Modelling Fluid Flow in Zone 1 of an Open Horseshoe Channel with Lateral Inflow Channels

Abstract

For many years, flooding has been a significant issue, especially after heavy rainfall. Engineers have constructed channels to direct water into rivers, lakes, and oceans, aiming to mitigate flooding. The challenge lies in designing drainage ditches, irrigation canals, and navigation channels that maximize hydraulic efficiency for water transport and electricity generation. Most studies have focused on rectangular, parabolic, trapezoidal, and circular open channels, leaving a knowledge gap in the study of horseshoe-shaped channels with lateral inflows. This research aims to model a uniform flow in Zone 1 with a horseshoe-shaped cross-section and lateral inflows.The study aimed to determine how variations in the angle of lateral inflow channels and the increase in lateral inflows in Zone 1 affect the main channel flow velocity. Governing equations were derived by applying conservation equations to the physical conditions of the flow. These equations were solved using the finite difference approximation method due to its precision, stability, and convergence. The results, presented graphically, revealed that the main channel velocity decreases as the number of lateral inflow channels increases. Ultimately, the main channel velocity decreases as the angles of the lateral inflows increase. Mitigating floods and collecting water for irrigation drive scientific, technological, and engineering progress by demanding creative remedies and infrastructure that enhance crop production and tolerance to climate changes. By enhancing food security and enabling sustainable farming practices, these developments promote economic growth and raise living standards in communities while also generating job possibilities. Water management that incorporates scientific and technological advancements allows society to more effectively utilize natural resources, which in turn promotes greater socioeconomic empowerment.

Keywords: Fluid, Open channel, Lateral inflow, Horseshoe Channel, laminar flow, Flow Area.

Nomenclature:

H **Radius**
h **Height**
Q **Discharge**
A **Area**
L **Length of the Channel**
K **Number of Lateral Inflows**
V **Velocity of the channel**
θ **Angle**

1.0 introduction

1.1 Background information

The study of open channel flow is common research area with studies carried out on Natural channels like rivers and man-made channels such as irrigation canals. In open-channel flows, gravity, viscosity, and inertia are the main forces at work, individually playing a key function.

“Manynaturalandengineeredsystemsinvolvebranchingnetworks,like bloodves-selsormicrofluidicchips.Researchisexploringhowtodesign thesenetworkstomaximizeflowefficiency byconsidering factorslikechannelwidthratios,channelshapeandanglesateachgenerationandatthe branchingpoints” (Ashish Garg 2024).

Insomecases,thechannelwallsthemselvescandeformunderpressureorflow forces. Thiscanimpactflow behaviorandpotentiallyenhanceflow bycreatingspecificchannelshapes. Furtherintheintroduction,wecanalsoincorporatetherecentadvancementofenhancedflowi ndeformable CNTtubes/channelstohighlighttheveryrelevantandbroaderrangeoffluid-CNTinteractions,wherethe fluidflowcharacteristicsareinfluencedbythetube’sflexibility,impactingpracticalareaslike drug delivery anddesalination

“Many researchers observed enhanced water flow through carbon nanotubes (CNTs) and attributed the reason to large slips. Even after taking significant slip effects into account, there remain unaddressed observations of significant improvements in flow rates. As CNTS are deformable, we represent nanotubes with a deformable-wall using a linear pressure–area relationship. We assume lubrication assumption, and using the properties of nanoconfined water, we derive the model for deformable-nanotubes”. (Garg, A. 2024).

Ashish Garg (2023) studied “Pulsatile pressure enhanced rapid water transport through flexible graphene nano/Angstrom-size channels: a continuum modeling approach using the micro-structure of nanoconfined water. The study investigated how the applied pulsating pressure influences the flow rate. Further more the study found out that due to the oscillatory pressure field, there is no change in the averaged mass flow rate in the rigid-wall channel, whereas the flow rate increases in the flexible channels with the increasing magnitude of the oscillatory pressure field. Also, in flexible channels, depending on the magnitude of the pressure field, either of the steady or oscillatory or both kinds of pressure field, the averaged mass flow rate dependence varies from Δp to Δp 4 as the pressure field increases”.

