MATHEMATICAL MODEL FOR MITIGATING THE COMMUNITY SPREAD OF COVID – 19 IN AFIKPO NORTH LGA OF EBONYI STATE, NIGERIA

Abstract

To mitigate the dynamics of covid-19 in Afikpo North Local Government Area of Ebonyi State in Nigeria, a mathematical model which incorporates the key compartment and parameters regarding covid-19 in Afikpo LGA. of Ebonyi State Nigeria is formulated. The basic reproduction number is obtained using the next generation matrix, while the disease free equilibrium is found to be locally stable and globally asyptically stable when Ro < 1. The model in calibrated using data obtained from Nigeria centre for Disease Control (NCDC) in Ebonyi State and key parameters of the model are estimated, sensitivity analysis is carried out to investigate the influence of the parameters in mitigating the disease.Our analytical and simulation results in Ebonyi State between March 2023 and October 19, 2023, are in good agreement.Mitigation strategies and its effectiveness in reducing the spread of covid-19 across Afikpo North LGA are considered.

Introduction

The Corona Virus pandemic of the 2019, popularly known as covid-19 has become a household name globally. Sadly, the virus mortality and virulence is still a stalk reality starring the world in the face. The virus that was first reported in the city of Wuhan in China towards the end of the year 2019, ravaged the entire world in less than a year.Since the advent of covid-19 pandemic, the entire world has not fully return to normalcy from its mortality, economic downturn and a reset to new pattern of living. The ripple effect of the pandemic in both developed and developing nations are still feelable.

Africa being populated with majority of developing nations is not speared from the negative economic downturns and morality caused by covid-19.

Nigeria as a nation is considered the giant of Africa due to its large population, economic prowess, with great number of illustrious citizens excelling in different facets of life scattered in diaspora and locally, is one of the few African nations greatly hit by the sting of covid-19. Nigeria recorded its first case of covid-19 on 27th February, 2020 when first discovered from an Italian immigrant [1]. Since then, the virus has spread to every state in the country with recorded number of infected persons as at October 2023 to be 267,146 with total mortality of 3,155, while total number of recovery was recorded as 259,953 and active 3,567 [2].

Ebonyi State is one of the least developed states in the eastern region of Nigeria amidst states like Enugu, Anambra, Abia and Imo. Ebonyi State is the newest created state in the region, created on 1st October 1996 and carved out of the present day Abia and Enugu States. [3]. Ebonyi State is made up of 13 LGAs with the state capital at Abakaliki. Afikpo North is the second largest LGA. It has its own culture and traditional ways of celebrating during ceremonies. Ebonyi state, Nigeria is also adversely affected by the menace of covid-19 pandemic. Afikpo North LGA being second largest heterogeneous town in the state has its own share of the pandemic too. The major contending problem now in fighting covid-19 in Afikpo north LGA is how to mitigate human-to-human transmission. The local government is known for its customary exuberant burial ceremonies and weddings that maintaining social distancing protocol is almost impossible to be adhered to. Their mentality and beliefs on covid-19 is timid. This is partly responsible for the increase in the new number of recorded infections. Many of the people in the community still believe that the disease does not kill Africans while some believe that 'covid-19 is just a scam to enrich some cabal' [4]. Many are still organizing/attending public gatherings and some are still not adhering to basic safety precautions of wearing of face-mask and washing of hands with running water or use of hand sanitizers. [5], [6], [7]. Many researchers have used mathematical models to assess the impact of the non-pharmaceutical control measures in mitigating the spread of covid-19. From the literature the authors were able to assess, non has reduced their study to consider the effect of NPIs to local community with different belief systems and cultures like Afikpo north local government in Ebonyi state, Nigeria. Therefore, this study assessed the impact of non-pharmaceutical control measures like social distancing, use of face-masks and incorporating collective, periodic burial and wedding ceremonies to spread persons who would have come together in one ceremony, thereby enhancing social distancing and mitigating community spread of covid-19 in Afikpo North LGA of EBonyi State, Nigeria.
The rest of the paper is organized as follows:
Section 2 is devoted to the formulation of the models,
Section 3 is for model analysis and parameter estimation
Section 4 is devoted to numerical simulation displayed graphically to show the effect of testing rates and intervention measures of covid-19 dynamics within the community.
Section 5 is devoted to conclusion.

2. Model Formulation

2.1 Model Assumptions

Before we present our deterministic model, we first of all present the assumptions on which our model formulation will be anchored viz:

i. We assumed that vaccinated individual can be re-infected with covid-19 due to loss of vaccine-acquired immunity, but at a slower rate than those not vaccinated since individual with multiple re-infections developed resistance to the virus quicker.

ii. All recruits into the population are susceptible to covid-19

iii. Recruitment into the population is via birth and immigration

iv. Deceased infectious humans can still infect susceptible humans before and during funerals.

v. There is vital dynamic attributed to the inflow of immigrants, as well as natural deaths. This allows a demographic process to take place.

vi. Infection is acquired through the eyes, inhalation, and ingestion of the virus from infected aerosols and contaminants

vii. For an individual to become infectious, he/she must pass through the latent/incubation stage.

2.2 Model Formulation

Here the deterministic mathematical model formulation of covid-19 is presented. Our total population N at time 't' is partitioned into compartment; susceptible 's', vaccinated 'v', exposed 'E', Quarantined 'Q', infectious 'I' and the Removed (that is the unburied infectious corpses) compartment 'R'. Thus the human population at any given time 't' is given by

$$N(t) = S(t) + V(t) + E(t) + Q(t) + I(t) + R(t) \qquad (1)$$

Humans are recruited into the susceptible compartment 'S(t)' through immigration and birth at a constant rate of 'Λ'. Also, the susceptible compartment gained population from the proportion 'θ' of recovered infectious humans and from the proportion 'γ' of recovered quarantined humans at the rates 'k_1' and 'k_2' respectively. While a proportions 'p' exit the compartment after being quarantined at the rate of 'α'. Also the unquarantined proportion '(1-p)' of the susceptible exit the compartment due to covid-19 infection with the force of infection 'λ'.

$$\lambda = \frac{\beta_1 I + \beta_2 D}{N}$$

Where β_1 and β_2 are the transmission rates from infectious humans and corpse respectively.

