In-Situ Measurement of Unsaturated Hydraulic Conductivity Function By A NEW POINT Source Field Dripper Method (NPSFDM)

ABSTRACT

Unsaturated hydraulic conductivity function (K_h) is an important soil parameter for drip irrigation design. There are many laboratory and in-situ measurement methods available for the measurement of K_h with associated limitations. A point source field dripper model (PSFDM) using **Wooding (1968)** theory was used by **Shani et al. (1987)** for K_h measurement in the field for the first time. Warrick (1985) gave an approximate equation for K_h estimation. Keeping the difficulty of detecting interface and steady state conditions a new model for in-situ measurement of K_h was developed. An experimental set up (micro-irrigation simulator) was developed for in-situ measurement of K_h which simulates real field drip conditions. Steady state saturated fronts diameters were measured against 2.02, 4.04, 7.56 and 8.31 lph dripper discharges maintaining one atmospheric pressure in drip line. K_s values calculated by PSFDM-Shani, PSFDM-Warrick and PSFDM-New were 3.60, 4.31 and 3.22 cm/hr and α as 0.1643, 0.1544 and 0.1468 cm^{-1}, respectively. Ks values measured by PSFDM are quite close to each other. K_s value measured by using inverse auger hole method (IAHM) was 0.60 cm/hr and by infiltrometer method (IM) as 0.79 cm/hr. The PSFDM is the only method available to measure any change in K_h value due to tillage or any other interventions. Newly developed experimental set up in combination to PSFD Models is the best combination for field application.

Key words: Field dripper, sodic soil, unsaturated hydraulic conductivity, saturated front

INTRODUCTION

Unsaturated hydraulic conductivity (K_θ) governs hydrologic processes on soils. The knowledge about unsaturated hydraulic conductivity enhances our understanding to water quantity and quality, the atmosphere-terrestrial relationship, nutrient cycling, soil erosion, flooding and landslides. The unsaturated hydraulic conductivity is highly nonlinear as function of soil moisture content. Researchers have proposed different types of unsaturated hydraulic conductivity functions from time to time (**Richards (1931)**, **Gardner (1958)**, **van Genuchten (1980)**, **Brooks and Corey (1966)**, **Clapp and Hornberger (1978)**, **Fredlund et al. (1994)**, **Singh and Verma (2010)** and **Saxena (2015)).** Field methods for the measurement of K_θ or K_h take a lot of money, labour and time. They also require a lot of water covering only a tiny volume of soil. **Phillip (1985)** and **Elrick and Reynolds (1992)** observed and reported that Guelph permeameter a bore hole method for in-situ measurements of K_h is unreliable and may result in physically unattainable values for soil parameters. Using **Wooding's (1968)** solutions of field dripper creating a pool or circular water, **Shani et al. (1987)** developed a protocol for the measurement of Gardener's K_h function which represents practical range of moisture content. Steady-state theory of a buried point source was used by **Singh (1999)** for estimation of subsurface K_h function of the soil.

Singh et al. (2001) also proposed a model for estimating K_h function using hemispherical water flow geometry on soil surface. These models covers small volume of soils and for obtaining a true representative values of Kh large numbers of measurements would be needed. **Ojha et al. (2018)** proposed to use of extrapolated saturated front radii estimates for the in-situ measurement of K_h with higher accuracy. **Warrick (1985)** approximate solution for calculating saturated width of progressing front under the line source field dripper was used for the estimation of K_h function **(Ojha et al. 2020;** Wang et al., 2024**).** Line Source Field Dripper Method covers large volume of soil hence may provide representative K_h value. **Gardner (1958)** exponential K_h function applicable for practical range of unsaturated soil moisture regime can be written as below.

$$K_h = K_h \exp(\alpha h) \tag{1}$$

$$K_h = K_s \exp\left(\frac{h}{\lambda_c}\right) \tag{2}$$

where,

K_s = saturated hydraulic conductivity of soil (LT^{-1})
$\alpha = 1/\lambda_c$ [L^{-1}]
λ_c = scaling parameter [L]
α is a relative measure of capillarity over gravity and inherent property of the soil.

