

# Effects of Po-Shen Loh's Method on Students' Achievement in Quadratic Equations

## Abstract

This paper presents Po-Shen Loh's method as an alternate method for solving quadratic equations, which is seen as an efficient and natural method for solving general quadratic equations. The study is action research involving a sample of forty – five (45) students of Kumasi Wesley Girls High School, randomly selected for the study. Prior to the study, it was observed that the students did not know about any other method of solving quadratic equation other than the four traditional methods (factorisation, formula, completing squares and graphical) in the mathematics curriculum at the senior high school level in Ghana. The pre-test and post-test scores obtained by the students were analysed quantitatively and an inferential and descriptive analysis were performed. Comparatively, the scores obtained from the pre-test and post-test showed a significant improvement on the students' ability to solve quadratic equation using the Po-Shen Loh's method. The authors therefore recommend for it to be introduced into the senior high school mathematics curriculum in Ghana.

**Keywords:** *Po-Shen Loh method; quadratic equations; factorisation; completing squares method; graphical method; quadratic formula.*

## 1. Introduction

The earliest solution method known in literature for quadratic equations is seen in the work of Diophantus around the year 250 titled in his series of books called Arithmetica (Zhou, 2023). Quadratic equations have practical applications in our daily lives (Bhattacharyya, & Chandra, 2022). Countless applications of this in the real world calls for more attention on the topic at the secondary schools in Ghana.

Prominent among its applications, projectiles are used in various fields such as military and law enforcement. They are designed to immobilize live targets using electrical discharge weapons (Bradshaw, 2023). Quadratic equations are commonly used in problem-solving, and

students are often required to solve them in standardized tests (Cheng, 2023). These equations can be solved digitally using iterative algorithms or analog solvers. Students are required to solve both applied and raw quadratic equation problems in almost every national standardised test.

“Quadratic equations are considered one of the most challenging topics in secondary school mathematics, according to various researchers. The limited time allocated for learning quadratic topics in the curriculum has been found to impede students' understanding and fluency with quadratics” (Kabar, 2023). “Students often face difficulties with critical prerequisite concepts, such as algebraic conventions, which hinder their success in working with quadratics” (Reid O'Connor and Norton, 2022). Additionally, students encounter challenges in algebraic procedures, particularly in factoring quadratic equations, and struggle to apply meaning to quadratics (Bora and Ahmed, 2019).

Many researchers have highlighted the need for more effective teaching and learning strategies to address students' difficulties in acquiring algebraic concepts, including quadratic equations (Didis and Erbas, 2015). Pre-service mathematics teachers' meanings for quadratics and their ability to draw on prior knowledge have been found to impact their teaching practices (Wilkie, 2021). Overall, quadratic equations pose challenges for students and require attention in terms of curriculum design and instructional approaches.

Quadratic equations are considered one of the fundamental topics in the secondary school mathematics curriculum. However, research indicates that there are challenges and difficulties associated with teaching and learning quadratic equations (Kabar, 2023; Reid O'Connor, 2022). Students often struggle with prerequisite concepts and algebraic

conventions, which hinder their understanding and success in working with quadratics (Samuel *et al.*, 2016). Additionally, there is a lack of sufficient research and scope in the field of quadratic equation studies, particularly in terms of considering different perspectives and knowledge of mathematics teachers and pre-service teachers (Wilkie, 2016). Students face various challenges in solving quadratic equations, such as difficulties in algebraic procedures, factoring, and applying meaning to the equations. It is important to address these challenges and provide suitable time and attention to develop conceptual understanding and procedural fluency in quadratic equations (Odutayo and Fonseca, 2024). Didis & Erbas (2015) posit that, “the inclusion of quadratic equations in the mathematics syllabus for secondary schools worldwide is indicative of its basic nature that has been expanding alongside the advancement of algebra”.