This yield

$$\frac{dS}{dt} = \Lambda + \theta I k_1 + yQk_2 - \rho\alpha S - (1-\rho)\lambda S - \mu S$$

The vaccinated compartment gain humans from the proportion of susceptible humans that were vaccinated at the rate of ‘α’ by yielding to government efforts of media sensitization, health enlightenment and availing the vaccine free of charge. Since vaccinated humans can still be re-infected with covid-19 due to weak vaccine-acquired immunity, thus the weak vaccine-acquired immunity by the vaccinated humans w(t)ε (0,1), slows the force of infection ‘λ’ and is time dependent. Thus humans exit the compartment at the rate ‘wλ’ to the exposed compartment. This yield

$$\frac{dV}{dt} = \rho\alpha S - w\lambda V - \mu V$$

The exposed compartment ‘E(t)’ gains population from covid-19 infected humans (both vaccinated humans with weak vaccine-acquired immunity and unvaccinated humans) at the rate of ‘wλ’ and ‘λ’ respectively. A proportion of humans ‘ϕ’ exit the compartment as a result of being quarantined at the rate of ‘τ’ due to contact tracing, immigrating from high risk region or voluntarily self-submission after noticing some symptoms. While the remaining unquarantined proportion ‘(1-ϕ)’ becomes infectious and exit the compartment at the rate ‘σ_1’. This yield

$$\frac{dE}{dt} = (1-\rho)\lambda S + w\lambda V - (1-\phi)\sigma_1 E - \phi\tau E - \mu E$$

The quarantined compartment ‘Q(t) gains population from the proportion ‘ϕ’ of exposed quarantined humans at the rate ‘τ’. A proportion ‘γ’ recovers at the rate ‘k_2’ due to boost in immunity and exit the compartment back to the susceptible compartment. While the remaining proportion ‘(1 - γ)’ becomes symptomatic at the rate of ‘σ_2’ and exit the compartment to the infectious compartment. This yield

$$\frac{dQ}{dt} = \phi\tau E - \gamma k_2 Q - (1-\gamma)\sigma_2 Q - \mu Q$$

The infectious compartment ‘I(t)’ gains population from a proportion ‘1 - ϕ’ of the exposed unvaccinated humans that developed symptoms at the rate ‘σ_1’.

Also, from the proportion ‘$(1 - \gamma)$’ of quarantined humans that becomes symptomatic at the rate ‘σ_2’. A proportion ‘θ’ of infectious humans recovered due to boost in immunity at the rate of ‘k1’ and exit to susceptible compartment. While the remaining proportion (1 - θ) eventually died from covid-19 at the rate of ‘d’ and exit to the removed/corpse compartment. This yield

$$\frac{dI}{dt} = (1 - \phi)\sigma_1 E + ((1 - \gamma)\sigma_2 Q - \theta I k_1 - (1 - \theta)dI - \mu I$$

The removed/corpse compartment ‘R’ gain population from the proportion ‘(1 - θ)’ of infectious humans that died from covid-19. Since the corpse of a person killed by covid-19 is infectious, thus the corpse will be exited from the compartment at the rate of ‘η’ through proper safety burial from specialize well-knitted mortician(s) or medical personnel(s).

The system diagram for the transmission of Covid-19 and the assumptions of the model are given below.

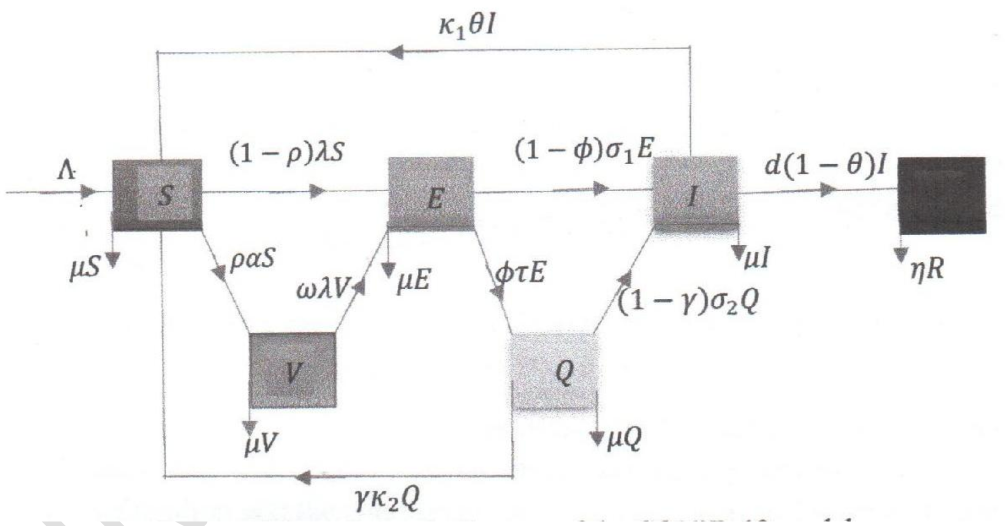


Figure 1: The systematic diagram of the COVID-19 model

Table 1: Description of Parameters of the Model

Variables	Description
A	Recruitment rate of susceptible humans
ρ	Proportion of vaccinated humans
1- ρ	Proportion of humans not vaccinated
λ	Force of infection
ω	Resistance rate of covid-19 due to weak vaccine-acquired immunity
φ	Proportion of quarantine humans

$1 - \varphi$	Proportion of humans unquarantined
τ	Quarantine rate
σ_1	Progression rate from exposed to infectious
σ_2	Progression rate from quarantine to infectious
d	Covid-19 induced death rate
γ	Proportion of recovered quarantine humans
$1 - \gamma$	Proportion of infectious quarantined humans
μ	Natural death rate
K_1	Recovery rate of infectious humans
K_2	Recovery rate of quarantined humans

