Method Article

Analysis of City Transportation Routes With Fleury Algorithm On Bus Route Network(Case Study of Trans *Mamminasata* Makassar City Indonesia)

ABSTRACT

Graph theory is a branch of mathematics that studies the structure and describes the relationships between vertices and edges. In general, graph theory is used to represent discrete objects (vertices) and the relationships between them (edges). A path that can pass through each edge exactly once in a graph is called a directed graph Euler path. One way to find Euler paths is by using Fleury's algorithm. Fleury's algorithm is designed to find Euler paths in directed graphs. This article examines the application of Fleury's algorithm to the determination of a transportation route in a city interpreted in a directed graph. The case study in this research focuses on the trans *Mamminasata*bus route in Makassar city Indonesia with the aim of implementing Fleury's algorithm in bus route generation. The result obtained from the simulation using Fleury's algorithm is that all edges can be visited exactly once, so that an Euler path is formed on the transportation route. The route formed from the Euler trajectory will be a comparison of the current route to determine the operational efficiency of the bus route network in the city.

Keywords: Directed graph; Eulerian path; Fleury algorithm; transportation route

1. INTRODUCTION

Urban transportation plays a crucial role and is a major component of daily life. One of the most commonly used modes of urban transportation is the inner-city bus, which is often the backbone of public transportation systems in many cities around the world. The efficiency and sustainability of urban transportation systems depend heavily on the operation of the inner-city bus route network.

Aspects that can affect the efficiency of bus routes involve optimal route and schedule planning. For example, the use of the Vehicle Routing Problem method to optimize trans-Pontianak Equator bus routes [1]. Integrated mass transportation plays a strategic role in the service and socio-economic development of the community. This shows that integrated bus transportation in the city is a vital transportation need. Therefore, determining the optimal bus route not only has an impact on operational efficiency but also affects the level of consumer demand for the service.

Research on the relationship between bus transportation routes and Euler trajectories provides an innovative and mathematical perspective for designing more efficient and sustainable transportation systems. The Euler trajectory concept presents a mathematical approach to determining the trajectory that involves each road section exactly once. Applying this concept to bus route networks can help optimize the use of each route, reduce waste, and improve operational efficiency. However, in the search for Euler trajectories, sometimes problems arise in the selection of edges, such as when the edge that has been bypassed turns out to be bypassed again because there is no marker for the edge that has been bypassed.

Fleury's algorithm, with its ability to find Euler paths on directed or undirected graphs, becomes an effective instrument in route planning. The distinctive feature of Fleury's algorithm is to mark the route that has been traveled. Research on Fleury's algorithm covers not only the theoretical aspects but also its applications in real-world contexts. For example, the use of Fleury's algorithm to find the nearest route for postmen in China [2]. In Indonesia, similar research was also conducted related to finding the shortest route for postmen using the Fleury algorithm [3]. This algorithm was also used in Vietnam to find the shortest path in garbage collection using garbage collection vehicles [4]. The results of these studies recommend using a graph theory approach to obtain optimal travel costs and times. Fleury's algorithm has been used in graph theory to find Euler trajectories in various contexts, including urban transportation route networks such as bus transportation. The application of Fleury's algorithm in inner-city bus transportation has the potential to optimize bus routes, reduce waiting time, reduce fuel consumption, and improve public transportation services.

Based on this description, this research aims to explore the potential use of Fleury's algorithm in improving the efficiency of urban transportation through optimal bus route planning. It is hoped that the results of this research can make a significant contribution to the improvement of urban transportation systems as well as help create a more sustainable environment and better mobility for city residents.

