

Modelling Long Memory Time Series for Groundnut prices in Andhra Pradesh with Autoregressive Fractionally Integrated Moving Average for Forecasting

ABSTRACT

The presence of long memory in time series is characterized by an autocorrelation function that decreases slowly or hyperbolically. The most suitable model for capturing this phenomenon is the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model, which is particularly useful for modeling historical prices in financial data analysis. This research aims to assess ARFIMA modeling of long memory processes using the Geweke and Porter-Hudak (GPH) parameter estimation method. The model was applied to the monthly prices of Groundnut in Andhra Pradesh, from the period January 2002 to December 2023. The best-fitted model identified was ARFIMA(1,0.43,1), which demonstrates a strong short-term forecasting ability, closely matching actual prices with lowest AIC, MSE and RMSE values when compared to SARIMA(1,1,3)(0,1,2)₁₂ model. The study concluded that the ARFIMA model forecasted better than the SARIMA model for forecasting of Groundnut prices of Andhra Pradesh.

Keywords: Autoregressive fractionally integrated moving average, Geweke and Porter Hudak method, Groundnut, Long memory, Prices.

INTRODUCTION

The Groundnut (*Arachis hypogaea* L.) is a leguminous plant extensively cultivated in tropical and subtropical regions between 40°N and 40°S latitudes. (www.fao.org/fileadmin/user_upload/inpho/docs/Post_Harvest_Compodium-Groundnut.pdf). Groundnut renowned for its high oil content and edible seeds, it ranks as the fourth most important source of edible oil and the third most significant source of vegetable protein globally. In India, it is not only a vital oilseed crop but also a significant agricultural export commodity. (Groundnut outlook reports of Andhra Pradesh, January to December 2022, ANGRAU, Lam, Guntur). Globally, groundnut is cultivated over 327 lakh hectares, producing 539 lakh tonnes with productivity of 1648 kg per hectare (FAOSTAT, 2021). India, with an annual all season coverage of 54.2 lakh hectares, ranks first in Groundnut cultivation area and is the second largest producer, achieving 101 lakh tonnes with a productivity of 1863 kilograms per hectare in 2021-22 (agricoop.nic.in) In India, Andhra Pradesh contributes an area of 5.94 lakh hectares, production 6.01 lakh tonnes and productivity 1012 kg/ha of during 2022-23 of groundnut. (des.ap.gov.in). Area and production Scenario of groundnut in India over several decades were depicted in figure.1.

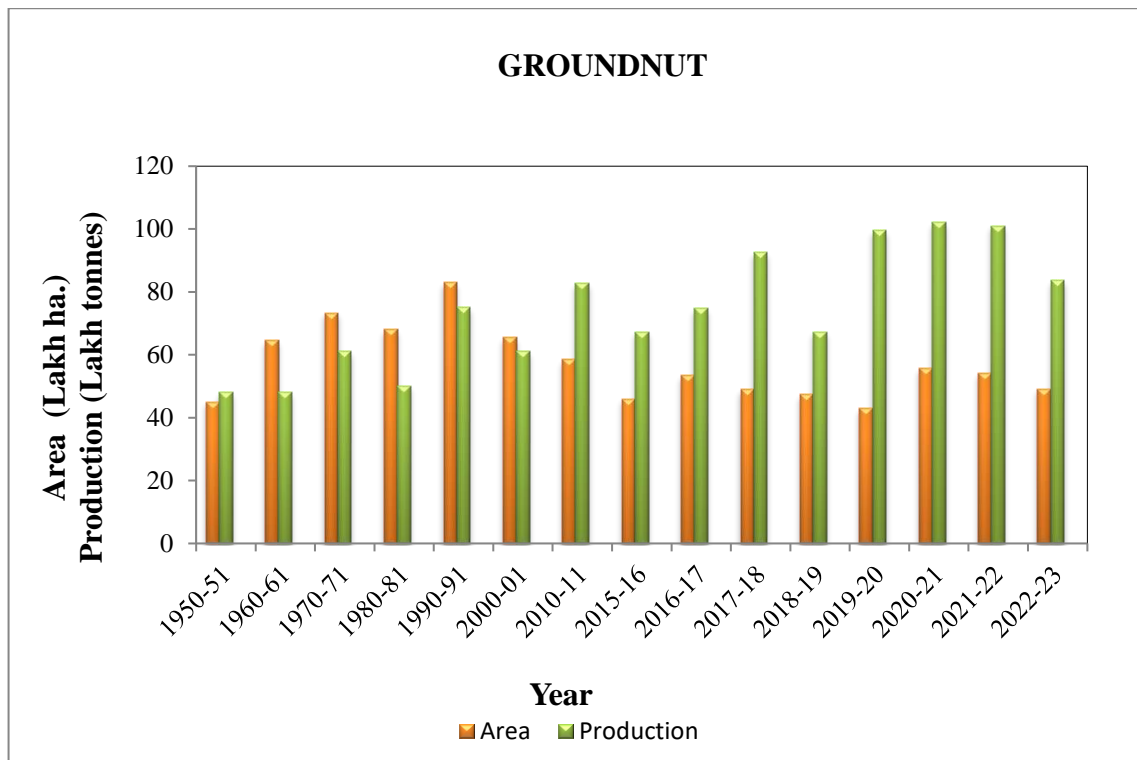


Fig.1. Area, Production and productivity of Groundnut in India

In addition to production, the prices of Groundnut were also fluctuated rapidly and were unpredictable, largely influenced by various factors that introduce significant risk and uncertainty into price modeling and forecasting. Agricultural commodity prices are vital for consumers' access to food, as they directly impact real income, especially among the poor who allocate a large portion of their income to food[13-14]. Given the critical role of food prices in combating hunger, policymakers require reliable forecasts of expected food prices to effectively manage food security. So, forecasting agricultural commodity prices is a crucial aspect in agricultural sector as it allows farmers, traders, and policymakers to make informed decisions regarding production, marketing, and policy implementation. The concept of long memory processes has evolved to provide substantial evidence for describing phenomena in time series data, particularly in the fields of finance and macroeconomics. The presence of long memory can be identified empirically by examining the persistent autocorrelations within observed time series data. This persistence is indicated by the stationarity of the data over time, characterized by autocorrelations that decrease slowly or hyperbolically, often associated with a class of autoregressive moving average (ARMA) models. (Granger and Joyeux 1980).