From the schematic diagram, the model equation is derived below:

1. $\frac{dS}{dt} = \Lambda + \theta I k_1 + yqk_2 - (\rho\alpha + [1-\rho]\lambda + \mu)s$

2. $\frac{dv}{dt} = \rho\alpha s - (w\lambda + \mu)\, v$

3. $\frac{dE}{dt}\ (1-\rho)\lambda s + w\lambda v - ([1-\varphi\ ]\sigma_1 + \varphi\tau + \mu)E = 0$

4. $\frac{dQ}{dt}\varphi\tau E - (yk_2 + (1 - y)\sigma_2 + \mu)Q$

5. $\frac{dl}{dt} = (1 - \varphi)\ \sigma_1 E + (1-y)\ \sigma_2 + \mu)Q$

6. $\frac{dR}{dt} = (1 - \theta)\ dl - \eta R$

With s(0)>0,(0)>0,E(0)>0,Q(0)>0,I(0)>0 and R(0)>0, as the initial conditions.

STABILITY ANALYSIS OF THE EQUILIVRIUM STATE OF A SOCIETY FREE FROM COVID -19 (SFC)

For the model to be stable, the model behavior can be examined near the equilibrium solution. This is only obtainable if there can be total eradication of the case of covid-19 in the society under consideration and the condition that will enable such reality shall be determined as clearly as done below.

1. $\wedge + \theta IK_1 + YQK_2 - (\rho\alpha + [1-\rho]\lambda + \mu)s = 0$

2. $\rho\alpha s-(w\lambda+\mu)\ v=0$

3. $(1-\ \rho)\ \lambda s + w\lambda v-([1-\ \varphi)\ \sigma_1+\varphi\tau+\mu)\ E = 0$

4. $\varphi\tau E-(yk_2+ (1-y)\ \sigma_2+\mu)\ Q$

5. $(1-\ \varphi)\ \sigma_1 E+ (1-\ y)\ \sigma_2 Q– (\theta k_1+ [1-]\ d+\mu)\ l$

6. $(1-\theta)dl-\eta R=0$

The above implies that,

$$\frac{dS}{dt} = \frac{dV}{dt} = \frac{dE}{dt} = \frac{dQ}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

Since the society is assumed to be free from the disease (Covid-19). Thus we chose a point. $E^0(S^0, V^0, E^0, Q^0, I^0, R^0)$ as the Disease-Free Equilibrium state which we chose to write as SFC (Society Free from Covid-19) otherwise recognized as DFE in some works.

Clearly, we deducted that S = $\Lambda/_{\mu} \equiv \left(\Lambda/_{\mu}, 0,0,0,0,0\right) \in R^6$

JACOBIAN TRANSFORM OF THE MODEL EQUATIONS

Now, by Jacobian matrix method to determine the Eigen-values, we need to linearize these system of equations to get the following:

The sensitivity analysis of this model parameter is studied by considering the basic Reproduction number R_0 which is given as p is the spectral radius and $G=FV^1$ is the next generation matrix. The reproduction number of case one of covid-19 will generate an average over the course of its introduction in a given area that has not been exposed to the disease before . this metric is used to determine whether or not a victim becomes so infectious in an identified area, then the computation of R^0 must account for this tendency . we simply judge this by considering when $R^0<1$ or when $R^0>1$. If R^0 is less than 1. Then the transmission/contact rate is low implying that the control/parameter/is /are effective whereas the reverse is the case otherwise.

Infact, with the SFC of the equilibrium state of this model, the equilibrium state of this model, the R^0 of the covid-19 under consideration has been found that

$E^0(S^0, V^0, E^0, Q^0, I^0, R^0) \varepsilon R^6 = (\frac{\Lambda}{\mu}, 0,0,0,0,0)$. The R^0, can be calculated with the help of the next generation matrix as ready explained.

In short, F= $[\frac{\sigma f_1}{\sigma x_j}]$ and v = $[\frac{\sigma v_j}{\sigma x_j}]$ where the $f_j^{(s)}$ are the incoming victim of the disease into

Compartment and the $V_J^{(S)}$are the transfers from one compartment to another, we deduce as follows:

At the SFC, the susceptible and the removed compartment are excluded . The $F_J^{(S)}$ becomes (1-ρ)λs which is the interactional force that brings about the exposed class.

Subsequently, the other compartment becomes zero each in their case.

The$v_j^{(s)}$are deduced as follows:

Vaccinated class (V) = - ($w\lambda$ +μ) v

Exposed class (E)= -([1-φ]δ_1+φ_r +μ)E

Quarantine class (Q) = - (-k_2+[1-y)δ_2+μ)Q

Infectious class (1) = - (θk_1+ (1-θ)d+μ)l

At the SFC, it is assumed that ρ is 1 implying that everybody is vaccinated or that no body contacted the disease which makes ρ to be zero (0) and we can write as follows:

$$F_J = \begin{pmatrix} \lambda s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ since } \rho\text{=0.(Nobody is vaccinated)}$$

$$\begin{pmatrix} \frac{df_1}{ds} & \frac{df_1}{v} & \frac{df_1}{dE} & \frac{df_1}{dQ} & \frac{df_1}{dI} & \frac{df_I}{dR} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{df_6}{ds} & \frac{df_6}{dv} & \frac{df_6}{dE} & \frac{df_6}{dQ} & \frac{df_6}{dI} & \frac{df_6}{dR} \end{pmatrix}$$

Where j is the jacobian matrix. On evaluating the jacobain, we shall have:

$$\begin{pmatrix} T_1 & 0 & 0 & YK_2 & \theta K_1 & 0 \\ \rho\alpha & T_2 & 0 & 0 & 0 & 0 \\ (1-\rho)\lambda & \omega\lambda & T_3 & 0 & 0 & \varphi\tau \\ 0 & 0 & \omega\pi & T_4 & 0 & 0 \\ 0 & 0 & (1-\varphi)\sigma_1 & (1-\gamma)\sigma_2 & T_5 & 0 \\ 0 & 0 & 0 & 0 & (1-\theta)d & \eta \end{pmatrix}$$