2. THEORETICAL FOUNDATION

The concept of using Fleury's algorithm on transportation networks is applied by using Euler graph theory and then marking the edges that have been passed on the graph so that it becomes an illustration of a transportation route. Locations that are grouped into a destination region undergo a process of selecting one main node from the region. Suppose the vertex is $v_i$with $i = \{1,2,3, \dots, n\}$, then $V = \{v_1, v_2, v_3, \dots, v_n\}$ is the set of destination location vertices. The vertices that have been selected must have an edge that connects them. Suppose the edge connecting vertex $v_i$to $v_j$is called edge e_k with $k = \{1,2,3, \dots m\}$, then $E = \{e_1, e_2, e_3, \dots, e_m\}$ is the set of all edges formed from vertex V. Each vertex $v_i$has a variety of edges, ranging from first-degree vertices, second-degree vertices, or even more than third-degree vertices. This will complicate the process of determining the optimal route because so many edges can be chosen.[5]

One way that can be used to optimize a route is to create a graph on the transportation network system that can generate Euler trajectories because it can pass through each edge E exactly once. In order to facilitate the process of creating Euler trajectories, a theorem is needed to determine the number of degrees at each vertex so that the graph formed can produce Euler trajectories. The following is the theorem used to help in the process of determining the number of degrees at vertex$V$.

Theorem 1[6]
A nontrivial connected graph G is Euler if and only if every vertex of G has an even degree.

An Euler graph is a graph that can pass through an edge exactly once; hence, it is definitely an Euler path.

Theorem 2[6]
A connected graph G contains an Euler path if and only if exactly two vertices of G have an odd degree. Furthermore, every Euler path of G starts at one odd vertex and ends at another odd vertex.

From Theorem 1 and Theorem 2, it is shown that Euler trajectories can be formed on graphs whose vertices have an even degree or have at most two vertices of an odd degree. The following is an example of a graph with every vertex of even degree and a graph with two vertices of odd degree in Figure 1.

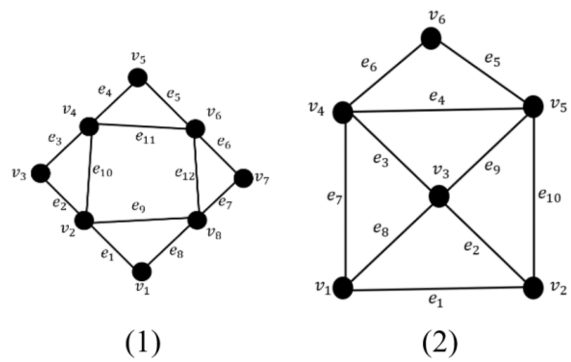


Fig. 1. (1) Graph with every vertex of even degree (2) Graph with 2 vertices of odd degree

In the Trans *Mamminasata* case study in Makassar city, Indonesia, the process of determining the number of buses used in each corridor is carried out by calculating the length of time taken by the bus for 1 round, or 2 times the total distance from the starting point to the end point in each corridor, then divided by the departure time of each bus.

If B is the number of buses, total distance is the sum of e_k, average speed is $\bar{x}$, gap time is w_t, and p is the number of spare buses, then the formula for determining the number of Bn becomes:

$$B = \frac{2 \times \sum_1^m e_k}{\bar{x}.w} + p \qquad (1)$$

Equation (1) obtained is then used to calculate the number of buses on the Trans *Mamminasata* route created using the Fleury algorithm.

2. RESEARCH METHOD

The following are the stages of using fleury's algorithm on the transportation network.

1. Define the graph in the transportation route
   a. Vertex
      Selecting destination location nodes as the first step in designing a transportation route network and grouping them in one area that has the same characteristics or transportation needs. Then take one node to become the main node and annotate the main node that has been selected as the destination location.
   b. Edge
      An edge is a path that connects two vertices or destination locations on a route.

c. Degree
All roads and edges connected to a vertex must have at most two odd-degree vertices.

2. Clarify the existence of euler trajectories on graphs.
   a. An Euler trajectory can be formed on a graph if it satisfies the condition that each vertex is of even degree (Theorem 1) or has at most two vertices of odd degree (Theorem 2).
   b. If the graph in the route satisfies condition (a), then the graph definitely generates an Euler trajectory.

3. Generating euler trajectories with fleury's algorithm on graphs
The following are the steps to determine the Euler path on graph R using Fleury's algorithm. [3]
Step 1 : Take one node to start.
Step 2 : From the vertex, take one edge to pass through.
Step 3 : Marking the edge is a reminder that the edge cannot be traversed anymore. The marking done is to delete the edge that has been bypassed.
Step 4 : Through the marked edge, the next vertex will choose an edge to skip again and repeat the second step until all edges are skipped.