The most notable definition of a long memory process was provided by Haslett and Raftery (1989), who stated that data exhibiting long memory are characterized by an autocorrelation function that does not decline exponentially, but rather decreases

slowly or hyperbolically. The concept of long memory in time series was initially introduced by Hurst (1951) through various data sets. Subsequently, Granger and Joyeux (1980) and Hosking (1981) developed a model suited for long memory processes, known as the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model. This model effectively explains time series by incorporating both short memory and long memory components, with the differencing parameter represented as a real number. The study of long memory processes, particularly in relation to the ARFIMA model, had been extensively developed in data analysis across both time and space. One of its most compelling features was its suitability for long-term predictions and assessing the effects of shocks within conventional macroeconomic frameworks. In this study, to demonstrate the long memory process using ARFIMA models by employing the Geweke and Porter-Hudak (GPH) differencing parameter estimation method on historical data of Groundnut monthly prices of Andhra Pradesh. The ARFIMA model was used in this study because its capability to directly estimate the differencing parameter, eliminating the need to initially know the order values of the autoregressive and moving average components. The study also compared the forecasting performance of SARIMA model with ARFIMA models.

METHODOLOGY:

Data: Groundnut monthly prices of Andhra Pradesh data was collected from Agricultural Market Intelligence Committee (AMIC), Lam farm, Guntur from the period January 2002-December 2023. The collected data was divided into training and a testing datasets. First 384 observations were used as a training dataset for model development and last 12 observations were used as testing dataset for model validation purpose.

Descriptive Statistics:

The summary statistics viz., mean median, standard deviation, skewness, kurtosis, minimum and maximum were used to study the behaviour of the monthly prices of Groundnut in Andhra Pradesh.

ARIMA Model:

The Autoregressive Integrated Moving Average (ARIMA) methodology developed by Box-Jenkins is the most widely used model for analysing time series data. The Box-Jenkins model-building process is used to fit a blended ARIMA model to

provided data. The basic purpose of fitting the ARIMA model is to accurately characterise and forecast the time series stochastic process (Box and Jenkins, 1970).

Initially, George Box and Gwilym Jenkins conducted substantial research on ARIMA models, and their names were frequently associated with the broad ARIMA method used in time series analysis, forecasting, and control. The two forms of stochastic processes are stationary and non-stationary. The ARIMA model can only be used with stationary data.

Stationarity and Non-stationarity:

A process that generates data in equilibrium around a constant value and has a constant variance around the mean throughout time is referred to as “stationary.” If the means shift over time and the variance is not roughly constant both mean and variance, the series is said to be non-stationary. To build the ARIMA model the series should be stationary in nature. If the original series is not stationary then it has to make stationary by differencing.

Autoregressive (AR) Model

An observed time series Y_t can be elucidate by linear function of its previous observation, Y_{t-1} and some unexplainable random error ε_t . Let us consider equally spaced time series $Y_t, Y_{t-1}, Y_{t-2} \dots$, over an equal period of time say $t, t-1, t-2, \dots$, then Y_t can be defined as;

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t \quad \dots(1)$$

If we represent the series in Backshift operator format, then it becomes

$$\phi(B) = 1 - \phi_1(B) - \phi_2 B^2 - \dots - \phi_p B^p \quad \dots(2)$$

Where, B is the backshift $BY_t = Y_{t-1}$ then the AR model can be written as $\phi(B)Y_t = \varepsilon_t$.

6.1. Moving Average (MA) Model

Another important model of great practical utility in the frame work of time series is finite moving average model. The MA (q) model is defined as;

$$Y_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad \dots(3)$$

In terms of backshift operator, the MA model of order q is given as follows;

$$\theta(B) = 1 - \theta_1(B) - \theta_2 B^2 - \dots - \theta_q B^q \quad \dots(4)$$

Where B is the backshift operator and the moving average model can be expresses as;

$$Y_t = \theta(B)\varepsilon_t \quad \dots \quad (5)$$

6.2. Autoregressive Moving Average (ARMA) model

In order to obtain the higher efficiency and greater flexibility in modeling we combine both autoregressive and moving average processes together. These models are called as "mixed models" and are represented as ARMA (p, q) models

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad \dots(6)$$

Generally, in Backshift operator it is expressed as follows;

$$\phi(B)Y_t = \theta(B)\varepsilon_t \quad \dots(7)$$

6.3. Autoregressive Integrated Moving Average (ARIMA) model

ARIMA is one of the most traditional methods of non-stationary time series analysis. In contrast to the regression models, the ARIMA model allows to explain by its past, or lagged values and stochastic error terms. ARIMA modes are also called as mixed family of models. Though, the mixed models make forecasting process more complicated, but these models yield accurate forecasts. The pure models mean, the models which contains only AR or MA components, but not both. The term integration (I) is the reverse process of differencing, to produce the forecast. An ARIMA model is represented as ARIMA ($p \ d \ q$). An ARIMA model is expressed as follows;

$$\phi(B)(1 - B)^d Y_t = \theta(B)\varepsilon_t \quad \dots(8)$$

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad \dots(9)$$

Y_t is the time series, ϕ_i and θ_j are model parameters, ε_t is random error, p is number of autoregressive terms, q is number of moving terms and B is the backshift operator such that, $BY_t = Y_{t-1}$ (Box and Jenkins 1994, Brockwell and Davis 1996).

The four-stage univariate Box Jenkins procedure is summarized schematically in Fig.2.