Where $T_1 = -(\rho\alpha+[1-\rho]\lambda+\mu)$

$T_2 = -(w\lambda+\mu)$

$T_3 = ([1-\varphi]\sigma_1+\mu)$

$T_4 = -(yk_2 + (1-y)\ \sigma_2+\mu$

$T_5 = -(\theta k_1+ (1-\theta)\ d+\mu)$

We need the characterized equation of the Jacobian Transformation which is evaluated at the SFC point thus:

$$
\upsilon - \lambda_1 1 = \begin{vmatrix} T_1 - \lambda_1 & 0 & 0 & YK_2 & \theta K_1 & 0 \\ 0 & T_2 - \lambda_2 & 0 & 0 & 0 & 0 \\ 0 & \omega\lambda & T_3 - \lambda_3 & 0 & 0 & \varphi\tau \\ 0 & 0 & \varphi\tau & T_4 - \lambda_4 & 0 & 0 \\ 0 & 0 & (1-\varphi)\sigma_1 & (1-y)\sigma_2 & T_5 - \lambda_5 & 0 \\ 0 & 0 & 0 & 0 & (1-\theta)d & \eta - \lambda_6 \end{vmatrix}
$$

The characteristic equation is then given as

$$
f(\lambda_i) = [\lambda_1 + T_1][\lambda_2 + T_2][\lambda_3 + T_3][\lambda_4 + T_4][\lambda_5 + T_5][\lambda_6 + \eta]
$$

At this point, we can conclude that these system of Equations with the parameters defined on them imply that the model is asymptotically stable.

SENSITIVITY ANALYSIS

Basic Reproduction Number of Covid-19

$$
Therefore\ F = \begin{bmatrix} 0 & \beta_1 & \beta_2 & \beta_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Where $\beta_1, \beta_2, \beta_3$ are the compartmental contact rates.

$$
\begin{pmatrix} -(\omega\lambda + \mu)v \\ -([1-\varphi]\sigma_1 + \varphi\tau + \mu)E \\ -(\gamma k_2 + [1-\gamma]\sigma_2 + \mu)Q \\ -(\theta k_1 + [1-\theta]d + \mu)I \end{pmatrix}
$$

$$
\text{Equivalently}\,\mathrm{Vj} = \begin{pmatrix} T_1 V \\ T_2 E \\ T_3 Q \\ T_4 I \end{pmatrix}
$$

$$
V = \begin{bmatrix} T_1 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 \\ 0 & 0 & T_3 & 0 \\ 0 & 0 & 0 & T_4 \end{bmatrix}
$$

With the help of Maple software, we were able to get R_0 using $G = FV^{-1}$. The normalized forward sensitivity index of R0 that depends differentiable on a parameter say ρis defined by $Y_{\rho}^{R_0} = \frac{dR_0}{d\rho}.\frac{\rho}{R_0}$ which is used to derived the analytical expressions with respect to each parameter in the model and this assisted us in obtaining our sensitivity index table that is used in our simulation of the research work as shown below.

Table 2 : PARAMETER VALUES USED IN THE SIMULATION OF THE MODEL

S/N	Parameter	Value	Source
1.	Λ	1% of the population	Primary and Secondary data
2.	λ	0.15	Primary and Secondary data
3.	ρ	0.08	Calculated
4.	ω	0.04	Primary and Secondary data
5.	φ	0.02	Primary and Secondary data
6.	τ	0.02	Calculated
7.	$\sigma_1\sigma_2$	ϵ [0, 1]	Calculated
8.	d	ϵ [0, 1]	Calculated
9.	γ	0.02	Calculated
10.	μ	$\frac{1}{53.6}$	Assumed
11.	k_1k_2	ϵ [0, 03,0.06]	Calculated
12.	θ	0.02	Calculated
13.	η	0.05	Calculated
14.	$\beta_{1,}\beta_{2,}\beta_{3,}$	ϵ [0, 1]	Calculated

Figs. 2-8. SIMULATED GRAPHICAL PLOTS FROM STATE VARIABLES

FIG. 2:

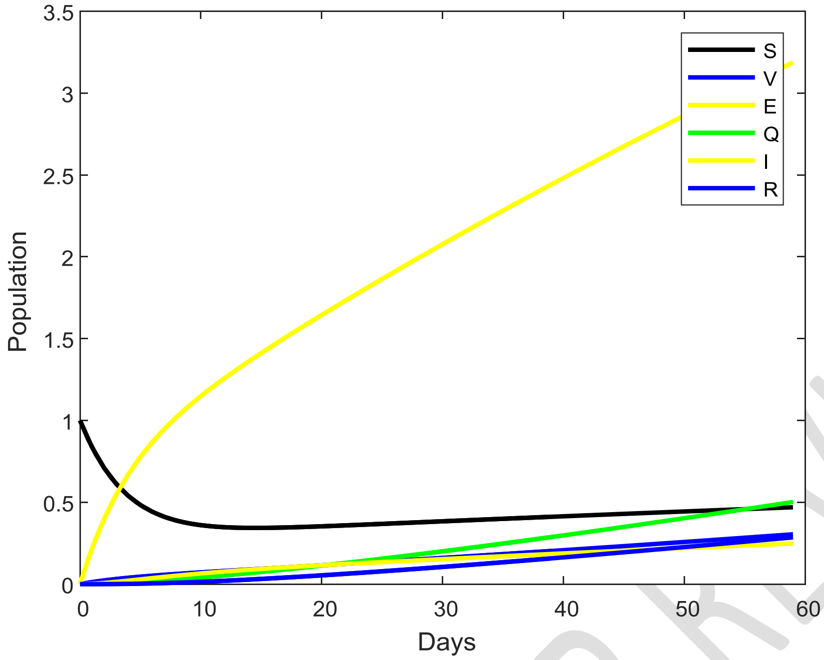


FIG. 3

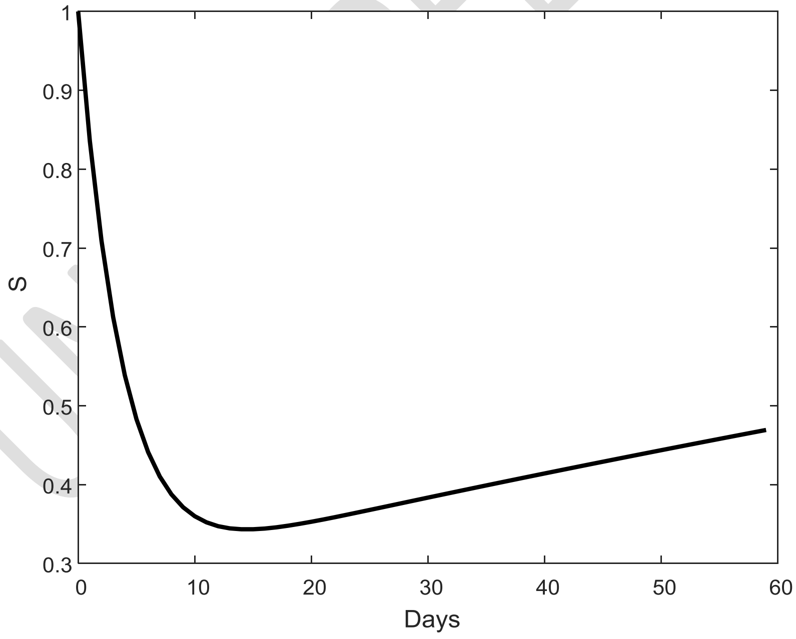


FIG. 4

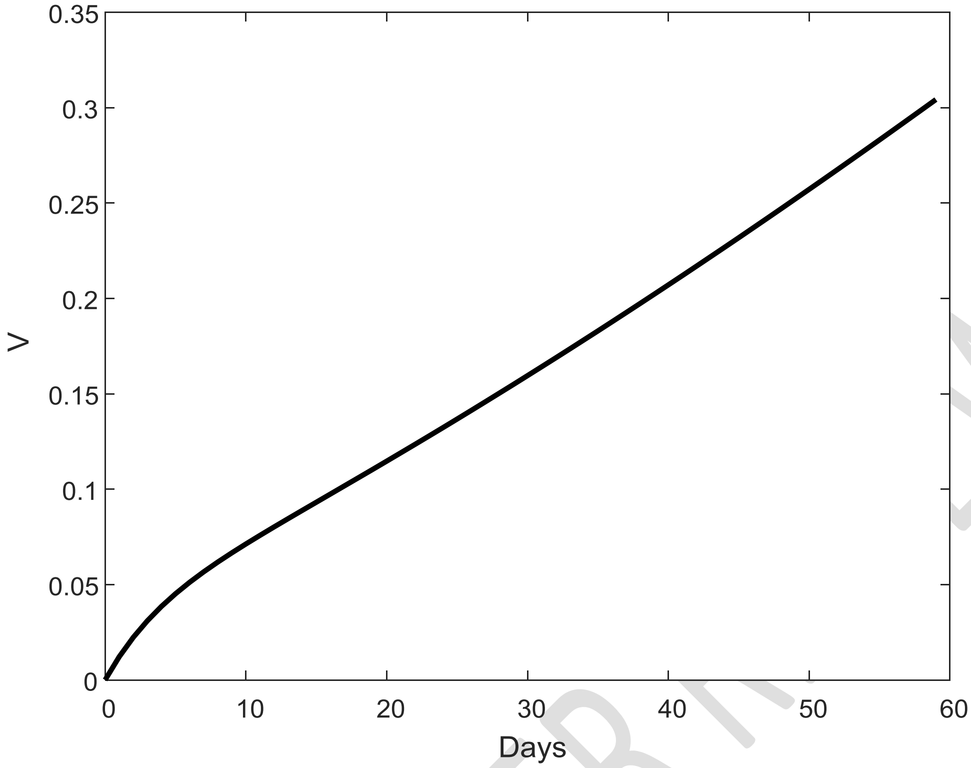

0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
V
0
10
20
30
40
50
60
Days