4. Comparing the route generated using fleury's algorithm with the existing route

3. RESULTS AND DISCUSSION

3.1 Trans *Mamminasata* route created using the Fleury algorithm

The research results obtained from the process of using Euler's graph theory with Fleury's algorithm on the Trans Mamminasata bus transportation network are the main bus stop points that have been determined based on important location points such as markets, settlements, educational places, malls, offices, commercial terminals, hotels, and sports facilities. The bus stop is denoted as a V node. The following 41 V nodes or bus stops represent a group of areas, namely $V = \{v_1, v_2, \ldots, v_{41}\}$ in table 1.

Table 1. Nodes designated as main stops

Vertex V	Main Stop Location	Vertex V	Main Stop Location
v_1	Tanjung Bayang	v_{22}	CitralandHertasning
v_2	Pantaiakarena	v_{23}	UIN Alauddin Samata
v_3	Trans Studio	v_{24}	Gedungfajar
v_4	Bundaran CPI	v_{25}	Mall Nipah
v_5	PantaiLosari	v_{26}	MakamPahlawan
v_6	Benteng Rotterdam	v_{27}	M'Tos
v_7	MTC	v_{28}	UIM
v_8	RS Palamonia	v_{29}	UNHAS
v_9	RujabKetua DPRD	v_{30}	DinKes Prov. Sul Sel
v_{10}	Mall Ratu Indah	v_{31}	CitraLandTallasa City
v_{11}	TokoBtng Veteran	v_{32}	PT.Penguin Indo
v_{12}	Toko Marmer Gran	v_{33}	PT. FKS Multi Agro
v_{13}	PasarSenggol	v_{34}	PIP Makassar
v_{14}	Kampus MIPA UNM	v_{35}	Golf Baddoka
v_{15}	Term. Malengkeri	v_{36}	Term.Daya

v_{16}	UNISMUH	v_{37}	Hotel Dalton
v_{17}	UIN I Alauddin	v_{38}	Bandara
v_{18}	Hotel Claro	v_{39}	Royal Penerbangan
v_{19}	Taman PakuiSayang	v_{40}	GorSudiang
v_{20}	Masjid Raya	v_{41}	Kampus II PNUP
v_{21}	Mall Panakukang		

From the vertex V that has been determined, the edge E is selected using Theorem 1, with each vertex having no more than two odd-degree vertices. The following 45 edges E are formed from the vertex with $E = \{e_1, e_2, \dots, v_{45}\}\}$ in table 2.

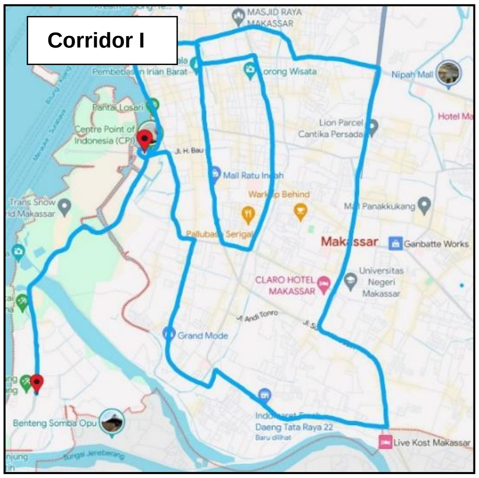
Table 2. The name of the road on each side E of the graph obtained from the trans Mamminasata route in Makassar City with two corridors divided into two corridors