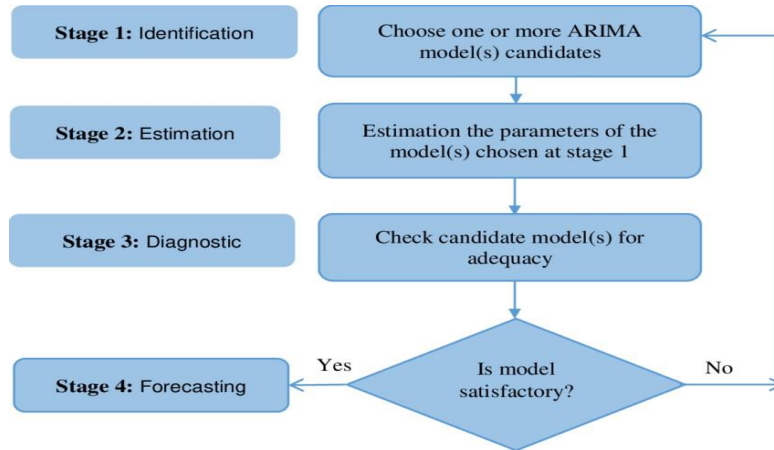


Fig.2. Flow chart of Box-Jenkins Methodology

The main stages in setting up a Box-Jenkins forecasting model are described below:

1. Identification:

The autocorrelation function (ACF) and partial autocorrelation function (PACF) are two graphical devices to measure the correlation between the observations within a single data series and they give an idea about the patterns and relationship in the available data. As the time series under study is a particular realization of the process, the theoretical ACF and PACF must resemble the estimated ACF and PACF of the data.

Table.1. Pattern of ACF and PACF for identification of AR, MA and ARMA process

PROCESS	ACF	PACF
AR	Decays towards zero	Cut off to zero (lag length of last spike is the order of the process)
MA	Cut off to zero (lag length of last spike is the order of the process)	Decays towards zero
ARMA	Tails off towards zero	Tails off towards zero

2. Estimation of parameters

At the estimation stage, coefficients of the identified models are estimated using method of least squares or maximum likelihood estimation methods are used to estimate the parameters. Stationarity and invertibility are checked for the coefficient obtained and at the same time diagnostic checking is done in order to know whether the model fit the data satisfactorily or not. The importance of the estimation coefficients is measured in terms of the statistical significance.

3. Diagnostic Checking

Different models can be obtained for various combinations of AR and MA individually and collectively. The best model is obtained with following diagnostics.

(a) Low Akaike Information Criteria (AIC)

AIC is given by $(-2 \log L + 2m)$ where $m = p + q + P + Q$ and L is the likelihood function. Since $-2\log L$ is approximately equal to $\{n(1+\log 2\pi) + n \log \sigma^2\}$ where σ^2 is the model MSE. Thus, AIC can be written as $AIC = \{n(1+\log 2\pi) + n \log \sigma^2 + 2m\}$ and because first term in this equation is constant, it is usually omitted while comparing between models. The model having lowest AIC is considered as the best model.

Bayesian Information Criterion

The BIC criterion is defined as

$$BIC = n^{k/n} \frac{RSS}{n} \quad \dots(10)$$

Where k = Number of regressors (including the intercept) and 'n' is the number of observations.

BIC imposes a harsher penalty than AIC, the lower the value of BIC, better is the model. Again, like AIC, SIC can be used to compare forecasting performance of the model

(b) Plot of residual ACF

Once the appropriate model has been fitted, the goodness of fit can be examined by plotting the ACF of residuals of the fitted model. If most of the sample autocorrelation coefficients of the residuals are within the limits $\pm 1.96/N$ where N is the number of observations on which the model is based, then the residuals are white noises indicating that the model is good fit.

(c) Box-Pierce or Ljung-Box texts

Box-Pierce statistic is a test to measure the overall adequacy of the chosen model by examining a quantity Q , whose approximate distribution is Chi-square.

$$Q = n \sum_1^k r_{(j)}^2 \quad \dots (11)$$

Where k as maximum lag considered, and is usually around 20, n = number of observations, $r_{(j)}$ is the estimated autocorrelation at lag j . Chi-square with $(k-m-1)$ degrees of freedom where $m-1$ is the number of parameters estimated in the model.

A modified Q statistics is the Ljung-box which is given by

$$Q = n(n+2) \sum \frac{r_{(j)}^2}{n-j} \quad \dots (12)$$

The critical value of Q statistic is compared with Chi-square $(n-1)$ degrees of freedom. Residuals should be uncorrelated and Q should be small if model is correctly specified. A significant value of test statistic indicates the chosen model is not a good fit.

4. Forecasting

The model that satisfies all the diagnostic checks is considered for forecasting. If the model is based on differencing / de-trending transformations, then the model must be represented with relevant expressions of original series. Then only, the forecasts can be made.

Seasonal ARIMA

In the time series analysis, seasonality is defined as the pattern of changes that repeats over S time periods, where S is the number of time periods between the repeats of the pattern. For quarterly data, $S = 4$ time periods per year and for monthly data $S = 12$ time periods per year are considered. As the regular differencing was applied to the series having non-stationary nature similarly seasonal differencing will be applied to the seasonal non-stationary series. The seasonal Autoregressive (SAR) and Seasonal Moving Average (SMA) are the parameters of seasonal ARIMA. In the seasonal ARIMA model, seasonal AR and MA terms predicts the x_t often with the lags that are multiples of S .

Seasonal ARIMA model is denoted by ARIMA (p, d, q) (P, D, Q) $_s$, where p represents the number of autoregressive terms, q represents the number of moving average terms and d denotes order of differencing to induce stationarity, P represents the number of seasonal Autoregressive components, Q represents the number of seasonal moving average terms and D represents the number of seasonal differences required to make the series stationarity. The seasonal ARIMA model expressed as follows;

$$\phi(B)\Phi(B)\nabla^d\nabla_s^D r_t = \theta(B)\Theta(B)\varepsilon_t \quad \dots (13)$$

$$w_t = \nabla^d\nabla_s^D r_t \quad \dots (14)$$

$\nabla^d = (1 - B)^d$ denotes the number regular differences and $\nabla_s^D = (1 - B^s)^D$ denotes number of seasonal differences.