For extremely dried soil h=--∞ and K_h becomes zero and for saturated soil, h=0 and finally K_h becomes K_s. A lot of time and effort are needed to estimate K_s and λ_c or α in the laboratory using conventional pressure plate apparatus. Large numbers of samples are required for an accurate and representative estimate. The saturated wetted front radii have to be created against various dripper discharge rates and must be measured in order to use the PSFDM developed by **Shani et al. (1987)** or newly developed PSFDM. **Shani et al. (1987)** used the **Wooding (1968)** theory of water front advance against a circular pool source. A constant-size circular water pond with a radius of r_s (saturated water pond radius, L) is produced at the point of discharge from a point-source field dripper for a given discharge and can be written below.

$$q = \frac{Q}{\pi r_s^2} = K_s\left(1 + \frac{4\lambda_c}{\pi r_s}\right), \quad \frac{r_s}{\lambda_c} \leq 10 \tag{3}$$

where,

Q = volume of water discharged by point source field dripper per unit time ($L^3\ T^{-1}$)

An approximate solution was also given by **Warrick (1985)** for steady state saturated front under point source field dripper as below.

$$q = 0.836 K_s + \left(\frac{1}{\alpha}\right).\frac{1}{r_s} \tag{4}$$

A new LSFDM was developed after developing a new hypothesis and applied in the fields for estimating K_s and α.

Experimental set ups used by earlier researchers have chances of fluctuations of dripper discharges under low head conditions prevalent in the water supply tank and twisting and angling of drip line. Installation of real drip system for in-situ measurement of K_h is neither advisable nor feasible. For accounting any fluctuations in dripper discharges due to pressure fluctuations in water supply a micro irrigation simulator need to be developed for experimentation. The system should have similar pressure range as that of normal drip irrigation system operating in the field. A point source field dripper system operated with micro-irrigation simulator is used in the present study for in-situ measurement of K_h. K_s value measured by other methods were compared with the value obtained by PSFDM.

Unsaturated hydraulic conductivity function (Kh) is a crucial soil parameter for designing drip irrigation systems. Various methods exist for measuring Kh, including soil particle distribution curves and the point source field dripper method (PSFDM). However, these methods have limitations in controlling dripper discharge. An experimental setup, a micro-irrigation simulator, was developed to simulate real field drip conditions for in-situ measurement of Kh.

MATERIALS AND METHODS

Theoretical Development

Hypothesis 1: The rate of change of water discharge within the saturated soil mass (dQ/dA) is directly proportion to saturated hydraulic gradient (i). The hypothesis can be mathematically written as below.

$$\frac{dQ}{dA} \propto i \tag{5}$$

Let us describe the soil moisture movement in unsaturated zone of the soil.

Hypothesis 2: The rate of change of water flux within unsaturated soil mass (dQ/dA) is directly proportional to hydraulic gradient (i), water flow geometrical shape (g_s) factor and air entry head ($h_e = 1/\alpha$). Mathematically it can be written as below.

$$\frac{dQ}{dA} \propto g_s \frac{i}{\alpha} \tag{6}$$

Combining Eqn. (5) and (6) one will arrive at common hypothesis of saturated and unsaturated flow under disc sourced water flow geometry.

$$\frac{dQ}{dA} \propto \left(i + g_s \frac{i}{\alpha} \right) \tag{7}$$

By introducing a proportionality constant λ in Eqn. (7) one will arrive at,

$$\frac{dQ}{dA} = \lambda \left(i + g_s \frac{i}{\alpha} \right) \tag{8}$$

The proportionality constant of water flow under disc source water flow geometry in the soil is nothing but saturated hydraulic conductivity (K_s).