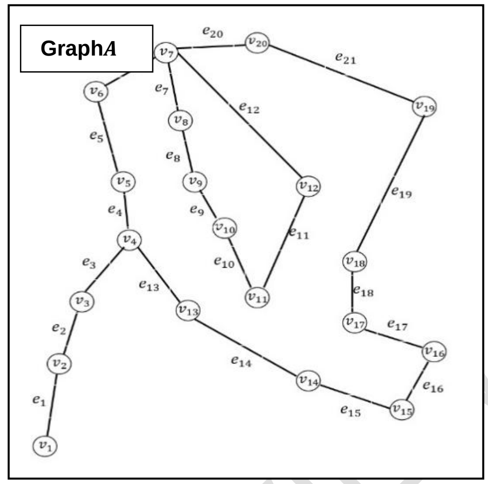
Corridor I		Corridor II	
Edge	**street name**	**Edge**	**street name**
$e_1\ (v_1, v_2)$	JlTanjungBunga	$e_{24}(v_{22}, v_{23})$	Jl. H.M. Yasin Limpo
$e_2\ (v_2, v_3)$	Jl. Metro TjBunga 1	$e_{23}(v_{21}, v_{22})$	Jl. LetjenHertasning
$e_3\ (v_3, v_4)$	Jl. Metro TjBunga 2	$e_{22}(v_{19}, v_{21})$	Jl. Boulevard
$e_4\ (v_4, v_5)$	Jl.Hl. SombaOpu	$e_{25}(v_{19}, v_{22})$	Jl. AP. Pettarani 3
$e_5\ (v_5, v_6)$	Jl. Penghibur	$e_{26}(v_{24}, v_{25})$	Jl. UripSumoharjo 1
$e_6\ (v_6, v_7)$	Jl. Ahmad Yani	$e_{27}(v_{25}, v_{26})$	Jl. UripSumoharjo 2
$e_7\ (v_7, v_8)$	Jl. JendralSudirman 1	$e_{28}(v_{26}, v_{27})$	Jl. Perintis Kemerdekaan1
$e_8\ (v_8, v_9)$	Jl. JendralSudirman 2	$e_{29}(v_{27}, v_{28})$	Jl. Perintis Kemerdekaan2
$e_9(v_9, v_{10})$	Jl. DR Ratulangi 1	$e_{30}(v_{28}, v_{29})$	Jl. Perintis Kemerdekaan3
$e_{10}(v_{10}, v_{11})$	Jl. DR Ratulangi 2	$e_{31}(v_{29}, v_{30})$	Jl. Perintis Kemerdekaan4
$e_{11}(v_{11}, v_{12})$	Jl. Veteran Selatan	$e_{32}(v_{30}, v_{31})$	Jl. JalurLingkaran Barat
$e_{12}(v_{12}, v_7)$	Jl. Veteran Utara	$e_{33}(v_{31}, v_{32})$	Jl. Ir. Sutami
$e_{20}(v_7, v_{20})$	Jl. Masjid Raya	$e_{34}(v_{32}, v_{33})$	Jl. DaengtaQalia
$e_{21}(v_{20}, v_{19})$	Jl. AP. Pettarani 1	$e_{35}(v_{33}, v_{34})$	Jl. Salodong
$e_{19}(v_{18}, v_{19})$	Jl. AP. Pettarani 2	$e_{36}(v_{34}, v_{35})$	Jl. Batara Bira
$e_{18}(v_{17}, v_{18})$	Jl. Sultan Alauddin 1	$e_{37}(v_{35}, v_{36})$	Jl. Underpass SimpangMandai
$e_{17}(v_{16}, v_{17})$	Jl. Sultan Alauddin 2	$e_{39}(v_{36}, v_{37})$	Jl. Perintis Kemerdekaan5
$e_{16}(v_{15}, v_{16})$	Jl. Malengkeri Raya	$e_{40}(v_{37}, v_{38})$	Jl. Poros Bandara Baru
$e_{15}(v_{14}, v_{15})$	Jl. Abdul Kadir	$e_{42}(v_{38}, v_{39})$	Jl. DaengRammang
$e_{14}(v_{13}, v_{14})$	Jl. Rajawali	$e_{42}(v_{39}, v_{40})$	Jl. Pajjaiang
$e_{13}(v_4, v_{13})$	Jl. Metro Tanjung B	$e_{44}(v_{40}, v_{41})$	Jl. Paccerakang
		$e_{45}(v_{30}, v_{41})$	Jl. BumiTamalanreaPermai
		$e_{38}(v_{30}, v_{36})$	Jl. PerintisKemerdiakaan 6
		$e_{43}(v_{36}, v_{40})$	Jl. Pajjaiang

The following is the result of the route created using Euler graph theory with the Fleury algorithm method, which is divided into 2 corridors, namely corridor I as graph A and corridor II as graph B.