FIG. 5

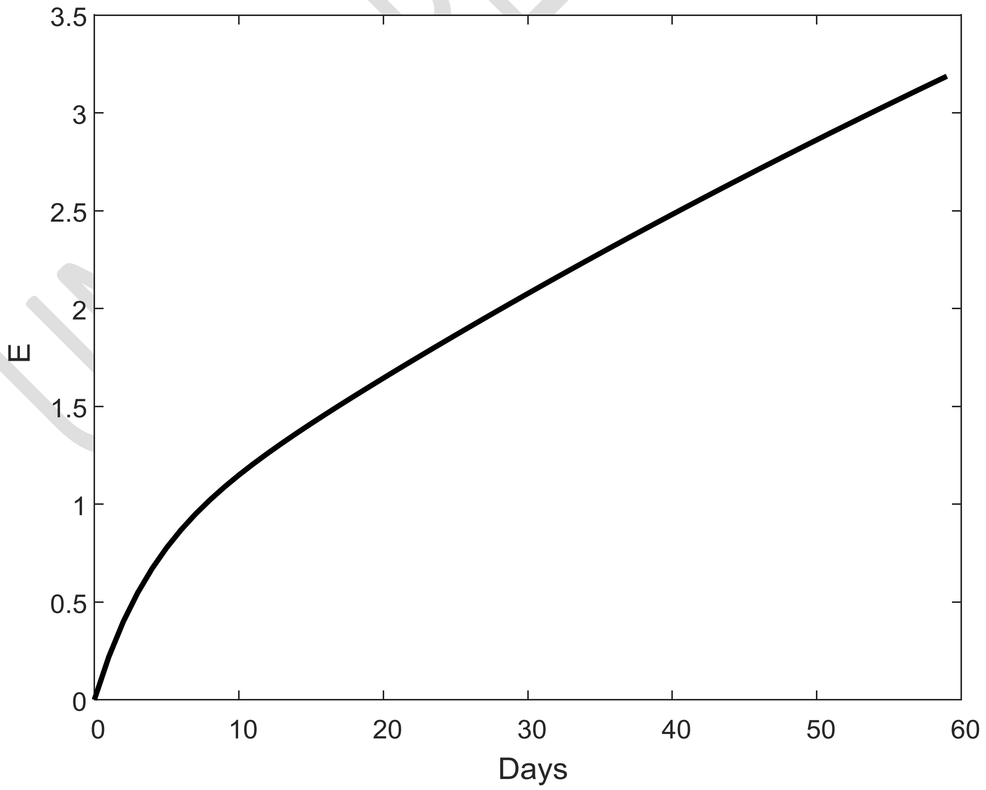

3.5
3
2.5
2
1.5
1
0.5
0
E
0
10
20
30
40
50
60
Days

FIG. 6

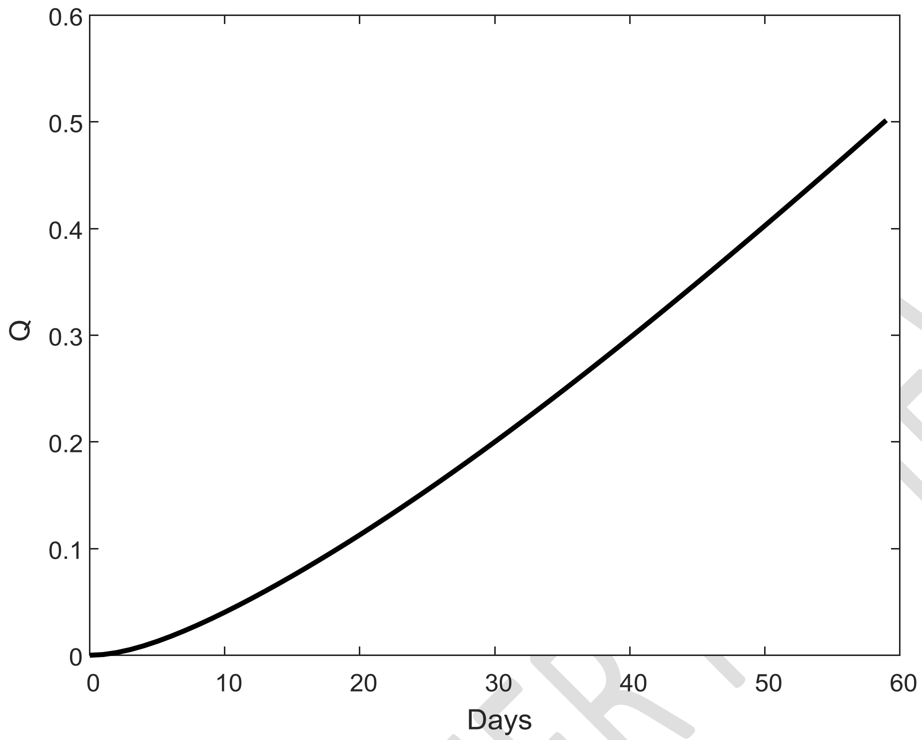

0.6
0.5
0.4
0.3
0.2
0.1
0
Q
0
10
20