Referring the water flow geometry under shallow circular pond on soil surface as shown in Fig. 1, let us consider a saturated wetting front at a distance of z below ground surface in the area where water infiltrates. Matric head suction of soil at this point is quite small, h_m. If the head of water pool on the soil surface is h, the head at soil surface becomes z+h and the head at the extreme end of wetted front of the soil becomes $z+h+h_m$. The head difference between a point at the depth z and a point on soil surface becomes $z+h+|h_m|$ and average hydraulic gradient (i) between the two points is,

$$i = \frac{z + h + |h_m|}{z} \tag{9}$$

Since h+|h_m| is quite small hence, the hydraulic gradient becomes unity (i=1). Substituting value i=1 and proportionality constant $\lambda = K_s$ in Eqn. (8), one will obtain following equation.

$$\frac{dQ}{dA} = K_s\left(1 + g_s \frac{1}{\alpha}\right) \tag{10}$$

The water flow under drip irrigations with various water flow geometries comprise of saturated and unsaturated flow. Moisture flow within the soil mass under point or disc source conditions is functions of K_s and K_h both. **Fig. 1** shows the definition sketch of water flow geometry of point source field dripper. Let us hypothesize the flow under saturated conditions first.

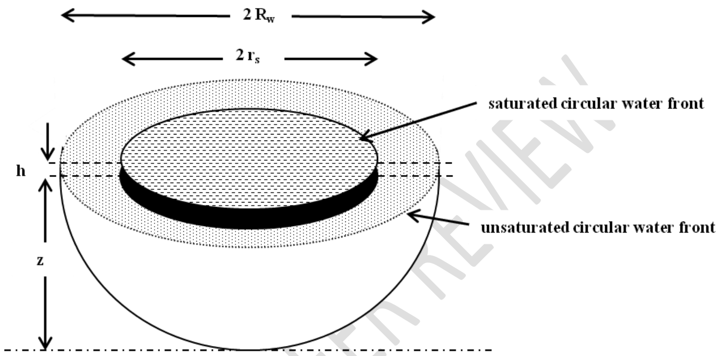

Fig. 1. Water flow geometry under shallow circular pond over soil surface.

The geometrical shape factor g_s is the ratio of characteristic length of saturated front (l_c) is the saturated diameter (2 r_s) of the circular pool of water to the average saturated area ($A_{avg\text{-}s}$) of the pool. The average saturated area of the circular pool is $(0+\pi r^2)/2 = \pi r^2/2$. Substituting these values in Eqn. (10) one will get,

$$\frac{dQ}{dA} = K_s\left(1 + \frac{2r_s}{\frac{0 + \pi r_s^2}{2}} \frac{1}{\alpha}\right) \tag{11}$$

which can be simplified as below.

$$\frac{dQ}{dA} = K_s\left(1 + \frac{4r_s}{\pi\alpha} \frac{1}{r_s}\right) \tag{12}$$

Rearranging the Eqn. (11) and integrating as below.

$$\int_0^Q dQ = K_s\left(1 + \frac{4r_s}{\pi\alpha} \frac{1}{r_s}\right)\int_0^A dA \tag{13}$$

$$Q = K_s\left(1 + \frac{4r_s}{\pi\alpha}\frac{1}{r_s}\right)A \tag{14}$$

Since A = r$_s^2$ above equation reduces to,

$$\frac{Q}{A} = \frac{Q}{\pi r_s^2} = K_s\left(1 + \frac{4r_s}{\pi\alpha}\frac{1}{r_s}\right) \tag{15}$$

$$q = K_s\left(1 + \frac{4r_s}{\pi\alpha}\frac{1}{r_s}\right) \tag{16}$$

Correction of Ks and α

Here q is the water flux of the saturated pool of the soil. A correction factor ξ_c should be introduced with K_s to correct the value of K_s against incomplete saturated front advance of the circular pool and difficulty in demarcating actual interface between saturated and unsaturated soil volume. First term once corrected it would be automatically taken into account for correcting α value. The final equation can be written as below.

$$q = K_c + \frac{4K_s}{\pi\alpha}\frac{1}{r_s} \tag{17}$$

Where,

$K_C=\zeta_c K_S$
ζ_c= correction factor for short duration test
The common range of ζ_c range between 1.00 to 1.25, hence taking value of ζ_c as 1.125.

