ASSESSMENT OF THE VALIDITY OF ASSUMPTION OF ZERO NET MIGRATION IN ESTIMATION OF ADULT MORTALITY USING PRESTON INTEGRATED APPROACH

Abstract

This study examines the implication(s) of ignoring net migration in estimation of adult mortalityfrom non-stable population using the Preston (1983) integrated method and proposes a method of assessing presence of net migration in a study data. In deriving the model for estimation of adult mortality from non-stable population, Preston (1983) has assumed that the study population is closed to migration, that is, the net migration is zero or negligible. However, in most developing countries this assumption is not necessarily true. In this study, the method proposed for assessing the need for adjustment for net migration is the ratio (AF) of the observedproportion of mid-period population reported as aged x years$\left(c(x,t)\right)$when net migration is not zeroto the corresponding proportion when net migration is zero$\left(\hat{c}(x,t)\right)$.The Preston (1983) approach to estimation of adult mortality is to relate characteristics of the observed population to some life table functions and using this relationship toobtain estimate of theimplied level of adult mortality. The results indicate that when net migration is actually zero, the ratio AF is equal to one and different from one when net migration is not zero. Data on age-sex distribution of populations of three selected developing countries were used to illustrate the methods. It has therefore, been recommended that when this ratio is not one an adjustment should be made for net migration.

Key words: Adult Mortality,Non-stable population, Life Table Functions, Adjustment factorand net migration

1 Introduction

In developing countries the problem of poor data quality has greatly hampered the modeling and control of adult mortality(Hill, 2001). As a result of poor vital registration records, data on adult mortality in most developing countries come from censuses and sample surveys (Ramachandran, 1989; Hill, 2001). Data from these two sources have been shown to be defective (Ekanem, 1972; Hill, 2001; Nwogu, 2006, 2011; Ohaegbulem, 2015; Bello, 2017; Nwogu and Okoro, 2017; Okoro, 2019; Okoro and Nwogu, 2019, 2020).

As a consequence demographic parameters, including adult mortality in developing countries are derived through indirect techniques. Among the indirect techniques for estimation of adult mortality methods based on data from two censuses age distributions appear very popular. These include those of Stolnitz (1956), Coale and Hoover (1958), Coale and Demeny (1967) projection and cumulation method (UN, 1967), Carrier and Hobcraft (1971), Preston (1983) integrated method, Preston-Bennett (1983) method and United Nations (2002) Synthetic Survival Ratio. While some of these indirect techniques based on two censuses assume that inter-censal period is five years or multiples of five years, Preston (1983) integratedmethod; Preston-Bennett (1983) method and UN (2002) Synthetic Survival Ratio method do not make such assumptions. These methods are of interest in this study because most developing countries belong to this category, whose inter-censal period is not five years or multiples of five years. Unlike the Preston (1983) method, the Preston-Bennett (1983) method and UN (2002) Synthetic Survival Ratio do not require a standard life table for their implementation. However, the synthetic survival ratio method, which is similar to the Preston-Bennett method, does not produce good estimates of life expectancy especially when the age-specific growth rates change substantially from one five-year age group to another.Preston-Bennett (1983) method, on the other hand is based on spurious interpolation at the last open age interval. The Preston (1983) assumes that there is a standard life table for its implementation and performs creditably well in estimation of adult mortality when appropriate standard life table for a study population is available.

In deriving the model for estimation of adult mortality from non-stable population, Preston (1983) has assumed that the study population is closed to migration. That is, the net migration is zero or negligible. However, in most developing countries this assumption is not necessarily true.When this assumption fails, these models cannot perform well. Preston (1983) made no provision for assessment of the validity of the assumption before applying them to any study data. Thus, there is need for a model that will help toassess the validity of this assumption before their use for estimation of adult mortality.

Therefore, the aim of this study is to provide a model for assessing the validity of the assumption of zero net migration in a study data before applying the Preston (1983) method for estimating adult mortality. The objectives are:

(i) to derive a non-stable population model that relates characteristics of the observed population to some life table functions;

(ii) to express the life table functions in terms of the observed population characteristics;

(iii) to provide an index that may be used to assess the validity of the assumption of zero net migration;

(iv) to assess the performance of the new models using empirical examples.