Quadratic equations are considered one of the fundamental topics in the secondary school mathematics curriculum in Ghana (Oppong-Gyebi, 2023). Teachers in Ghana view the math curriculum as rigid, abstract, and disconnected from practical applications, which they find non-realistic and irrelevant (Adjei, 2023). However, there is limited research on the teaching and learning of quadratic equations in mathematics education literature (Oppong-Gyebi, 2023). The current studies in algebra education show an upswing in interest in quadratic equation research, but the number of studies is still quite scarce (O’Connor and Norton, 2022). In Ghana, basic school teachers lack knowledge and understanding of curriculum concepts and the curriculum development process, which hinders their involvement in curriculum design (Tetteh and Khumi-Agbasa, 2019). Overall, while quadratic equations are recognized as important in the secondary mathematics curriculum, there is a need for more research and support for teachers in effectively teaching and learning this topic.

Quadratic equations are of the form  $ax^2 + bx + c = 0$  with the caveat that,  $a \neq 0$ . Four solution methods are provided for in the Ghanaian mathematics curriculum at the senior secondary school level namely;

- a) Factorization method
- b) Graphical method
- c) Use of formula method
- d) Completing squares method

The first three methods are treated in Core Mathematics, which is taken by every secondary school student in Ghana. However, the last method is reserved for students pursuing Elective Mathematics, which is a more advanced form of the Core Mathematics. All the four methods come with their own challenges, making learners experienced difficulty in appreciating the topic.

The first method is prone to factorization challenges. Given a quadratic equation  $ax^2 + bx + c = 0$ , many learners are unable to come out with two factors that add up to  $-b$ , and multiplies to give  $ac$ . According to Hoch & Dreyfus (2004), students without “structure sense” may not realise that the quadratic trinomial  $6x^2 - x - 6$ , and its factorised equivalent  $(2x - 3)(3x + 2)$ , are “different interpretations of the same structure”. This view is held by Vaiyavutjamai, et al (2005) and Lim (2000), who found that “when faced with  $(x - 5)(x + 2) = 0$ , for example, many secondary school students, choose to write the left side as  $x^2 - 3x - 10$ , then re-factorise before applying the null factor law”.

Graphical method for solving quadratic equations probably was envisaged to be the easiest approach but the devil lies in the substitution of values into the quadratic equation, and being able to come out with an appropriate scale to plot the curve coupled with unsteady hands among many secondary school students. When the roots of a quadratic equation are not

integers, the graphical method becomes a lot of work and often land on yucky numbers that cannot be read on the graph (Xuhui, 2011). When it comes to the use of the quadratic formula, students encounter difficulty in recalling the formula, as well as making correct substitutions. Again, learners are challenged in substituting negatives numbers and believing in the possibility of taking square roots of negative numbers.

As far as the method of completing squares is concerned, the syllabus acknowledges its complexity and thus shifted it to Elective Mathematics. For this reason, this method is reserved only for learners perceived to be more competent in Mathematics. Again, students are unable to appreciate the method. One fact is that, the ability to appreciate this method gives the learner the confidence to prove the quadratic formula. Hence, they are more likely to recall the quadratic formula than their colleagues who do not take Elective Mathematics. This is the best choice when the factorization method fails.

In an attempt to overcome the above challenges, Po-Shen Loh's method for solving quadratic equation needs to be presented as an alternative method to Ghanaian secondary school students to eliminate the confusions they face in solving quadratic equation. This research therefore seeks to test the Po-Shen Loh's method on some students in Ghana to test its feasibility and appropriateness in enhancing learner achievement in quadratic equations.

## **2. The General Po-Shen Loh method**

The general form of the quadratic equation is  $ax^2 + bx + c = 0$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

We look for two numbers such that their sum is  $-\frac{b}{a}$  and their product is  $\frac{c}{a}$ .

### ***Sum of roots***

We look for two numbers that add up to  $-\frac{b}{a}$ . Clearly,  $-\frac{b}{2a}$  repeated are the two most likely outcomes. Next, we look for  $u$  which goes to zero if the solution is a repeated root, and gives a value otherwise.