Where, $\phi(B)$ is stationary Autoregressive operator, $\theta(B)$ is a stationary moving average operator, ε_t is a white noise (Brockwell and Davis, 1996).

ADF (Augmented Dickey Fuller) test for Stationarity

In a stochastic process for a time series data the assumption of stationarity holds prime importance. It refers to the mean and variance constant over time and covariance between the two time periods depends only on the distance or gap or lag between the two time periods then the process is said to be stationary.

For time series data y_t , the regression equation of ADF (Dickey and Fuller, 1981) test is represented as follows:

$$\Delta y_t = \alpha_1 + \alpha_2 t + \delta y_{t-1} + \sum_{i=1}^h \beta_i y_{t-i} + \varepsilon_t \quad \dots (15)$$

Where $\Delta y_t = y_t - y_{t-1}$, α_1, α_2 and β_i are parameters of regression model, h is lag length.

$$\delta = \rho - 1 \text{ and } -1 \leq \rho \leq 1.$$

In the ADF test, $\delta = 0$ indicates the time series under consideration is nonstationary. In the tests null hypothesis (H_0) and alternative hypothesis (H_1) are as follows:

H_0 = Unit root is present in a time series data (Time series is nonstationary)

H_1 = Time series is stationary

Q-S test for Seasonality

The Q-S test is a variant of the Ljung-Box test computed on seasonal lags, where we only consider positive auto-correlations. More exactly,

$$QS = n(n+2) \sum_{i=1}^k \frac{[\max(0, \hat{\gamma}_{r,l})]^2}{n-i.l} \quad \dots (16)$$

The current implementation still considers that the statistics is distributed as a $\chi^{(k)}$.

Long memory model:

Long memory in time-series can be defined as autocorrelation at long lags. According to Jin and Frechette memory means that observations are not independent (each observation is affected by the events that preceded it). The ACF of a time-series x_t is defined as

$$\rho_k = \text{COV}(x_t, x_{t-k}) / \text{VAR}(x_t) \quad \dots (17)$$

for integer lag k . A covariance stationary time-series process is expected to have autocorrelations such that $\lim_{k \rightarrow \infty} \rho_k = 0$. Most of the well-known class of stationary and invertible time-series processes have autocorrelation at decay at the relatively fast exponential rate, so that $\rho_k \sim |m|^k$ where, $m < 1$ and this property is true, for example, for the well-known stationary and invertible ARMA (p, q) process.

For long memory processes, the autocorrelations decay at an hyperbolic rate

which is consistent with $\rho_k \approx, \rho_k \approx ck^{2d-1}$, k increases without limit, where C is a constant and d is the long memory parameter. ACF is considered to be the time domain analogue of spectral density. In terms of the auto covariance sequence (ACVS), i.e. $\{S_{x,k}\}$ for $\{x_t\}$, $\{x_t\}$ is a stationary long memory process if there exist $\lim_{k \rightarrow 0} S_{x,k} / (C_S k^b)$ constants b and C_S satisfying $-1 < b < 0$ and $C_S > 0$ such that the standard time-series models such as stationary autoregressive processes have ACVS s such that $S_{x,k} \approx c\phi^k$ for large k , where $C \geq 0$ and $|\phi| < 1$. For a long memory process, $S_{x,k} \approx C_S k^b$ for large k . In both the cases $S_{x,k} \rightarrow 0$ as $k \rightarrow \infty$, but the rate of decay toward zero is much slower for a long memory process, implying that the observations that are widely separated in time can still have a non-negligible covariance i.e. the current observations retain some ‘memory’ of the distant past. An alternative definition can be stated as if $\{x_t\}$ is a stationary process with the spectral density function (SDF) denoted by $S_x(\cdot)$, then $\{x_t\}$ is a stationary long memory process if there exist constants a and C_S satisfying $-1 < a < 0$ and $C_S > 0$ such that

$$\lim_{f \rightarrow 0} S_x(f) / (C_S |f|^a) = 1 \quad \dots\dots(18)$$

Where, $a = -b-1$. In other words, a stationary long memory process has an SDF $S_x(\cdot)$ such that $S_x(\cdot) \approx (C_S |f|^a)$, with the approximation improving as f approaches zero.

GPH estimator:

The approximated regression equation is used in this method, which was calculated by logarithmic transformation of the spectral density function (SDF). This method is based on least squares regression in the spectral domain (Geweke and Hudak, 1983), utilize the sample form of the pole of the spectral density at the origin, $f(\eta) \sim \eta^{-2d}$, as $\eta \rightarrow 0$. The SDF of a stationary model y_t , $t=1, \dots$, can be expressed as,

$$f(\eta) = [4\sin(\eta/2) - d] f_\varepsilon(\eta) \quad \dots(19)$$

where $f_\varepsilon(\eta)$ is the spectral density of ε_t , assumed to be a finite and continuous function on the interval $[-\pi, \pi]$. After logarithmic transformation of the SDF, the log-spectral density can be written as,

$$\log(f_y(\eta)) = \log(f_\varepsilon(0)) - d \log[4\sin^2(\frac{\eta}{2})] + \log \frac{f_\varepsilon(\eta)}{f_\varepsilon(0)} \quad \dots(20)$$

Let, (η_i) be the periodogram obtained at the Fourier frequencies, $\eta_i = 2\pi lk$, $l=1, 2, \dots, t$; k is the total number of observations and t is the number of considered Fourier frequencies, that is the number of periodogram series which is utilized in

regression,

$$\log(I_y(\eta_l)) = \log(f_\varepsilon(0)) - d \log[4 \sin^2(\frac{\eta_l}{2})] + \log\left(\frac{f_\varepsilon(\eta_l)}{f_\varepsilon(0)} + \frac{I_y(\eta_l)}{f_y(\eta_l)}\right) \dots (21)$$

where, $\log(f_\varepsilon(0))$ is being constant, $\log[4 \sin^2(\frac{\eta_l}{2})]$ is the exogenous variable and