2 Methodology

2.1 Review of Preston integrated method for estimation of adult mortality

Preston (1983) integrated method for estimating adult mortality is based on two age distributions from censuses with arbitrary inter-censal period.According to Preston and Coale (1982), at a point in time t, the proportion of a population aged x years ($c\ (x,t)$) can be expressed in terms of birth rate ($b(t)$), rate of population growth ($r'(a,t)$), rate of net migration ($\mathrm{v(a,t)}$) and the survival probability from birth ($p(x,t)$) as

$$c\ (x,t) = b(t)\exp\left[-\left(\int_0^x \mathrm{r(a,t)} - \mathrm{v(a,t)}da\right)\right]p(x,t) \qquad (2.1)$$

Where

$r'(a,t) = \mathrm{r(a,t)} - \mathrm{v(a,t)}$ is the rate of natural increase (the rate of population growth adjusted for rate of net migration).

Preston (1983) showed that for a closed population, $\mathrm{v(a,t)} = 0$, that is net migration is zero, then $r'(a,t) = \mathrm{r(a,t)}$ and hence,

$$c(x,t) = b(t)\exp\left(-\int_0^x (r(a,t)da\right) p(x,t) \quad (2.2)$$

$$\frac{1}{p(x,t)} = \frac{b(t)}{c(x,t)}\exp\left(-\int_0^x (r(a,t)da\right) \quad (2.3)$$

Since every term in (2.1) through (2.3) refers to the same time t, we drop t from the subsequent Equations. Furthermore, since $p(x,t) = l_5 \, {}_5p(x,t)$ where, l_5 is the probability of survival to age 5 from birth and ${}_5\,p(x,t)$ is the probability of survival to age x from age 5 years. Thus,

$$\frac{1}{l_5 \, {}_5p(x)} = \frac{b}{c(x)}\exp\left(-\int_0^x (r(a)da\right)$$

Or

$$\frac{1}{{}_5\,p(x)} = \frac{b\,l_5}{c(x)}\exp\left(-\int_0^x (r(a)da\right) \quad (2.4)$$

By choosing survival probabilities from age 5 to x years (${}_5P^s(x,t)$) from the appropriate standard life table and using the Brass (1971) logit transformation, Preston (1983) showed that

$$Ln\left(\frac{1 - {}_5P(x)}{{}_5P(x)}\right) = \alpha + \beta\, Ln\left(\frac{1 - {}_5P^s(x)}{{}_5P^s(x)}\right) \quad (2.5)$$

Hence,

$$\frac{1 - {}_5P(x)}{{}_5P(x)} = e^{\alpha}\left(\frac{1 - {}_5P^s(x)}{{}_5P^s(x)}\right)^{\beta} \quad (2.6)$$

$$\frac{1}{{}_5P(x)} \cong 1+k\left[\frac{{}_5q^s_{(x)}}{{}_5P^s_{(x)}}\right] \qquad \text{if } \beta=1 \qquad (2.7)$$

Where ${}_5P^s(x)=1-{}_5q^s(x)$ is survivorship functions from a chosen standard life table and

$k=e^{\alpha}$ is an index of level of mortality in the study population relative to the standard life table.

Equating $\frac{1}{{}_5P(x)}$ in Equations (2.4) and (2.7), we have

$$\frac{1}{{}_5P(x)} = \frac{bl_5}{c(x)}\exp\left(-\int_0^x r(a)\,da\right) = 1+k\left[\frac{{}_5q^s_{(x)}}{{}_5P^s_{(x)}}\right] \quad (2.8)$$

$$\frac{l_5}{c(x)}\exp\left(-\int_0^x r(a)\,da\right) = \frac{1}{b} + \frac{k}{b}\left[\frac{{}_5q^s_{(x)}}{{}_5P^s_{(x)}}\right] (2.9)$$

Equation (2.9) is equivalent to a simple linear regression,

$$y = A + B\mathbf{w} \quad (2.10)$$

Where

$$y = \frac{l_5}{c(x)}\exp\left(-\int_0^x r(a)\,da\right) (2.11)$$

$$w = \frac{{}_5q^s_{(x)}}{{}_5P^s_{(x)}}$$

$$= \frac{1-{}_5P^s(x)}{{}_5P^s_{(x)}} (2.12)$$

with intercept $A=\frac{1}{b}$ and slope $B=\frac{k}{b}$. Hence, the fitted lineis

$$\hat{y} = \hat{A} + \hat{B}\mathbf{w} (2.13)$$

with

$$\hat{y} = \frac{l_5}{\hat{c}(x)}\exp\left(-\int_0^x r(a)\,da\right) \quad (2.14)$$