3.1.1. Corridor I

Trans *Mamminasata* bus routes for corridor I are obtained by selecting important locations such as markets, settlements, educational places, malls, offices, commercial terminals, hotels, and sports facilities so that the selection of route locations is more optimal. Then the route of corridor I is illustrated in the form of a directed A graph in Figure 2.





(1) (2)

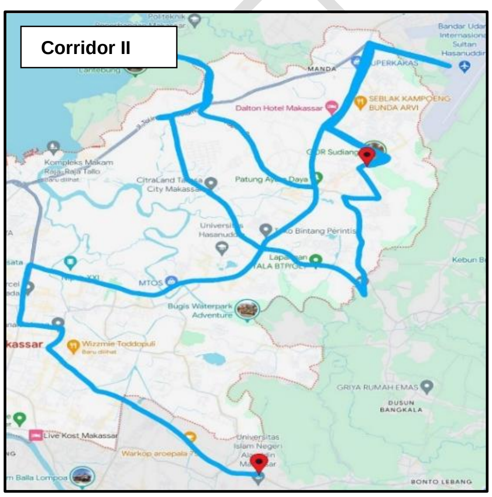
Fig. 2. (1) Corridor I route on the map; (2) Corridor I route on the directed A graph

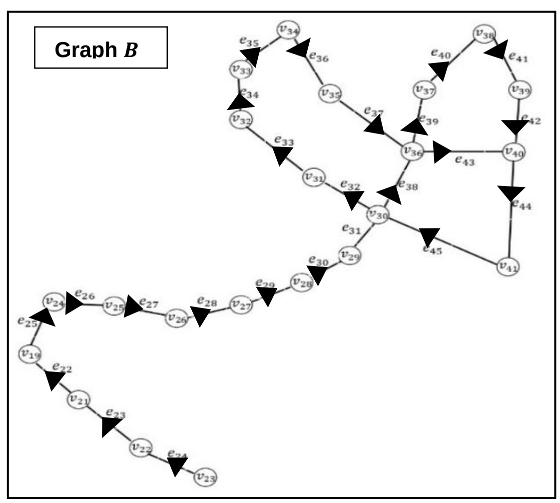
The form of Euler path L for graph A with v_1 as the start vertex and v_4 as the end vertex is as follows:.

$$L(A)= \quad (e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{12}, e_{20}, e_{21}, e_{19}, e_{18}, e_{17}, e_{16}, e_{15}, e_{14}, e_{13})$$

3.1.2. Corridor II

Trans Mamminasata bus routes for corridor II are obtained by selecting important locations such as markets, settlements, educational places, malls, offices, commercial terminals, hotels, and sports facilities so that the selection of route locations is more optimal. Then the route of corridor II is illustrated in the form of a directed A graph in Figure 3.





(1) (2)

Fig. 3. (1) Corridor II route on the map (2) Corridor II route on the directed B graph

The Euler path form L for graph B with v_{23} as the start vertex and v_{40} as the end vertex is as follows:.

$$L(B) = (e_{24}, e_{23}, e_{22}, e_{25}, e_{26}, e_{27}, e_{28}, e_{29}, e_{30}, e_{31}, e_{32}, e_{33}, e_{34}, e_{35}, e_{36}, e_{37}, e_{39}, e_{40}, e_{41}, e_{42}, e_{44}, e_{45}, e_{38}, e_{43})$$

3.2. Comparison of Trans Mamminasata routes created using Fleury's algorithm with existing ones

This research uses two different route data sets to compare Trans *Mamminasata*routes created using Fleury's algorithm with existing routes. The first route data is used to compare routes from road usage sessions because it is more effective than the second route data. However, the first route data has never been applied, so to compare routes in terms of optimizing road usage, the second route data currently used by Trans *Mamminasata* Makassar City is needed to operate. The following are the results of the comparison of Trans *Mamminasata* routes made using the Fleury algorithm with existing routes in terms of effective use of roads and optimization of bus use.

3.2.1. Effectiveness of road use

The following are roads that are passed by the route created using Fleury's algorithm but are not passed by the existing Trans *Mamminasata* route based on important location nodes such as markets, settlements, educational places, malls, offices, commercial terminals, airports, hotels, and sports facilities.

Table 3. Road data that is passed by the route using Fleury's algorithm but not passed by the existing Trans Mamminasata route.