30
40
50
60
Days

FIG. 7

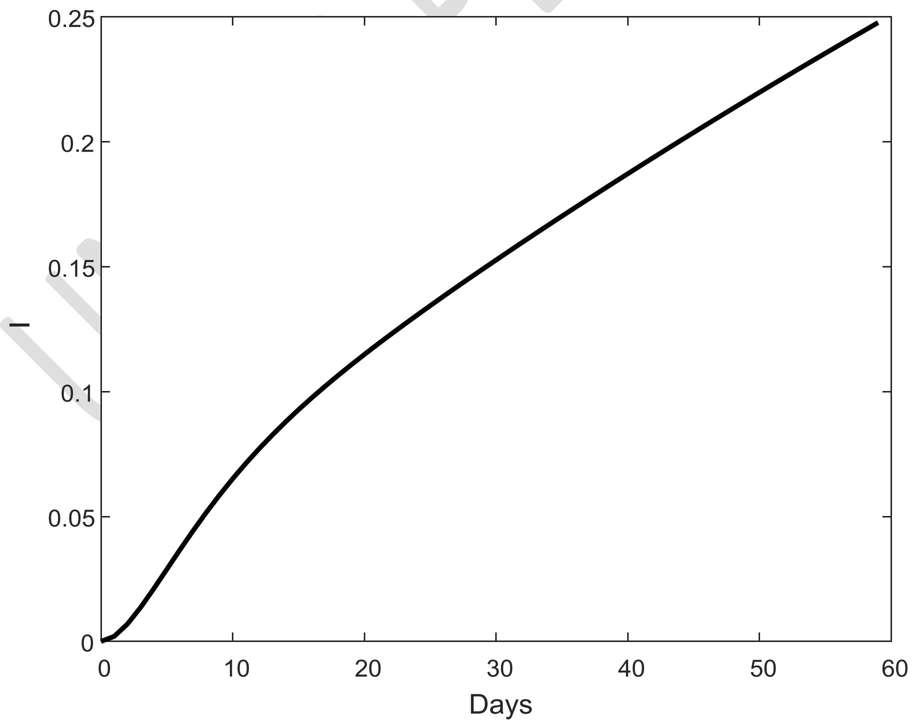
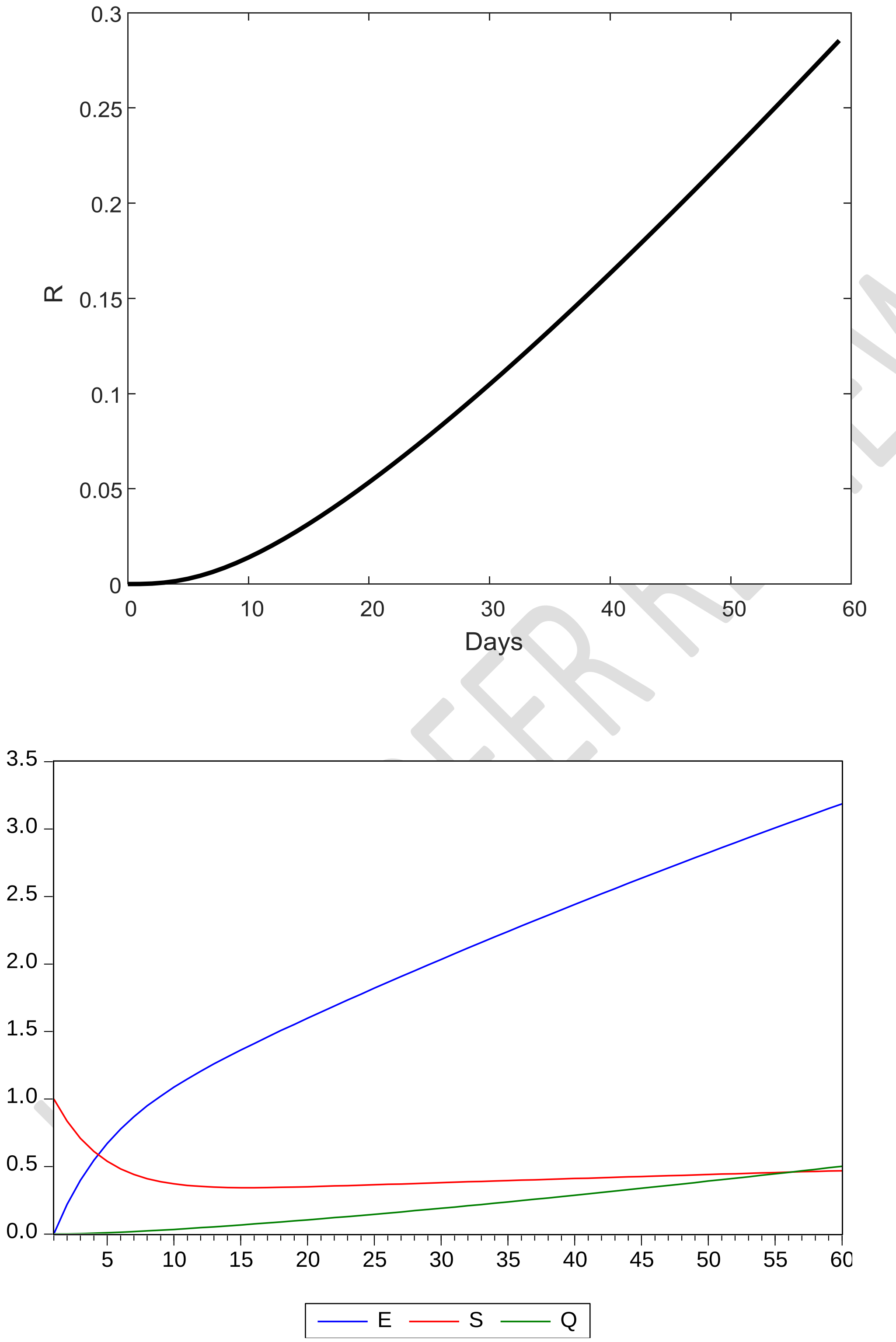
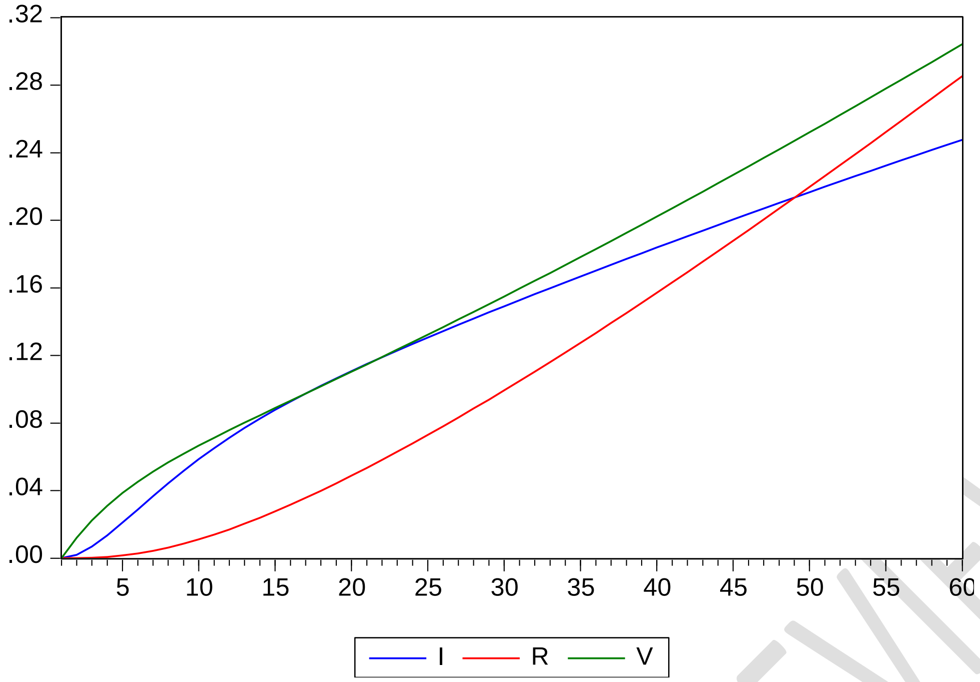
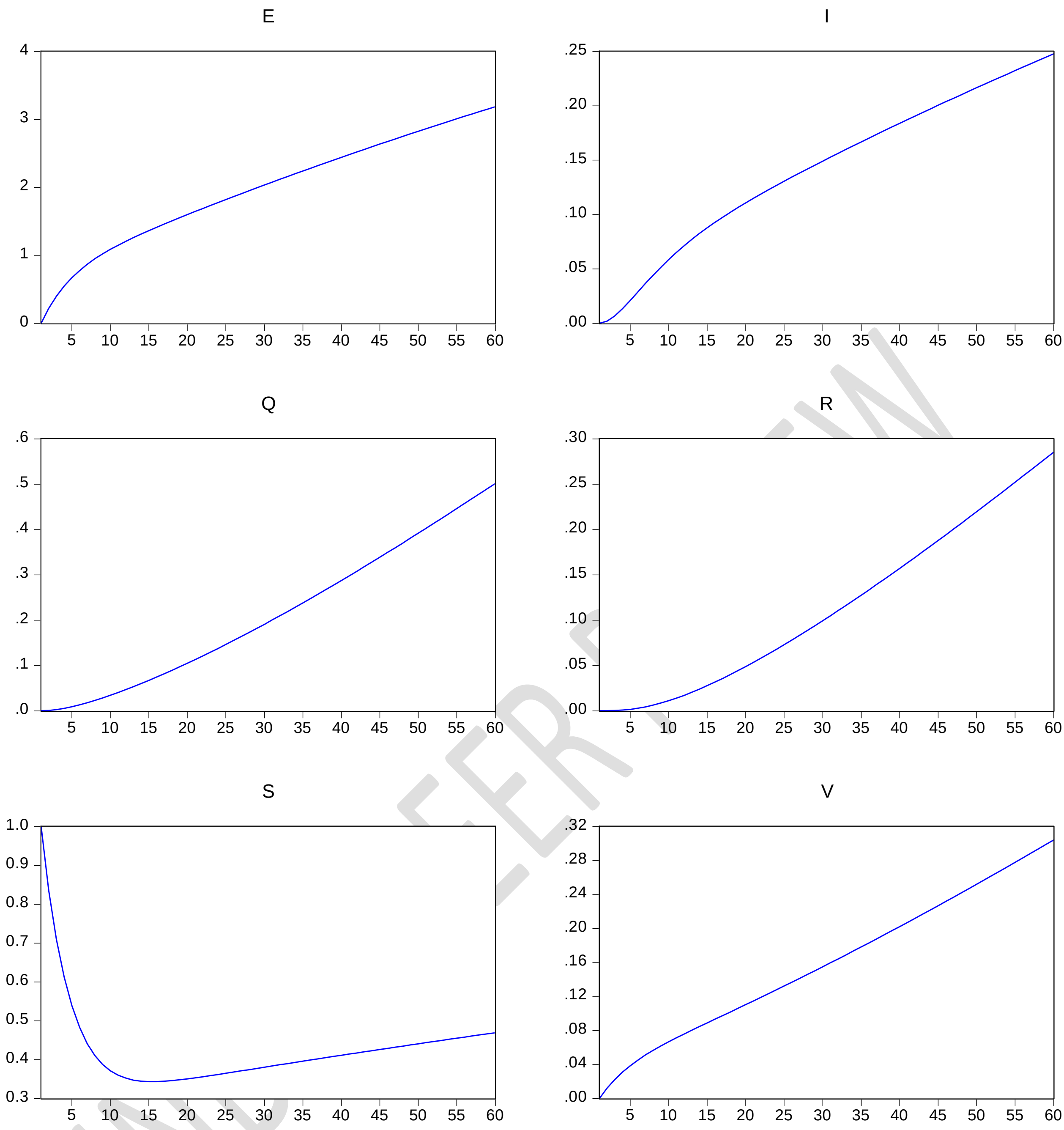