Hence, Preston (1983) obtained his estimate of adult mortality in a closed population as

$$\frac{1}{{}_5\hat{P}(x)} = \hat{b}\left[\frac{l_5}{c(x)}\exp\left(-\int_0^x r(a)\,da\right)\right] = \hat{b}\,\hat{y} \quad (2.15)$$

$$= \frac{\hat{y}}{\hat{A}} \quad (2.16)$$

2.2: Preston Integrated method for estimation of adult mortalitywhen net migration is not zero.

Recall from (2.1)

$$c(x) = b(t)\exp - \left(\int_0^x (r(a,t)da - \int_0^x \nu(a,t))da\right) \; p(x,t)$$

Preston (1983) set $\nu(a) = 0$ when net migration is zero and (2.1) was reduced to (2.2)

$$c(x) = b\exp\left(-\int_0^x (r(a)da\right) \; p(x)$$

From the regression Equation (2.14) ,

$$\hat{y} = \frac{l_5}{\hat{c}(x)}\exp\left(-\int_0^x r(a)\,da\right)$$

Hence,

$$\hat{c}(x) = \frac{l_5}{\hat{y}}\exp\left(-\int_0^x r(a)\,da\right) \quad (2.17)$$

$$b\,\hat{c}(x)\,p(x) = \frac{b\,l_5}{\hat{y}}\exp\left(-\int_0^x r(a)\,da\right)p(x)$$

If truly, net migration is zero the $\hat{c}(x)$ in (2.17) will be equal or approximately equal to $c(x)$ in (2.1). Otherwise,

$$c(x) = b\exp - \left(\int_0^x (r(a)da\right) \; p(x) \; \left\{\exp - \left(\int_0^x \nu(a))da\right)\right\}$$

or

$$c(x) = \hat{c}(x)\,\exp\,\left(\int_0^x \nu(a))da\right)$$

Hence,

$$\exp\left(\int_0^x \nu(a))da\right) = \frac{c(x)}{\hat{c}(x)} \quad (2.18)$$

Or

$$\int_0^x \nu(a))da = \ln\left(\frac{c(x)}{\hat{c}(x,)}\right) \qquad (2.19)$$

Thus, either (2.18) or (2.19) may be used to assess the validity of the assumption of zero net migration in a study population.

As noted earlier, in the absence of net migration, the factor in (2.18) in Preston (1983) method for estimation of adult mortality will be one (unity) if $\int_0^x \nu(a))da = 0$ and /or $\nu(a) = 0, \forall\ a$, or the deviation from one is zero. Thus, the value of the ratio or its deviation from one may be used as a measure of evaluation of the claim that a population is closed to migration. The closer the values of this ratio are to one or their deviations from one are to zero the more the assumption is supported.

Hence, our procedure is, using the populations aged x to x+5 at the first and second censuses.

${}_5N_x^{(1)}$ and ${}_5N_x^{(2)}$ we calculate for the age group [x, x+n)

(i) the population reported at midpoint of inter-censal period,

$${}_5\bar{N}_x = \frac{1}{2}\left({}_5\bar{N}_x^{(2)} + {}_5\bar{N}_x^{(1)}\right) \qquad (2.21)$$

(ii) the age specific population growth rate for the age group,

$${}_5r_x = \frac{1}{t}\ln\left[\frac{{}_5N_x^{(2)}}{{}_5N_x^{(1)}}\right] \quad (2.23)$$

(iii) the populations aged x

$$N(X) = \frac{1}{10}\left({}_5\bar{N}_x^{(2)} + {}_5\bar{N}_{x+n}^{(1)}\right)(2.22)$$

(iv) the proportion of the population aged x,

$$c(x) = \frac{N(x,t)}{\sum_{x-0}^{\omega+} {}_5\overline{N}_x} \quad (2.20)$$

Substituting these in Equation (2.14), we obtain, for ages$x$ = 10, 15, 20, ………,

(a) $$\hat{y} = \frac{l_5}{\hat{c}(x)} \exp\left(-\int_0^x r(a)\,da\right) \approx \frac{l_5}{\hat{c}(x)} \exp\left(-5\sum_{i=0}^{x-5} {}_5 r_i\right)$$

