5	4	**1**
7	1	**11**
9	2	**7**
11	1	**17**
13	3	**5**
15	1	**23**
17	2	**13**
19	1	**29**
21	6	**1**
23	1	**35**
25	2	**19**

The complete convergence of number 7 is shown in Bx EX2arranged in a one-lead-one-follower table using the parametric formulas. CSMB and CDMB clusters in this chain can be checked using the formulas stated in Boxes C1, C2, and C3. For example,a 5M CSMB cluster occurs starting at *L*2 = 319.

Checking formula Q for the calculation of $L2_0$ gives:

$$L2_0 = 2^{M+2} + 2^{M+3} - 1 = 319 \quad (27)$$

In the convergence mode, moving from the starting L_0, branches join at specific numbers (Nodes) to form "Thicker branches." This process continues until all branches meet at the base blocks, which are extensions of the root block [1/2/1], e.g. [5/4/1], [85/8/1]. Numbers of the series 21, 1365, 87381, … (All of them belong to *L*3) converge to 1 in one block only with an increasing number of the middle even numbers by 6.

Boxes EX7 and EX8 include parts (Stretches of converging chain with no apparent pattern of their block lengths. Proposed formulas are included for the reoccurrence of such a sequence of the block lengths in similar stretches. In addition to the examples studied here, many other cluster patterns and types occur. Expanding the study of the block followers, clusters, and stretches may make it possible to generalize their governing formulas with systematic derivations.

Summary

Blocksof positive integer series numbersare formed according to the three plus one conjecture rules. Blocks are bound by odd numbers (left hooks, *L* and right hooks, *R*). They include even numbers between these boundaries and are divided into families based on the *R*1 and *R*2 family (*R*1 = 5, 11, 17, … and *R*2 = 1, 7, 11, …) and groups based on their group length (The number of the middle-even numbers).Blocks are taken individually and placed beside each other according to the governing rules.They reach number one in the convergence mode or continue generating odd positive numbers infinitelyaccording to the generation mode. The final convergence to number one is reached because these blocks (Domino tiles) have all the positive integers included as left hooks, and all the *F*2 and *F*2 odd positive numbers are included as right hooks (*R*1 and *R*2). Block rules mandate that a single left hook produces only one right hook. This concludes that no looping would occur. The same R does not reoccur in the same branch or other branches or chains because of the nature of the formed series. Statistics show that R cannot increase infinitely in the convergence procedure. Oscillation around the same R-value would only occur if a single R is produced from different *L* values, which does not happen. Infinite repetitive up and down *R*-values without reaching number one would also violate the statistics as this means the absence of specific block lengths. Statistics show that blocks of various lengths (Different numbers of middle-even numbers) have a strict occurrence and repetition sequence along the positive integer series. Block lengths can extend infinitely, and each block length repeats its occurrence infinitely. Blocks connect in chains, leading to the final convergence to number one. In the generation mode, separate odd number series as divisions of the odd positive number series are generated in branches, which continue infinitely. The sum of these series adds up to the whole positive integer series. Any positive integer can be generated according to the rules starting from number one. The procedure can be reversed along the convergence mode to reach the final convergence at number 1. The positive integer series, the positive odd integer series, and the positive even integer series disappear and are replaced with an infinite number of Series of blocks as defined by the conjecture rules and their own rules.

Conclusions

1- Studying the odd positive number series in the generation mode is sufficient to study and prove or disprove the three-plus-one conjecture for the whole series of natural numbers.
2- The block format helped generalize formulas and rules governing the convergence and the generation chains.
3- Clusters of repeated 1*E* block length occur in converging chains, causing the *R*-value to increase continuously. However, the rank of these clusters (Number of consecutive single middle even number blocks) is determined and limited by specific rules. They can not continue infinitely. Repeated clusters may appear but have a countable number of members.
4- The alternation between *L*1, *L*2, and *L*3 and *R*1 and *R*2 and the different block lengths cause the joining of blocks having various lengths and the appearance of the up and down movement of the *R* and *L* value along the convergence or the generation mode.
5- Members of the *F*3 family can start converging chains or occur in a generation chain. They can not happen in the middle of a converging chain.
6- Reaching any odd number in the positive odd number series as a left hook (*L*) starting from 1 as the right hook (*R*) in the generation mode is possible. It is alsopossible to converge to 1 starting from any odd or even integer by following the conjecture rules(A mirror image of the generation mode).
7- Statistical study leads to predetermining specific block length occurrence and repetition along the positive odd integer series.
8- Statistics show that the sum of the ratios of all block lengthsreaches one at infinity.
9- Blocks with a right hook belonging to *F*1 (*R*1) always have an odd number of middle-even numbers.
10- Blocks with a right hook belonging to *F*2 (*R*2) always have an even number of middle-even numbers
11- Following the generation rules creates an infinite number of infinite series. Members of these chains are unique and do not repeat in the same or other chains. Looping does not occur within a chain. Entanglement does not happen between chains.
12- The sum of all chains resulting from block connections adds up to the natural number series.

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