Micro Irrigation Simulator

A micro-irrigation simulator was developed with the provision of increasing, decreasing or maintaining water pressure in the drip pipe line. There was pressure gauge to monitor pressure inside the chamber continuously. Water vessel has an airtight lid for closing the vessel. Filtered water is filled inside the vessel and closed with airtight lid. Pressure of the water vessel can be increased or decreased as and when required. A water outlet provided at the bottom of water vessel was connected with manifold for distributing water in one or more drip or pipe lines for simultaneous observations. A drip lateral of 11 mm diameter was connected to the manifold keeping outer outlet closed. The end of drip line was plugged off. Four drippers having discharge rate of 2.02, 4.04, 7.56, and 8.31 litres per hr were fixed on drip lateral at 0.50 m interval.

Experimental Site

Experiments were conducted at ICAR CSSRI- Regional Research Station, Lucknow. The site is located at 26^{o} 48$^{/}$13$^{//}$ N and 80^{o} 55$^{/}$ 25$^{//}$ E above 124 m above mean sea level. The climate of the area is semi-arid, subtropical and monsoonic receiving an average annual rainfall of 817 mm. Maximum rainfall is received between 23 to 40 standard weeks (June-October) amounting to 741 mm, which is 91% of the total annual rainfall. The remaining 9% rainfall is received between 41 to 19 standard weeks (November-May). The average annual evaporation is 1580 mm. During the rainy seasons between 23-40 weeks (mid-June-Oct) evaporation rate gradually decreases following rains. Further, up to 52 weeks (December), the evaporation decreases gradually due to low temperature. The period from 23-40 weeks (mid-June-mid October) remains is water surplus. The remaining period between 1-22 and 41-52 weeks remains in water deficit due in lower rains and higher evaporation rate. Mean maximum temperature of 39°C in the month of May and mean minimum temperature of 7.1°C in the mouth of January indicate a seasonal climate. Mean annual temperature and the mean winter soil temperature were 31°C and 18°C respectively. Thus, the temperature regime is hypothermic. The moisture regime of the soil is mainly ustic.

Inverse Auger Hole Method

For the measurement of in-situ saturated hydraulic conductivity of upper surface layer inverse auger hole technique was employed (**Hoorn, 1979**). An auger hole of 11.0 cm diameter was made to an average depth 50.5 cm. The hole was saturated for 24 hours by filling water in it intermittently up to the soil surface. Next day water was filled in the hole to the desired level and drop in water level was monitored for the in situ measurement of K_s. The equation used for making calculation for K_s is written as under.

$$K_s = 1.15 r \tan\alpha \tag{18}$$

$$\tan\alpha = \frac{\log_{10}\left(h_0 + \frac{r}{2}\right) - \log_{10}\left(h_t + \frac{r}{2}\right)}{t - t_0} \tag{19}$$

where,

r= radius of auger hole
t= time

Infiltrometer Test

Cylindrical infiltrometer of 30 cm diameter was driven in to the soil by 10 cm and crack around the cylinder was pressed filled. Water was filled inside the cylinder and drop in water levels was recorded. With the help of drop in water levels against measured time infiltration rates were calculated. Basic infiltration rate (K_s) was recorded when infiltration rate became almost the constant.

Measurement of Saturated Front Radii and Water Flux Density

For the measurement of in-situ K_h function at the site the land was levelled first to avoid water to flow in one direction. Steady state saturated fronts were created by running the system for 60 minutes in the field un-interrupted in soil of the premise of ICAR Central Soil Salinity Research Institute Regional Research Station, Lucknow. Saturated wetted fronts advances were measured at 1, 6, 16, 30, and 60 minutes. The saturated front moved quickly initially and rate of advance declined with the passage of time. A steady state conditions reached after about 60 minutes of time. Wetted front kept on advancing for long with time. By spotting the existence of a glossy appearance on the soil surface, the saturated front region was clearly distinguished. Average saturated front diameters were calculated for all four dripper discharges for estimation of K_h.