(b) $$\hat{c}(x) = \frac{l_5}{\hat{y}} \exp\left(-5\sum_{i=0}^{x-5} {}_5 r_i\right)$$ and

(c) $\exp\left(\int_0^x v(a))da\right) = \frac{c(x)}{\hat{c}(x)}$ in (2.18)by dividing $c(x)$ obtained in (2.20) by $\hat{c}(x)$ obtained in (b) above

l_5 is the estimate of probability of survival from birth to age 5 derived using Brass (1964) method from the information on number of children dead among children ever-born

b is the crude birth rate estimated from $\hat{b} = \frac{1}{\hat{A}}$

3 Result and Discussion

Empirical examples

In this section, we present some empirical examples to illustrate the applicability of the proposed model. The empirical examples are drawn from three developing countries which include Nigeria, Kenya and Zambia from Africa. The data, consisting of the distribution of populations by sex and 5 - year age groups from the selected countries.The values of this ratio (or the adjustment factors) for the study populations computed following the procedure outlined above or their deviations from one in the selected study populations are given in Table 1 while the corresponding graphs are shown in Figures 1 through 3 for Nigeria, Kenya and Zambia respectively.As Table 1 and Figure 1 show, within the age interval 5 to 65 years, the ratios ranged from 0.77 at age 5 to 1.06 at age 30 for Nigeria. Similar observations can be made for Kenya and Zambia. For Kenya, Table 1 and Figure 2 show that AF ranged from 0.90 at age 5 to 1.42 at age 65 years. The values of the adjustment factor AF are for Zambia, lowest (0.85) at ages 5 and 60 and highest (1.07) at age 25. However, the deviations from one appear much smaller in Kenya and Zambia than in Nigeria. The values of the AF are far from one and their deviations from one are clearly not zero.These indicate that the populations may not be closed to migration as assumed by Preston (1983). Thus, for a more accurate estimate of adult mortality the effect of net migration need to be considered. From the forgoing, it is clear that there is need for modification once the ratios are not one (unity) for all x in any of the countries

Table 1: Estimate of Adjustment Factor $AF = \frac{c(x)}{\hat{c}(x)}$ in Preston Integrated Method whenNet Migration is not zero

Age	AF			Abs (Deviation of AF from 1)		
	Nigeria	Kenya	Zambia	Nigeria	Kenya	Zambia
0	-	-	-	-	-	-
5	0.770	0.903	0.849	0.23	0.10	0.15
10	0.772	1.006	0.930	0.23	0.01	0.07
15	0.768	1.028	0.997	0.23	0.03	0.00
20	0.881	0.993	1.018	0.12	0.01	0.02
25	1.018	1.006	1.066	0.02	0.01	0.07
30	1.055	1.071	1.058	0.05	0.07	0.06
35	0.995	1.010	1.017	0.01	0.01	0.02
40	0.958	0.991	0.933	0.04	0.01	0.07
45	0.962	0.990	0.906	0.04	0.01	0.09
50	0.934	0.985	0.908	0.07	0.02	0.09
55	0.916	0.940	0.856	0.08	0.06	0.14
60	0.867	1.079	0.847	0.13	0.08	0.15
65	1.050	1.415	0.955	0.05	0.42	0.05
70	1.084	1.336	1.059	0.08	0.34	0.06