No	Street Name	Location Passed
1	Jl. Dr. Ratulangi	Mall, Perkantoran, Hotel, danPusatTokokebutuhan ATK
2	Sebagian Jl. Sultan Alauddin	Kampusdan Hotel
3	Jl. OpuDaengRisadju	PasarTradisional , PusatTokokebutuhansehari-hari, danperkantoran
4	Jl. JalurLingkar Barat	PusatGudangterbesar di kota Makassar
5	Jl. BumiTamalanreaPermamai	Pasar, Sekolah, dan UMKM

The roads marked in red are all roads that are important to pass but are not passed by the old Trans *Mamminasata* route, so the use of buses for important roads and locations is not effective, as seen in Figure 4.

Fig.4. Roads that are not passed by the old Trans *Mamminasata* Route

The following shows that the route created using Fleury's algorithm covers more areas that are considered important and can be done for all areas to be created routes, so that the route generation technique with Euler graph theory using Fleury's algorithm is more effective in terms of road usage (Figure 5).

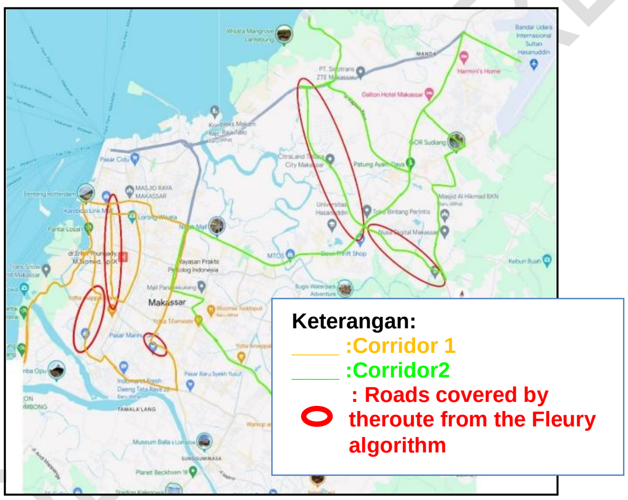


Fig. 5. Trans Mamminasata Route Created Using the Fleury Algorithm

3.2.2. Optimization of bus usage

Calculation of the number of buses B in corridor I and corridor II using equation (1) with waiting time $w = 10\ minutes$, average bus speed $\bar{x} = 28{,}75\ km/h$, and many spare buses $p = 5\ units$as follows:.

- Calculation of the number of buses in Corridor I with a route distance of $35{,}25\ km$

$$B = \frac{2 \times 35{,}25}{28{,}75 \times \left(\frac{1}{6}\right)} + 6$$

$$B = 19{,}71304$$

The results of B are then rounded up, so the number of buses used for corridor 1 is 20 units.

- Calculation of the number of buses in Corridor II with a route distance of $81\ km$

$$B = \frac{2 \times 81}{28{,}75 \times \left(\frac{1}{6}\right)} + 6$$

$$B = 38{,}8087$$

The results of B are then rounded up, so the number of buses used for corridor II is 39 units.

From the calculation results obtained using equation (1),

Table 4. Number of buses used on routes created using Fleury's algorithm

	Corridor I	**Corridor II**
Bus Quantity	20 units	39 units

The data in Table 4 is then compared with the number of buses used for the existing Trans *Mamminasata* route, obtaining a total of 88 units for 5 corridors, while the Trans *Mamminasata* route created using the Fleury algorithm is only 59 units for 2 corridors, so that the Trans *Mamminasata* route created using the Fleury algorithm is more optimal in terms of bus usage compared to the existing Trans *Mamminasata* route (Table 5).

Table 5. Data table of the number of buses on the Trans Mamminasata route created with Fleury's algorithm and the previous route.

	Fluery algorithm route	**Existing routes**
Number of corridors		5
Bus Quantity	9	88

4. CONCLUSION

Based on the results of the analysis, it is concluded that the use of Fleury's algorithm on the transportation network can be done with the condition that the graph G cannot have more than two odd degree vertices, and the Trans Mamminasata route created using Fleury's algorithm is better than the existing route in terms of efficient use of roads and optimization of bus use.

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