Water flux densities $\left(q = \frac{Q}{\pi r^2}\right)$ were calculated and plotted against the inverse of saturated front radii radii $\left(\frac{1}{r_s}\right)$. A straight line was obtained for which related slopes and intercepts were measured. The slopes and intercepts of the line can be also calculated by using linear regression protocol. In order to calculate saturated hydraulic conductivity (K_s) and scaling parameter ($\lambda=1/\alpha$) using following set of equations.

a) Shani Model

$$K_s = C \tag{20}$$

$$\alpha = \frac{4K_s}{\pi m} \tag{21}$$

b) Warrick Model

$$K_s = \frac{C}{0.836} \tag{22}$$

$$\alpha = \frac{K_s}{m} \tag{23}$$

Where,

C= intercept of plotted line
m = slope of plotted line

c) New Model

$$K_s = \frac{C}{1.125} \tag{24}$$

$$\alpha = \frac{4K_s}{\pi m} \tag{25}$$

The scaling parameter can be computed simply by inverting the value of α. For comparison, the saturated hydraulic conductivity values were also determined using inverse auger hole method.

RESULTS AND DISCUSSION

Saturated Front Advance

Saturated front diameter against dripper discharges of 2.02, 4.04, 7.56, and 8.31 litres per were measured as 20.17, 31.25, 41.25 and 50.25 cm while wetted front diameters were and 31.0, 44.12, 53.25 and 52.0 cm, respectively in the recently tilled normal soil at Central Soil Salinity Research Institute Regional Research Station Campus, Lucknow. The specific field dripper discharges against observed dripper discharge rates were 6.31, 5.26, 5.65 and 4.19 and cm h^{-1}.

Calculation of K_S and α by PSFDM

Fig. 2 shows the linear plots between point-source emitter discharge flux and inverse of observed saturated front radii. The slope of the plotted line was observed as 27.916 and the intercept was 3.60. The K_s values obtained by PSFDM-Shani, PSFDM-Warrick and PSFDM-New models were 3.60, 4.31 and 3.22 cm/hr, respectively. The value of α was calculated as 0.1643, 0.1544 and 0.1468 cm^{-1} by by PSFDM-Shani, PSFDM-Warrick and PSFDM-New models, respectively. The values of K_s calculated by PSFDM-New and PSFDM-Warrick deviated by -10.56 and 19.72% compared to the values obtained by PSFDM-Shani. The deviations of K_s value obtained by New PSFDM deviated least and fairly close to the values obtained by PSFDM-Shani. The α value obtained by PSFDM-New and PSFDM-Warrick deviated by -10.71 and -6.03%. All the values of α are fairly close to each other. The PSFDM derived from the new hypothesis gave the similar set of equations with correction factor and can be used for field applications.

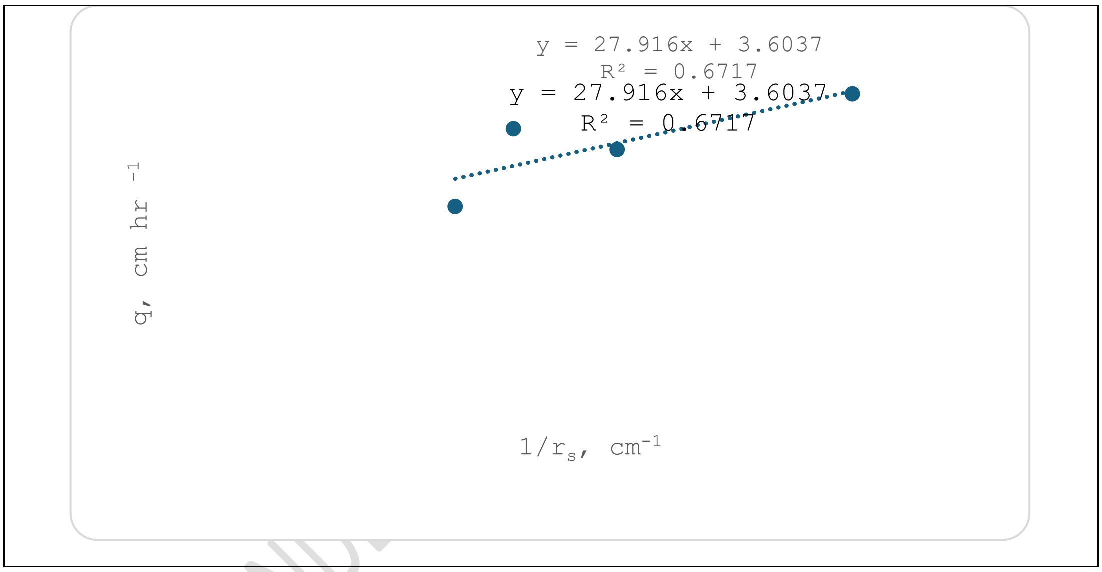

Fig. 2. Variation of flux density with inverse of saturated front radii.