We therefore arrive at  $\left(-\frac{b}{2a} + u\right)$  and  $\left(-\frac{b}{2a} - u\right)$  since  $\left(-\frac{b}{2a} + u\right) + \left(-\frac{b}{2a} - u\right) = -\frac{b}{a}$

### ***Product of roots***

The product of the roots is equal to  $\frac{c}{a}$

$$\left(-\frac{b}{2a} + u\right)\left(-\frac{b}{2a} - u\right) = \frac{c}{a}$$

$$\left(\frac{b}{2a}\right)^2 - u^2 = \frac{c}{a}$$

$$u^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$u = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\therefore u = \frac{\sqrt{b^2 - 4ac}}{2a}$$

Hence, the solution which yields the results  $\left(-\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right)$  and  $\left(-\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right)$

### **3. Application of Po-Shen Loh Method**

#### **Example 1**

$$x^2 - 6x - 27 = 0$$

Look for two numbers such that their sum is 6

$3 + u$  and  $3 - u$  are the clear possible answers

The next is to realise that their product must be  $-27$

$$(3 + u)(3 - u) = -27$$

$$9 - u^2 = -27$$

$$u^2 = 36$$

$$u = 6$$

Thus, our answers are  $(3 + 6)$  and  $(3 - 6)$  which reduces to 9 and  $-3$ .

#### **Example 2**

$$3x^2 + x - 14 = 0$$

$$x^2 + \frac{1}{3}x - \frac{14}{3}$$

Look for two numbers such that their sum is  $-\frac{1}{3}$

$-\frac{1}{6} + u$  and  $-\frac{1}{6} - u$  are the clear possible answers

The next is to realise that their product must be  $\frac{-14}{3}$

$$\left(-\frac{1}{6} + u\right)\left(-\frac{1}{6} - u\right) = \frac{-14}{3}$$

$$\frac{1}{36} - u^2 = \frac{-14}{3}$$

$$1 - 36u^2 = -168$$

$$36u^2 = 169$$

$$u^2 = \frac{169}{36}$$

$$u = \frac{13}{6}$$

Thus, our answers are  $\left(-\frac{1}{6} + \frac{13}{6}\right)$  and  $\left(-\frac{1}{6} - \frac{13}{6}\right)$  which reduces to 2 and  $-\frac{7}{3}$ .

## 4. Methodology

### 4.1 Research Design

Action Research was used as the research design for this study. In essence, action research is created to address a practice scenario in a classroom. The main focus of this study is the use of an alternate algorithm to help students solve quadratic equations. The researchers adopted an Action research design since it generates knowledge around inquiry in practical educational contexts. From related literature, Action research allows educators to learn through their actions with the purpose of developing personally or professionally. Due to its participatory nature, the process of action research is also distinct in educational research. According to Panda (2022), “an action plan is particularly significant in a situation whereby things are not running are expected or there is a need in the change of strategy. Practical

solutions might be very much needed in such cases. Action research therefore ensures that a practical solution into whatever social situation is found. By using action research, the learner understands the situation deeply besides finding the most practical solution to it”.

#### **4.2 Population and Sampling**

The population for the research study was students of Wesley Girls’ Senior High School, which is located in the Ashanti Regional capital of Ghana, Kumasi. The school is a single – sex institution (all females), which has a student population of onethousand eight hundred and forty – five (1845). For the purpose of this research, the researchersrandomlyselected forty - five (45) firstyear students offering various programmes from General Science, General Arts, Business, Hospitality and Catering, and Visual Arts programme in the school.

#### **4.3 Instrumentation**

The main research instruments used in conducting this research study were the pre–test and post–test methods. The instruments were well designed for easy collection, interpretation, analysis and organisation of the data collected. The pre–test and the post–tests task were given to students to carry out in order to know their level of performance in terms of their understanding of the Po-Shen Loh’s alternate method of solving quadratic equations. The pre–test and post – test served as the basis for evaluating and assessing the students and to also know their level of performance. The researchers wanted to have an in–depth knowledge of the situation using a small population size and hence used purposive sampling strategy for this study.

## 5. Results

The results of the study obtained by the students were analysed and discussed in relation to the objectives of this study. The analysis of the data was divided into two parts. The first part dealt with the descriptive analysis and the second part also looked at the inferential analysis of the data. Statistical Package for Social Scientist (SPSS) was used for the inferential analysis of the study.