Manning (1895) developed “manning formula which has been proven to be the most used formula in the study of the open channel.The Manning equation makes uses of the coefficient of roughness called Manning constant in the open channel flow”. “This has made the equation very reliable and more desired for the design of open channels. The Manning coefficient considers the degree of irregularity of the channel, channel size, bed material, and variation in shape and comparative effect of channel obstruction, meandering, and growth of vegetation in a channel” (Chadwick &Morfeit, 1993).

1.2 Literature review

Kwanza *et al.*, (2007) investigated “the effects of the slope of the channel, the width of the channel, and channel discharge for both trapezoidal and rectangular channels. The results demonstrated that, hydraulically, trapezoidal open channel flows are more efficient than rectangular cross-sectional open channels. According to the study, increasing the identified parameters results in an increase in flow volume. Research on the impacts of lateral input on the main flow is necessary because the study did not address these effects on the velocity of the main flow”.

Tsombe*et al,.(*2011) investigated “fluid flow in open channels with a circular cross-section. The study discovered that a decrease in fluid velocity results from an increase in flow depth. Moreover, a rise in channel slope causes the flow velocity to increase. Increasing the channel's radius also causes the flow velocity to decrease. Additionally, because the slope and the flow velocity are directly related, the data showed that a decrease in the channel's slope also causes a drop in flow velocity. Moreover, for a fixed flow area, the maximum velocity happens just below the free surface as the flow velocity increases with depth from the channel bottom to the free stream. The finite difference approach was employed in this study to solve the flow equations, however for more accurate results there is need to use finite element method”.

Thiong’o *et al* (2011) investigated “fluid flow in an open rectangular and triangular channel. The results showed that hydraulically, open channels with rectangular cross-sections are more efficient than those with triangular cross-sections. Additional research revealed that an increase in the energy coefficient, top width, and slope of the channel causes a rise in flow velocity for both rectangular and triangle channels. Moreover, the flow velocity increases with depth and reaches its maximum just below the free surface. The rectangular channel moves more water faster than an open triangular channel at the same depth and width, according to the velocity profiles for both types of channels”.

Thiong'o (2013) focused“on open rectangular and triangular channel flows. The objective was to ascertain the hydraulic efficiency of the open rectangular and triangular channels. There are non-linear partial differential equations as a result of the conservation of mass and momentum rules. The finite difference approach was adopted since such equations cannot be solved analytically. The depth and velocity of the flow are crucial variables in figuring out discharge. Studies have been done on how altering different parameters affects velocity. It has also been studied how fluid velocity changes with depth. The finite element approach can yield findings that are more accurate than those obtained by the finite difference method utilized in this study to solve its equations”.

Macharia *et al.,(*2014) studied “the flow of fluids in an open rectangular channel with lateral inflow channels and discovered that increasing the channel's lateral inflow angle does not increase the velocity of the fluid in the core channel. As the cross-sectional area of the lateral inflow increases, the main channel's flow speed decreases. When the lateral inflow channel length rises, the flow velocity in both channels drops, while the flow velocity in the open rectangular channel increases as the lateral inflow channel velocity increases”.

Ojiambo et al.,(2014), developed “a Mathematical model of fluid flow in an open channel with a circular cross-section. The study's findings showed that, for a static area of flow, flow velocity increases as flow depth increases from the lowest part of the channel to the free stream, and that maximum velocity is reached just below the free surface. The results also demonstrated that decreasing the channel's cross-sectional area and flow depth causes flow velocity to increase. As the lateral inflow rate per unit length of the channel decreases, the flow increases. Since this study concentrated on a flow without lateral inflow, a study with lateral inflow is required”.