Calculation of K_S by Inverse Auger Hole Method (IAHM) and Infiltrometer Test

In-situ value of K_s of the same site was calculated by using IAHM with flat bottom. The variation of (h_t+r/2) was calculated for against elapsed time (**Fig. 3**). The variation is linear having line slope (tan α) of 0.0021. The calculated value of K_s was 19.13 cm/day (0.79 cm/hr). The K_s values obtained by PSFDM-Shani, PSFDM-Warrick and PSFDM-New are 3.60, 4.31 and 3.22 cm/hr and 4.56, 5.46 and 4.08 times higher than those obtained by IAHM with flat bottom which is quite obvious as the PSFDM measures K_s value for completely tilled surface soil while auger hole measures K_s value for both tilled and untilled soil.

Infiltration rates were calculated by infiltrometer test as 210.24, 84.38, 65.88, 44.88, 35.29, 27.73 and 24.79 cm per day against time 1, 10, 20, 60, 120, 240 and 360 min (**Fig. 4**). The basic infiltration or K_s of the campus soil was recorded as 24.79 cm/day. The value recorded after 1440 min was 14.40 cm/day (0.60 cm/hr). The K_s value obtained by infiltrometer method deviated by -83.33, 86.07 and -25.00% compared to the K_s obtained by PSFDM-Shani, PSFDM-Warrick and IAHM, respectively. The K_s values obtained by IAHM and Infiltrometer method are quite comparable. Infiltrometer measures the K_s value of restricting soil layer i.e.

sub-soil which is having high pH, highly dispersed, compacted with very poor water transmission characteristics. IAHM measures K_s value of deeper soil profile which is having high pH, highly dispersed, compacted with very poor water transmission characteristics too.

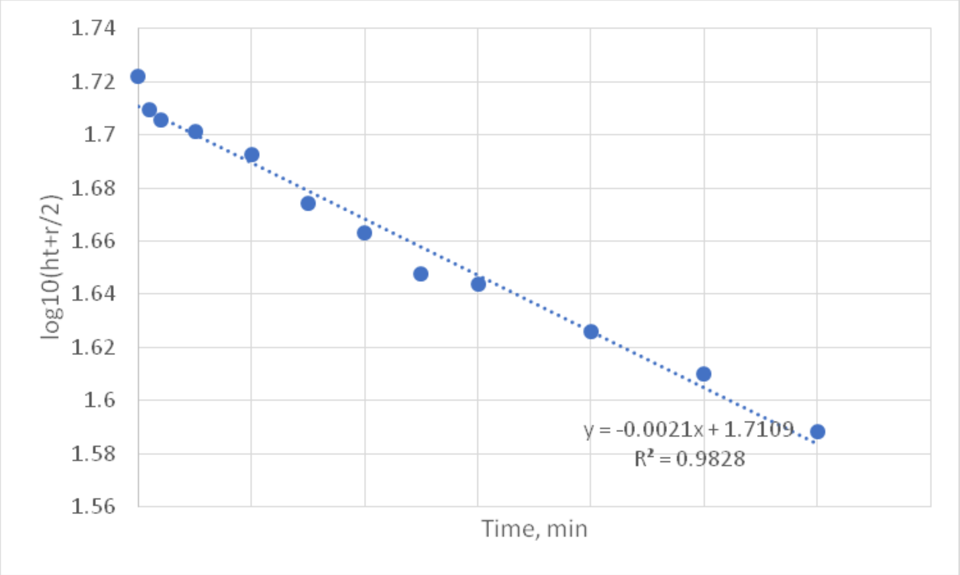

Fig. 3. Variation of $\log_{10}(h_t+r/2)$ against elapsed time.