After introducing the Po-Shen Loh's alternate method of solving quadratic equations to the students, the researchers tested their understanding of it with a test. The post-test aimed at finding out the effectiveness of the Po-Shen Loh's method as an alternate method for solving quadratic equations. In the pre-test, a total of ten (10) questions which were marked out of hundred (100%) percent and was administered in one (1) hour. With this pre-test, students were asked to use any of the traditional methods to solve quadratic equation (factorisation, quadratic formula, completing squares, and graphical) which they find convenient. Table 1 indicates the frequency distribution in percentages of the scores obtained by the students in the pre – test.

**Table 1: Frequency distribution of the Pre-Test Scores (in percentages)**

<b>Class Score</b>	<b>Frequency</b>	<b>Percentage (%)</b>
1 – 10	5	11.1
11 – 20	17	37.8
21 – 30	12	26.7
31 – 40	6	13.3
41 – 50	3	6.7
51 – 60	2	4.4
61 – 70	0	0.0
71 – 80	0	0.0
81 – 90	0	0.0
91 – 100	0	0.0
<b>Total</b>	<b>45</b>	<b>100</b>

After administering the pre-test, the researchers observed a generally low performance of the students in terms of solving quadratic equations, which was below the average score. The indication was that the students couldn't comprehend the use of the traditional methods of solving the quadratic equations. With the use of the quadratic formula, some of the students who tried using it had difficulty in recalling the formula in addition to making correct substitution of values into the formula. The graphical method which is probably envisaged to be the easiest approach also became a challenge to the students who adopted it. They had difficulty in choosing an appropriate scale to plot the curves and lacked the steady hands skill to draw effective graphs, as well as reading correctly from the graph. The method of completing the squares is perceived to be abstract and even the mathematics syllabus for Mathematics (Elective) in Ghana acknowledges its complexity. Most of the students felt that this method is reserved only for learners perceived to be more competent in the subject. The students who were able to solve some of the questions used the method of factorization. They however had difficulty when the roots of some of the quadratic equations had irrational numbers.

To address these challenges of the students, the researchers introduced the Po-Shen Loh's method to the learners as intervention. In order to ascertain whether this alternate method had gone down well with the students, the researchers administered a post-test for them. The post-test also involved a total of ten (10) questions which were marked out of hundred (100%) percent and was administered in one (1) hour due to its high similarity to the pre-test. The students were made to adopt any method of their choice. The observation made by the researchers during the post-test was that, most of the students tried using the Po-Shen Loh's method of solving the quadratic equations. Table 2 is the frequency distribution in percentages of the marks obtained by the students in the post – test.

**Table 2: Frequency distribution table for Post-test Results**

<b>Class Score</b>	<b>Frequency</b>	<b>Percentage (%)</b>
1 – 10	0	0.0
11 – 20	0	0.0
21 – 30	0	0.0
31 – 40	2	4.4
41 – 50	5	11.1
51 – 60	7	15.6
61 – 70	11	24.4
71 – 80	10	22.2
81 – 90	6	13.3
91 – 100	4	9.0
<b>Total</b>	<b>45</b>	<b>100</b>

The post – test scores indicated a change in performance of the students as compared to that of the pre –test scores. The researchers attributed the improvement in the students’ performance to the anxiety and change in the method of solving quadratic equations. With the introduction of the Po-Shen Loh’s method as an alternate method of solving quadratic equations, the students were exposed to a new algorithm that was convenient and appreciated by them.

A descriptive statistic, from Table 3 indicates that, the mean value of the pre – test scores and the post – test scores were 23.96 and 68.71 respectively. The standard deviation of the pre – test scores was also 13.384 while that of the post – test was 14.952.

**Table 3: Descriptive Statistics of Pre-test and Post-test Scores**

		<b>Mean</b>	<b>N</b>	<b>Std. Deviation</b>	<b>Std. Error Mean</b>
Pair 1	Pre-test Scores	23.96	45	13.384	1.995
	Post-test Scores	68.71	45	14.952	2.229

Comparatively, a conclusion can be drawn from the mean score values of both the pre – test scores and the post – test scores. The mean scores showed a significant improvement in students’ performance in solving quadratic equations. The introduction of the Po-Shen Loh’s method as an alternate method to the traditional method has helped the students in solving quadratic equations.