$\log\left(\frac{I_y(\eta_l)}{f_y(\eta_l)}\right)$ is the unforeseen term. The GPH estimate is based on two assumptions which

are reason behind the asymptotic behaviour of the equation. These are,

H1: For low frequencies, we suppose that $\log(\eta_l)/f_\varepsilon(0)$ is negligible

H2: The random variable $\log(\eta_l)/f_y(\eta_l)$, $l=1,2,\dots,t$ are asymptotically iid

Under the hypotheses H1 and H2, we can write the linear regression,

$$l(I_y(\eta_l)) = \alpha - d \log[4 \sin^2(\frac{\eta_l}{2})] + e_l \dots (22)$$

where, $e_l \sim iid(-c, \pi^2/6)$. Considering the equation (42) as a regression equation of

$l(I_y(\eta_l))$ on α and y_l , where $y_l = \log[4 \sin^2(\frac{\eta_l}{2})]$. The OLS estimate of d is obtained as,

$$d_{GPH} = \frac{\sum_{i=1}^t (y_i - \bar{y}) l(I_y(\eta_l))}{\sum_{i=1}^t (y_i - \bar{y})^2} \dots (23)$$

List 1 : According to the value of d long memory process can be sub-divided into 4 groups and these are:

Value of d	Names
$d \in (-1/2, 0)$	Intermediate Memory and Anti-persistence
$d = 0$	White noise (Short-Memory)
$d \in (0, 1/2)$	Stationary and Persistence Long Memory
$d \in [1/2, 1)$	Non-stationary and Persistence Long Memory

ARFIMA model:

Fractional integration is a generalization of integer integration, under which time-series are usually presumed to be integrated of order zero or one. Hosking and Reinsen described autoregressive fractionally integrated moving average (ARFIMA) process in details. x_t is called an ARFIMA(p, d, q) process with degree of differencing as d , if it satisfies

$$(1 - L)^d \Phi(L) x_t = \Theta(L) \varepsilon_t \dots (24)$$

where, ε_t is an independently and identically distributed (i.i.d.) random variable with zero mean and constant variance, L denotes the lag operator; and $\Phi(L) = 1 - \varphi_1 L - \dots - \varphi_p L^p$ and $\Theta(L) = 1 - \theta_1 L - \dots - \theta_q L^q$ denote finite polynomials in the lag operator with roots outside the unit circle For $d = 0$, the process is stationary, and the effect of a shock to ε_t on $x(t+j)$ decays geometrically as j increases. For $d = 1$, the process is said to have a unit root, and the effect of a shock to ε_t on $x(t + j)$ persists into the infinite future. In contrast, fractional integration defines the function $(1 - L)^d$

for non-integer values of the fractional differencing parameter d . For $0.5 < d < 0.5$ the process $x(t)$ is stationary and invertible. For such processes, the effect of a shock ε_t on $x(t+j)$ decays as j increases, but the rate of decay is much slower than for a process integrated of order zero. More precisely, the ACF for zero-integrated processes decays geometrically, whereas the ACF for a fractionally integrated process decays hyperbolically, with the sign of the autocorrelations being the same as the sign of d . In this sense, fractional integration captures long memory dynamics more parsimoniously than non-integrated ARMA processes. In the use of ARFIMA (p, d, q) models, correct specification of p and q is important. According to Robinson, under-specification of p or q leads to inconsistent estimation of AR and MA coefficients, but also of long memory parameter d , as does over-specification of both, due to a loss of identifiability.

RESULTS AND DISCUSSIONS:

The actual prices scenario of monthly prices of Groundnut in Andhra Pradesh was plotted and depicted in figure.3 to identify the basic behaviour of the price series.

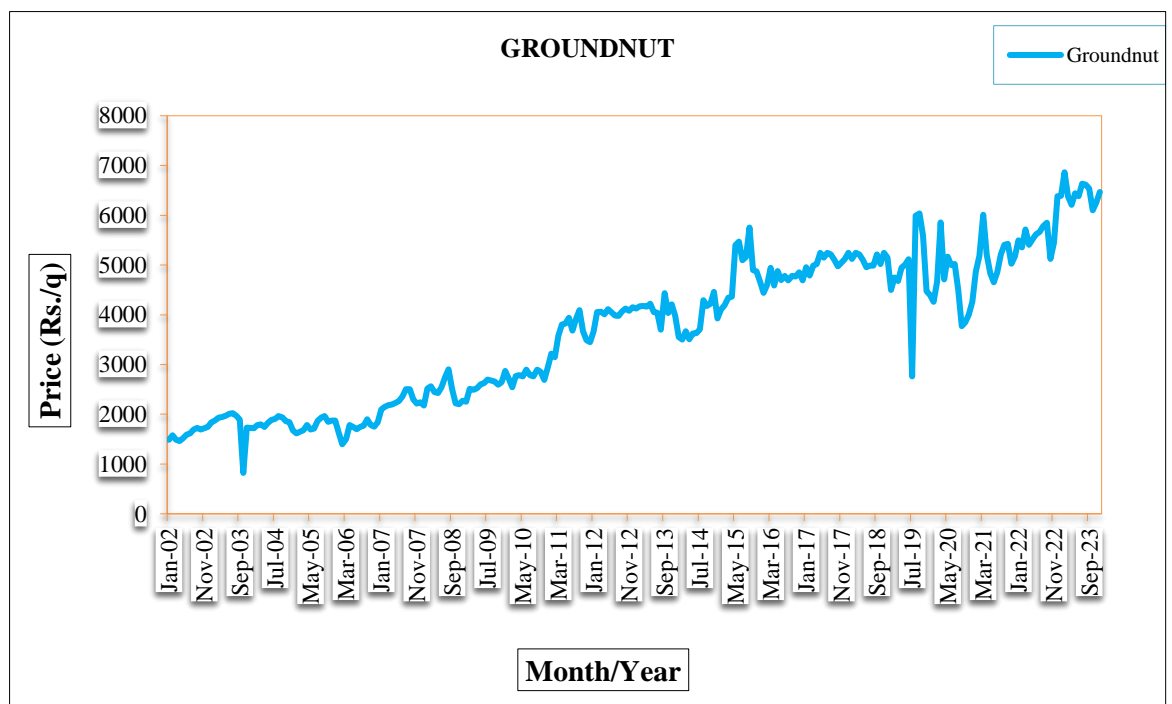


Fig.3. Groundnut Actual Prices Scenario in Andhra Pradesh during 2002-2023

Descriptive Statistics:

Descriptive Statistics were conducted to examine the behaviour of Groundnut monthly prices of Andhra Pradesh. The findings were depicted in Table.2, provided

valuable insights in to the characteristics of the data. It was observed that the prices of Groundnut during the study period had varied from Rs.822/q to Rs.6864/q with an average of Rs.3679.19/q. Standard Deviation was recorded as 1491.03, which indicates that the prices were dispersed highly over the months. It was also revealed that the data was positively skewed and platykurtic in nature. Further lists the summary statistical measures which were self-explanatory. The price series were also verified for the presence of outliers by Grubb's test. It was confirmed that there were no outliers detected from the Grubb's test during the study period.

BDS (Brock - Dechert- Scheinkman) test for non-linearity:

To test the linearity characteristics of the price series BDS test was conducted and results of the test was presented in Table.3. The probability value for both dimensions was less than 0.05 (at 5 % LOS) which shows that the data under consideration was nonlinear in nature.

Table.2. Descriptive statistics of Groundnut prices of Andhra Pradesh

Statistic	Groundnut
No of observations	264.00
Mean	3679.19
Median	3959.96
Standard Deviation	1491.03
Minimum	822.00
Maximum	6864.00
Skewness	0.06
Kurtosis	-1.24
Outliers detected (Grubbs test)	No

Table.3. BDS test for non-linearity in Groundnut prices of Andhra Pradesh

Sample	Dimension	Groundnut	
		Statistics	Probability
eps (1)	<i>m</i> =2	255.82	p<0.0001
	<i>m</i> =3	463.02	p<0.0001
eps (2)	<i>m</i> =2	246.57	p<0.0001
	<i>m</i> =3	320.99	p<0.0001
eps (3)	<i>m</i> =2	86.06	p<0.0001
	<i>m</i> =3	96.80	p<0.0001
eps (4)	<i>m</i> =2	50.62	p<0.0001

	$m=3$	51.18	$p<0.0001$
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Autocorrelation (ACF) and Partial Autocorrelation (PACF) plots for Groundnut prices of Andhra Pradesh:

The Autocorrelation (ACF) and Partial Autocorrelation Function (PACF) plots of Groundnut price series were depicted in the below figure.4. The figure shown that the prices were autocorrelated, which was supported by Box-Jung test statistic as the probability value was less than 0.05. It indicates that data under consideration was autocorrelated in nature. Once the price series were autocorrelated, the ARIMA model was built for the series. Further, the Groundnut prices contains seasonal component which was confirmed by Q-S test as the probability value obtained was 0.02 ($p<0.05$) which indicates the presence of seasonality in the dataset. So, SARIMA model was built for the price series.