0.25
0.2
0.15
0.1
0.05
0
I
0
10
20
30
40
50
60
Days

FIG. 8



R
0.3
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
60
Days
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
5
10
15
20
25
30
35
40
45
50
55
60
E
S
Q


.32
.28
.24
.20
.16
.12
.08
.04
.00
5
10
15
20
25
30
35
40
45
50
55
60
I
R
V



Conclusion

A deterministic model for mitigating community spread of Covid-19 in Afikpo North Local Government Area of Ebonyi State is formulated in this study. The theoretical study carried out on the basis of stability analysis showed that the model has a globally asymptically stable disease-free equilibrium if the basic reproduction number of new corona viruses is less than one. The model is used for assessing the community-wide impact of the various control and mitigating strategies implemented in the Local Government Area, to effectively curtail the pandemic. The model was parameterized using Covid-19 data published by the Nigeria centre for Disease Control (NCDC) in Ebonyi State for sixty (60) days.

It is found that based on limited testing facilities and lack of man-power of health personnel, it will be easier to achieve strict compliance with social distancing and wearing of face masks with the periodic ceremonies (wedding and burial), Covid -19 will be eradicated in the Local Government Area since the basic reproduction number R_0, will be kept below one. Implementation of the recommended non-pharmaceutical interventions (NPIs) can be greatly achieved through educating the citizens, traditional rulers, religious leader and enlightenment campaign by health workers.

REFERENCES

1. Ibio, E., Sharomi, O., Ngonghala, C. and Gumel, A.B. Mathematical Modelling and Analysis of Covid-19 Pandemic in Nigeria. Mathematical Biosciences and Engineering, 17(6), pp. 7192 – 7220, 2020.

2. NCDC, COVID-19 Situation Report. https//ncdc.gov.ngsitreps, Accessed on April 1, 2023.

3. Nkechi Emma-Echiegu; Ebonyi People and Culture 2018. Induction Programme for International Students.

4. Okuonghae, D. and Omame, A. Analysis of a Mathematical Model for COVID-19 Population Dynamics in Lagos Nigeria, Chaos, Solutions and Fractals, 139(2020): 110032, 2020.

5. Ibrahim, O.M. and Ekundayo, D.D., Covid-19 Pandemic in Nigeria: Misconception among individuals, impact on Animal and the role of Mathemtical Epidemologists. Preprints 2020040492, 2020, https://doi10.20944/preprints2020024.0492.VI.

6. Scharping, N., Could we be living with COvid-19 forever? https://www.discovermagazine.com/health/could-we-be-living-with-covid-19-forever. Accessed on APril, 20, 2023.

7. Vanden Driessche, P. and Watmough, J., Reproduction numbers and sub-threshold Endemic Equilibrium for compartmental Models of Disease transmission. Mathematical Biosciences. 180(1-2) pp 29 – 48. 2002