### 5.1 Testing the Hypothesis

“Hypothesis concerns comparing means of two small dependent samples, when the same respondent or person is measured under the two conditions or when matched paired are measured under the same condition” (Asiedu – Addo *et al*, 2004). In this study, the researchers compared the mean value of the pre –test and the post – test. The means of the dependent samples’ pre – test and post – test, are given in Table 4 below. The P – value for the statistical test for the hypothesis was set at  $P < 0.05$ .

The hypothesis designed to direct and guide the research study is follows:

**Null hypothesis,  $H_0$ :** There is no significant difference in the mean scores of the pre–test and the post–test of the students at  $\alpha = 0.05$  level of significance ( $P < 0.05$ ). i.e.  **$H_0: \mu_1 = \mu_2$**

**Table 4: Paired Sampled Test of Pre-test and Post-test Scores**

		Paired Differences (95% Confidence Interval of the Difference)							
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	Pre-test Scores	-44.756	7.039	1.049	-46.870	-42.641	-42.650	44	0.000
	Post-test Scores								

## **5.2 Decision:**

The results from the statistical test for the hypothesis from Table 4 yielded a p-value of 0.000, which indicated that the difference in the means was significant. The test statistic was set at  $P < 0.05$  and since  $P$  is less than 0.05 (level of significance), we reject the Null Hypothesis and accordingly accept the Alternate Hypothesis.

## **6. Discussion**

It can be observed from Table 1 that, as many as forty (40) students out of the forty-five (45) selected students had marks below 40 with only ten having marks above 40. This shows that students' performance in the pre-test was very low. In the pre-test, the mean score was 23.5% with a standard deviation of 12.7 which was not a good performance. This shows that almost seventy-six percent (76%) of the students had marks below the mean mark of 23.5%. Again, a mean mark of 23.5% out of a total of 100% is very low. Only two (2) out of the forty-five (45) students had a mark above 50%. Comparing this with the students' performance in the post-test, the researchers observed that there was a significant improvement. Thirty – eight (38) representing 84.5% scored above the average score. The improvement in the performance of the students, which was evident in the post – test scores they obtained, was not by chance, but through the introduction of the Po-Shen Loh method as an alternate method of solving quadratic equation that the researchers employed during the intervention activities. With this method, the researchers designed a well – planned intervention activity in the lessons with the students. This method and approach enabled the students to participate actively in the lessons since the student needed not to memorize any formula as well as

having any difficulties with any graph work. In the event of all these, the researchers found out that the students were motivated by this new algorithm and method and were also inspired by the way the lessons were taught. The Po-Shen Loh method used in this research study enabled the student to comprehend the conceptual knowledge and the procedural understanding of the solving quadratic equations.

## **7. Conclusion**

Mathematics has been an intimidating subject for many students, especially in the area of algebra. Solving quadratic equations has been a stumbling block for many students. The research study revealed that, until the intervention stage, the students did not know about any other method of solving quadratic equation other than the four traditional methods in the mathematics curriculum in Ghana at the senior high school level. With the Po-Shen Loh's method as an alternative approach of solving quadratic equations, students will refrain from their usual memorization of formulas and avoid the difficulties in using the graphical method. The Po-Shen Loh's alternate method is an interesting way to solve quadratic equations since it focuses on the conceptual understanding for students to learn about the quadratic equation structure, not just computation. It offers an opportunity for students who have trouble memorizing the standard quadratic formula. The method is also related to concepts like the sum of roots and the product of roots, which are useful in advanced mathematics. The authors are of the view that this study could add to the existing knowledge in the use of the Po-Shen Loh method in the teaching and learning of quadratic equations. The study also promoted active participation and intellectual involvement of learners in solving quadratic equations since they found the Po-Shen Loh method to be more effective as compared to the traditional methods of solutions.

## Acknowledgment

The researchers acknowledge with appreciation the assistance offered by the first-year students of Kumasi Wesley Girls' Senior High School, selected for the research study. The researchers are grateful to the entire staff as well. They also express their gratitude to unknown reviewers for their useful comments.

## Competing Interest

Researchers have declared that no competing interests exist.