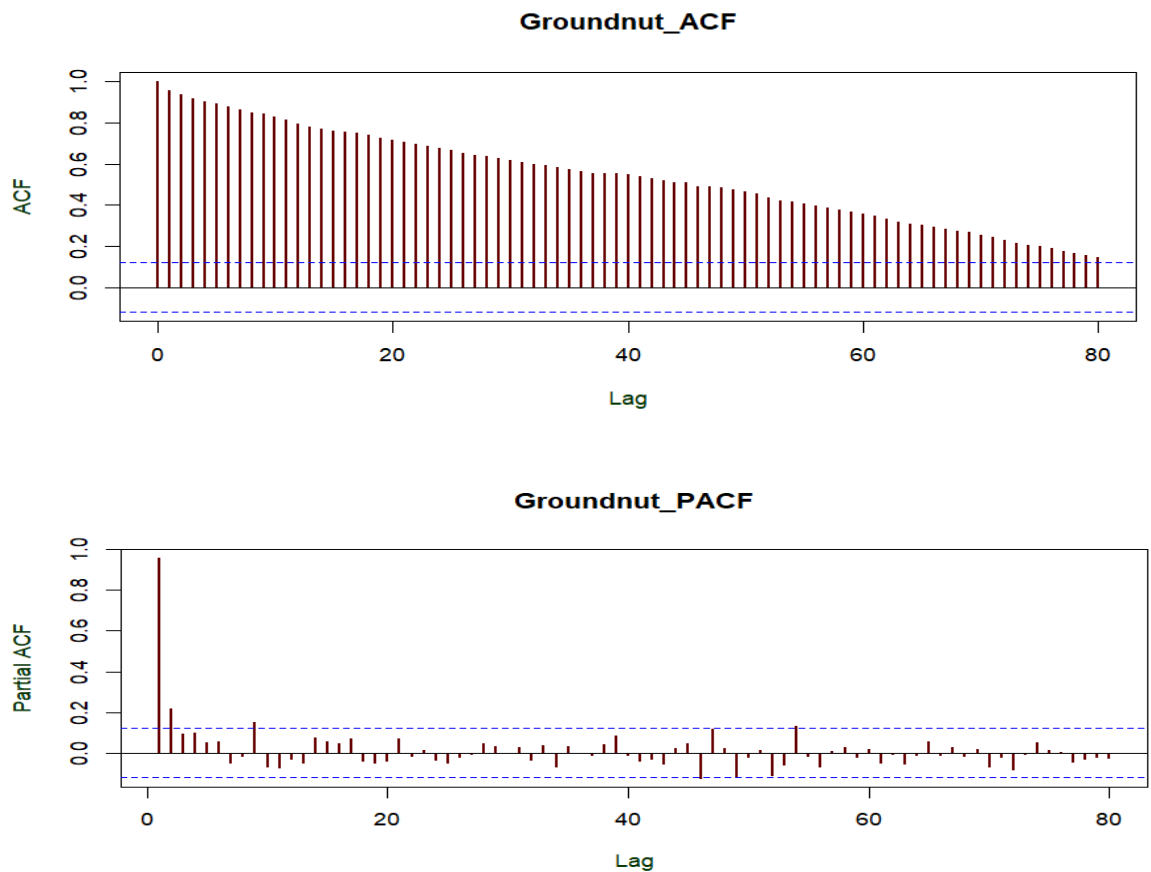


Fig.4. ACF and PACF plots for Groundnut prices of Andhra Pradesh

To develop ARIMA modelling, Augmented Dickey-Fuller test (ADF) was used

to check the stationarity of the data and the results were presented in Table.4. The Groundnut price series shows the probability value was 0.18 ($p>0.05$), confirmed that the data under consideration was non- stationary and became stationary at first difference as probability value was 0.01 ($p<0.05$). Before model estimation to ensure that the data for model consideration was autocorrelated by applying Box- Pierce non-correlation test and it was found significant as the probability value was 0.05 (at 5% LOS) that the data was autocorrelated in nature. All the possible combinations of SARIMA models were developed and best performed SARIMA models were presented in Table.5. Out of the best performed models, the final SARIMA model order i.e., SARIMA (1,1,3)(0,1,2)₁₂ model was selected based on least RMSE, MAE, MAPE and AIC values.

The results of the final selected SARIMA (1,1,3)(0,1,2)₁₂ model parameter specification viz., AR, MA, SAR and SMA were presented in Table.6. After determining the SARIMA model order, the model parameters were estimated using maximum likelihood method. After fitting of the model, the diagnostic checking of the residuals by Box- Pierce non-correlation test and it was showed that the residuals were non-autocorrelated in nature as probability value was 0.90 ($p>0.05$). The performance criteria of RMSE, MSE values for both training and testing data sets were illustrated in in Table.8 and Table.9 respectively.

Table. 4. ADF test for stationarity in Groundnut prices of Andhra Pradesh

Groundnut	Data type	ADF statistic	P-value	Decision
	ADF at level	-2.96	0.18	Non-Stationary
	ADF at 1 st Difference	-8.04	0.01	Stationary

Table.5. SARIMA models and Error values for Groundnut prices of Andhra Pradesh

SARIMA	RMSE	MAE	MAPE	AIC
SARIMA(1,1,1)(1,0,1) ₁₂	341.85	198.56	6.04	3685.21
SARIMA(1,1,1)(1,0,2) ₁₂	340.93	197.23	6.02	3645.20
SARIMA (1,1,1)(1,0,3) ₁₂	336.78	195.63	5.98	3642.12
SARIMA (2,1,1)(1,1,1) ₁₂	336.23	195.23	5.97	3643.23
SARIMA(2,1,1)(2,0,1) ₁₂	336.27	196.67	5.94	3646.45
SARIMA (2,1,2)(0,1,1) ₁₂	335.98	195.84	5.92	3642.89
SARIMA (1,1,3)(0,1,2)₁₂	335.94	194.27	5.79	3642.02
SARIMA (1,1,3)(0,1,1) ₁₂	336.39	195.98	5.84	3643.25

SARIMA (1,1,2)(1,0,2) ₁₂	338.25	196.49	5.94	3643.89
SARIMA(2,1,3)(1,1,0) ₁₂	338.25	196.49	5.85	3642.23
SARIMA(2,1,3)(0,1,1) ₁₂	336.78	194.63	5.98	3642.69
SARIMA(3,1,3)(0,1,2) ₁₂	336.55	196.67	5.99	3642.96

Table.6.Parameter estimation of SARIMA model for Groundnut prices of Andhra Pradesh

Model	Parameters	Estimation	S.E.	Z-value	Probability	Model fitting	
SARIMA (1,1,3)(0,1,2) ₁₂	AR1	-0.97	0.08	-21.66	p<0.0001	log likelihood	-1830.1
	MA1	-0.65	0.08	-8.44	p<0.0001		
	MA2	-0.20	0.07	-2.51	p<0.05		
	MA3	-0.15	0.06	-2.11	P<0.05	AIC	3642.02
	SAR1	0.16	0.05	2.75	p<0.0001		
	SMA1	-0.80	0.05	-16.07	p<0.0001		
Box-Pierce test for non-correlations of				Actual series	$\chi^2 = 242.15, p<0.0001(<0.05)$		
				Residuals	$\chi^2 = 0.02, p=0.90 (>0.05)$		

To develop ARFIMA model, first step was to ensure that the data exhibiting long memory by employing Geweke-Porter-Hudak (GPH) test and the differencing parameter (d) value obtained was 0.43 (d<0.5) indicated a strong evidence of existence of long-memory. It was also confirmed by observing the autocorrelations function plot does not fall exponentially but decreases slowly or hyperbolically as shown in figure 3. Before model estimation, to ensure that the data under consideration was autocorrelated by applying Box- Pierce non-correlation test found significant as the probability value was p< 0.0001 (<0.05) and concluded that the data was autocorrelated in nature. The differenced series was used to fit the ARFIMA (1, d, 1) model and the parameter specification of AR and MA parameters were presented in Table.7. After model development the diagnostic checking of the residuals by Box- Pierce non-correlation test found non-significant as the probability value was 0.35(>0.05). The modelling and forecasting performance of the training and testing data sets were given in Table.8 and Table.9 respectively. With the best performed model forecasted future values for a period of 12 months. For the month of December 2024, the Groundnut price was forecasted Rs.6589.52/q. which was depicted in Table 10.The actual and fitted values graph was plotted in Fig.5.