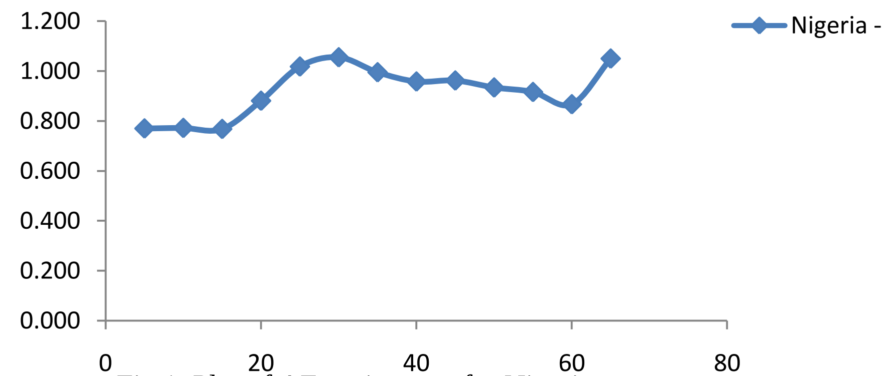

Fig 1: Plot of AF against age for Nigeria -

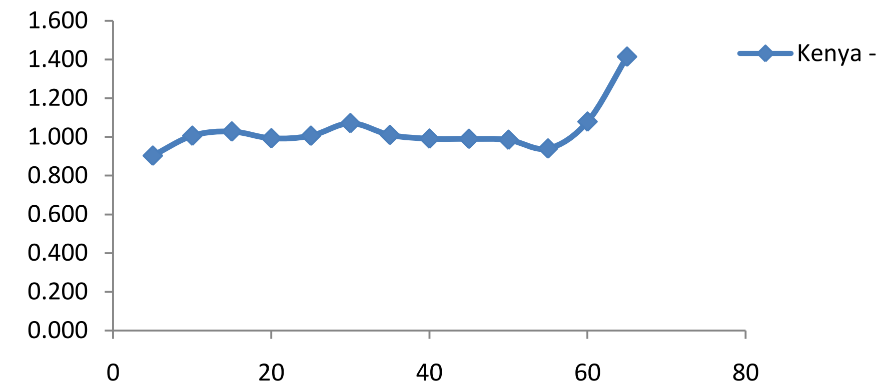

Fig 2: Plot of AF against age for Kenya -

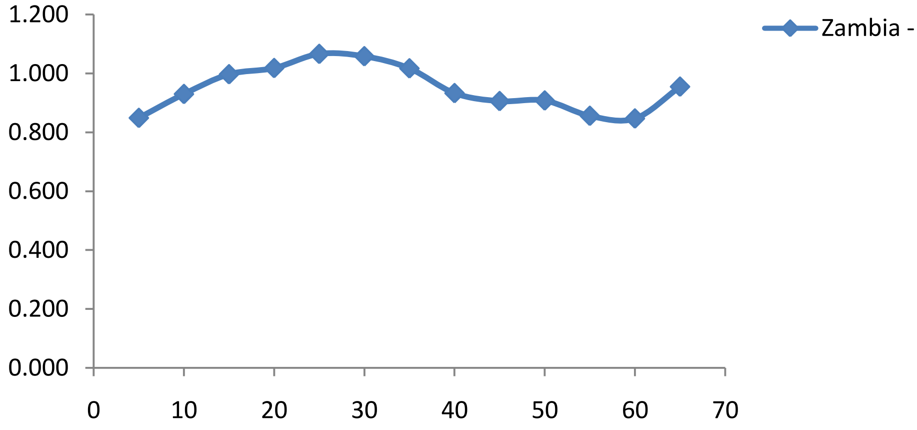

Fig.3: Plot of the AF against age for Zambia -

4 Summary, Recommendation and conclusion

The purpose of this study is to provide a model for assessing the validity of the assumption of zero net migration in a study data before applying the Preston (1983) method for estimating adult mortality. In deriving the model for estimation of adult mortality from non-stable population, Preston (1983) has assumed that the study population is closed to migration, that is, the net migration is zero or negligible. However, in most developing countries this assumption is not necessarily true.

The Preston (1983) approach to estimation of adult mortality is to relate characteristics of the observed population to some life table functions and using this relationshipto obtainestimate of the implied level of adult mortality. In this study, this procedure was adopted to obtain the proportion of mid-period population reported as aged x years when net migration is zero $\left(\hat{c}(x,t)\right)$. The method proposed for assessing the need for adjustment for net migration is the ratio (AF) of the observed proportion of mid-period population reported as aged x years $\left(c(x,t)\right)$ when net migration is not zero to the corresponding proportion when net migration is zero $\left(\hat{c}(x,t)\right)$. The results indicate that when net migration is actually zero, the ratio AF is equal to one and different from one when net migration is not zero.

Data on age- sex distribution of populations of three selected developing countries (Nigeria, Kenya and Zambia) were used to illustrate the methods. The results show that within the age interval 5 to 65 years, the ratios ranged from 0.77 at age 5 to 1.06 at age 30 for Nigeria, ranged from 0.90 at age 5 to 1.42 at age 65 years for Kenya and are for Zambia, lowest (0.85) at ages 5 and 60 and highest (1.07) at age 25. It has therefore, been recommended that when this ratio is not one an adjustment should be made for net migration.

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