## References

1. Adjei, J. K. (2023). Are They Good Enough for Student-Centered And Constructivist Class? Investigation into Teachers' and Students' Perception towards Mathematics Textbooks for Junior High Schools in Ghana. *Journal of Educational and Practice*, 14 (17), 20 – 59.
2. Asiedu-Addo, S. K. and Yidana, I. (2004). Mathematics Teachers' Knowledge of Subject Content and Methodology. *Mathematics Connection*, Vol. 4, 45-47. Winneba: Mathematical Association of Ghana.
3. Bhattacharyya, P. C., & Chandra, P. (2022). A NOVEL CONCEPT IN THEORY OF QUADRATIC EQUATION. *J. Mech. Cont. & Math. Sci*, 17(3), 41-63.
4. Bora, A., & Ahmed, S. (2019). School students' algebra learning: approaches and barriers. *International Research Journal of Humanities, Engineering & Pharmaceutical Sciences* (2019).
5. Bradshaw, J. L. (2023). Projectile motion with quadratic drag. *American Journal of Physics*, 91(4), 258-263.
6. Cheng, H. C., Su, S., Palaria, M., Zhang, Q., Yang, C., Hossain, S., ... & Chen, M. S. W. (2023, April). A Memristor-Based Analog Accelerator for Solving Quadratic Programming Problems. In *2023 IEEE Custom Integrated Circuits Conference (CICC)* (pp. 1-2). IEEE.

7. Didis, M. G. and Erbas, A. K. (2015). Performance and difficulties of students in formulating and solving quadratic equations with one unknown. *Educational Sciences: Theory & Practice*, 15(4), 1137-1150.
8. Güner, P. (2017). High school students' achievement of solving quadratic equations. *Bartın University Journal of Faculty of Education*, 6(2), 447-467.
9. Hoch, M., and Dreyfus, T. (2004). Structure sense in high school algebra: The effects of brackets. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 49–56). Bergen, Norway: PME.
10. Kabar, M. G. D. (2023). A Thematic Review of Quadratic Equation Studies in The Field of Mathematics Education. *Participatory Educational Research*, 10(4), 29-48.
11. Li, X. (2023). Mathematical knowledge for teaching algebraic routines: A case study of solving quadratic equations. *Journal of Mathematics Education*, 4(2), 1-16.
12. Lim, T. H. (2000). The teaching and learning of algebraic equations and factorization in O-level Mathematics: A case study. Unpublished M.Ed. dissertation, Universiti Brunei Darussalam.
13. Oppong-Gyebi, E., Amoako Atta, S., Kwadwo, A. A., Belbase, S., Bonyah, E., & Peprah Opoku, M. (2023). High School Teachers' Perceptions and Practices of Mathematics Curriculum in Ghana. *Education Research International*, 2023.
14. Panda, I. (2022). *The Importance of Action Research*. <https://ivypanda.com/essays/the-importance-of-action-research>.
15. Samuel, K., Mulenga, H. M., & Angel, M. (2016). An Investigation into Challenges Faced by Secondary School Teachers and Pupils in Algebraic Linear Equations: A Case of Mufulira District, Zambia. *Journal of education and practice*, 7(26), 99-106.
16. Tetteh, A. and Khumi-Agbasa, P. (2019). Basic School Teachers' Knowledge in Fundamental Curriculum Concepts and Curriculum Development Process in Ghana. *Journal of Education and Practice*, 10 (9), 95 – 104.
17. Vaiyavutjamai, P., Ellerton, N. F., and Clements, M. A.(2005). Influences on student learning of quadratic equations. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Research, theory and practice* (Vol. 2, pp. 735–742). Sydney: MERGA.
18. Wilkie, K. J. (2016). Using challenging tasks for formative assessment on quadratic functions with senior secondary students. *Australian Mathematics Teacher*, 72(1), 30-40.

19. Wilkie, K. J. (2021). Seeing quadratics in a new light: Secondary mathematics pre-service teachers' creation of figural growing patterns. *Educational Studies in Mathematics*, 106(1), 91-116.
20. Zhou, Z. (2023). Solution of the diophantine equation. *International Research Journal of Modernization in Engineering Technology and Science*, doi: 10.56726/irjmets33480