Table 7. ARFIMA model parameter estimation for Groundnut prices of Andhra Pradesh

Model	Parameters	Estimation	S.E.	Z value	Probability	Model fitting	
ARFIMA (1,0,1)	d	0.43	0.14	3.07	p<0.05	Log-likelihood	-1489.5
	Φ (AR)	0.65	0.12	5.33	p<0.0001		
	Θ (MA)	0.46	0.06	7.74	p<0.0001	AIC	2988.3
Box-Pierce test for non-correlations: $\chi^2= 0.88$, p=0.35 (>0.05)							

Table 8. Performance metrics of Training set for Groundnut prices of Andhra Pradesh

Training set	SARIMA	ARFIMA
AIC	3642.02	2988.33
RMSE	335.94	186.22
MSE	112855.68	34677.89

Table.9 Performance metrics of Testing set for Groundnut prices of Andhra Pradesh

Testing set	ACTUAL	SARIMA	ARFIMA
Jan-23	6391	6045.906	6211.29
Feb-23	6864	5945.865	6088.19
Mar-23	6380	5847.585	5946.46
Apr-23	6207	5873.262	5846.46
May-23	6443	5882.055	5799.52
Jun-23	6384	5907.222	5884.42
Jul-23	6634	5916.509	5871.78
Aug-23	6613	5941.196	5965.28
Sep-23	6540	5950.949	5962.53
Oct-23	6099	5975.184	5964.57
Nov-23	6255	5985.375	5978.96
Dec-23	6470	6009.186	6098.59
RMSE		541.01	516.27
MSE		292691.82	266534.71

Table 4.10. Sample of Predicted values using ARFIMA model for Groundnut price series of Andhra Pradesh

Month/Year	Actual (Rs./q)	Forecasted (Rs./q)	Forecast Error (%)
Jan-24	6752	6687.25	0.96
Feb-24	6647	6774.63	1.92
Mar-24	6511	6363.42	2.27
Apr-24	6437	6589.41	2.37
May-24		6605.63	
Jun-24		6625.12	
Jul-24		6658.48	
Aug-24		6809.01	
Sep-24		6809.52	
Oct-24		6454.15	
Nov-24		6582.23	
Dec-24		6589.52	

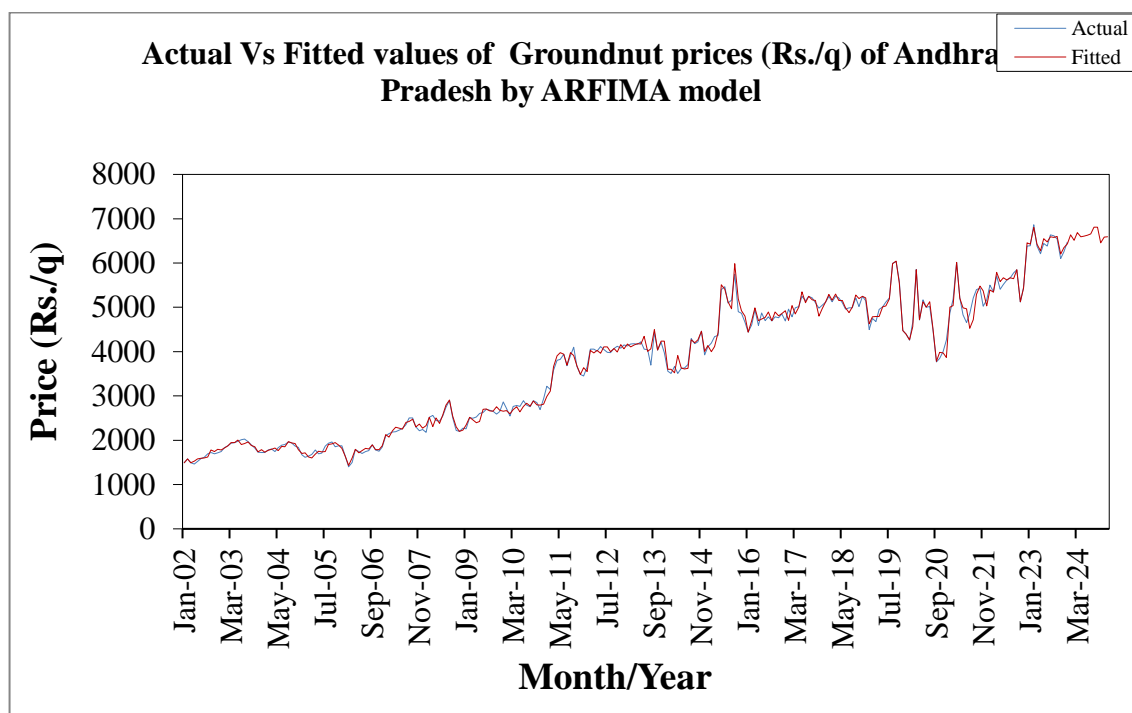


Fig. 5. Actual and Fitted values of Groundnut price series of Andhra Pradesh by ARFIMA model.

CONCLUSION:

The present study forecasted long memory time series data for the monthly prices of Groundnut in Andhra Pradesh using the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model. This model was denoted as ARFIMA (p,d,q), incorporates a fractional differencing operator with d as a real number in the range $0 < d < 1/2$. The ARFIMA (p,d,q) model demonstrated superior performance compared to the SARIMA model, based on performance metrics of both training and testing data sets. This model was particularly effective for short-term predictions of long memory time series, where the forecasting results closely match the actual data. The study concluded the ARFIMA model shown predictive capability compared SARIMA model for forecasting Groundnut monthly price series of Andhra Pradesh. With the best performed model forecasted future values for a period of 12 months. For the month of December 2024, the Groundnut price was forecasted Rs.6589.52/q.

Disclaimer (Artificial intelligence)

Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

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Details of the AI usage are given below:

1. No AI tools were used for this study.
- 2.
- 3.

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