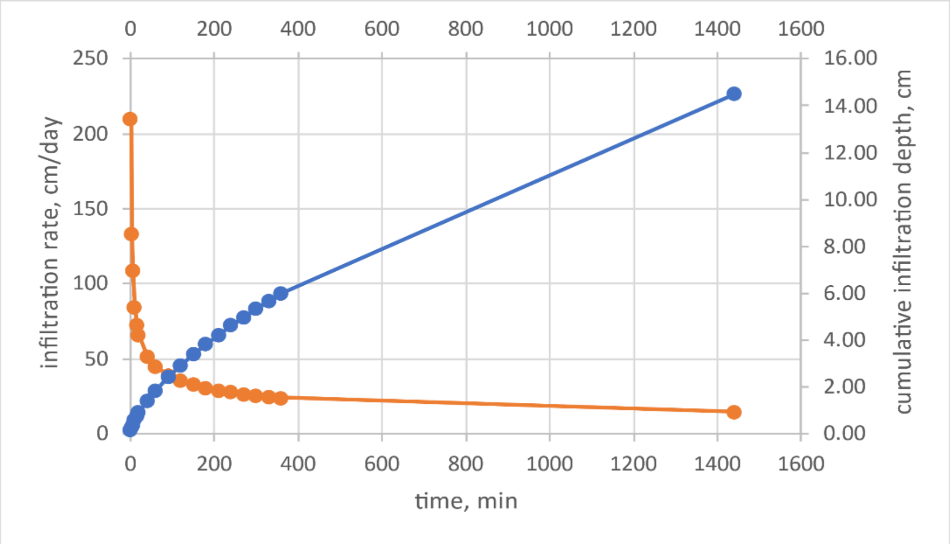

Fig. 4. Measured values of infiltration rate and K_s by infiltrometer test.

The measured values of K_s in ascending order are 0.60, 0.80, 3.22, 3.60 and 4.31 cm/hr by infiltrometer, IAHM, PSFDM-New, PSFDM-Shani and PSFDM-Warrick respectively. Similarly α in ascending order are 0.1467, 0.1544 and 0.1643 cm^{-1} by PSFDM-New, PSFDM-Warrick and PSFDM-Shani, respectively. The values of K_s and α are fairly close to each other for all PSFD Methods indicating good validity of new PSFD method. Infiltrometer test measure the K_s value of the most impeding soil layer that the soil layer immediately below the tilled soil while inverse auger hole method measure K_s value of deeper soil profile which is untilled and compacted since long. Infiltrometer and inverse auger hole methods fail to capture actual K_s signature of surface soil. Hence any change in plough zone due to soil reclamation, plowing, inter-culture operation, weeding, irrigation or compaction goes un detected. PSFD Method has capability to capture signature changes in soil transmission characteristics of the soil due to any alteration. Newly developed micro-irrigation simulator must be used for in-situ measurement of K_h more precisely compared to the traditional small supply head drip operating system.

CONCLUSIONS

Available in-situ as well as laboratory methods for estimating K_h have serious associated drawbacks. PSFDM of Shani and Warrick have been used by the researchers for in-situ measurement of K_h in combination with a low pressure head responsive experimental set up. Low pressure head variation in supply tank is subjected to changes in dripper discharges due to any bend or turn or torsion in the plastic pipe lines. Higher pressure in dripper line would minimize such detrimental discharge variations while experimentation. A new hypothesis was used to develop a new equation for in-situ measurement of K_h for correcting against saturated front diameter due to short time and invisibility of the interface. A micro-irrigation simulator was developed for coupling PSFD to it for the measurement of K_h in the field. The set up worked quite well in the field. K_s and α values obtained by PSFDM of Shani et al. (1985) was 3.60 cm/hr and 0.1643 cm^{-1}, by PSFDM of Warrick (1985) 4.31 cm/hr and 0.1544 cm^{-1} and newly developed PSFDM as 3.22 cm/hr and 0.1468 cm^{-1}, respectively. All the method consistently gave the comparable values of K_s and α for field applications. Inverse auger hole method resulted K_s value of 0.60 cm/hr while infiltrometer test gave a value of 0.79 cm/hr which is quite close to each other. Newly developed model and experimental set can be satisfactorily used for field application.

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