Jomba*et al.,* (2015) investigated “a mathematical model of fluid flow in an open channel with a Horseshoe cross-section. The results of the investigation showed that for a fixed flow area, the depth increases towards the free stream as the flow velocity increases. Furthermore, it was shown that higher roughness coefficients and hydraulic radii cause greater shear stresses, which in turn cause a decrease in velocity. Since flow velocity and channel slope are directly related, a decrease in the slope of the channel causes a corresponding drop in flow velocity. The flow velocity decreases as the cross-sectional area of the flow increases. This research endeavor focused on the horseshoe cross section without lateral input; hence, a related study incorporating lateral inflow is required".

Yadav et al., (2021) studied "State of art of different kinds of fluid flow interactions with piezo for energy harvesting considering experimental, simulations and mathematical modelling".The study studied the different kinds of fluid flow interactions with piezo smart materials have been discussed for energy harvesting. The present work has been classified into the following categories:(i) experimental investigations (ii) simulation and (iii) mathematical modelling. In section (i) different experimental set-ups such as harvesting of energy with the help of vortex flow, turbulent flow, cross flow, flow in an open channel and closed channel and flow through nozzles have been examined. In section (ii) simulations studies performed with different tools/software like ANSYS/fluent, COSMOL etc. have been detailed. Lastly, in section (iii) different mathematical equations such as Navier-Stokes equation of motion, Continuity equation, finite element method, numerical methods, transport equations, Bernoulli equation, equation of linear elasticity, fluid structural equations, piezoelectric equations and coupled-wave equations are described for generation of energy with fluid's interaction".

Marangu*et al.*, (2016) developed"a model of open channel fluid flow with trapezoidal cross-section and a segment base. The study found that a decrease in flow velocity is caused by increasing the cross-section area of the flow. Moreover, a decrease in flow velocity is brought on by an increase in the cross-section and the flow channel radius, as well as by an increase in the roughness coefficient. According to the study's findings, increasing the segment's circle's radius causes the flow velocity to decrease.The findings were that increase in depth of flow, channel radius, and the cross-sectional area produces a corresponding decrease in fluid velocity. Also, the results were that an increase in the bed slope of the waterway resulted in an increasing flow velocity. This study has employed finite difference method for solving the equations governing the flow but for more accurate results finite element method can be used in solving the same equations".

Omari *et al.*, (2018) Modelled"circular closed channels for sewer lines. As the cross-sectional sewer flow area increases, the sewer depth decreases, according to the data. An increase in sewer flow velocity has been seen when the friction slope is reduced. Furthermore, the sewer velocity was observed to increase with an increase in the tunnel angle of inclination. The present study concentrated on circular closed channels for lateral inflow sewer lines; hence, similar research on lateral inflow sewer lines is required".

Mohit Yadav et al., (2023) Modelled and Simulated of Piezo-beam Structure Mounted in a Circular Pipe using Laminar Flow as Energy Harvester. Modelling and simulation have been done by ANSYS software by the striking of laminar flowing water with a mean velocity of '30 m/s' on a circular cylindrical structure of length '1000 mm' with diameter '200 mm', containing a rectangular beam and PZT piezoelectric plate at a distance of '202.698 mm' from an inlet of the cylinder. The phenomena of vortex shedding occurred behind the bluff body by striking flowing fluid. Consequently, the piezoelectric beam structure started to oscillate. As a result of

this vibration, a voltage of 0.026V has been generated within the beam and piezoelectric patch arrangement, and it can be stored in batteries for future purposes.
Karimi (2018) studied flow in an open rectangular channel with a lateral inflow channel. The findings demonstrated that the results were consistent with earlier studies at zero degrees of the lateral rectangular channel. It was found that the velocity of the main channel increases when the area and length of the lateral inflow channel increase, whereas the velocity of this channel decreases when the velocity of the main channel increases. Lastly, an increase in the lateral inflow channel's angle does not always translate into a rise in the main channel's velocity. The governing equations in this study were solved using the finite difference approach; however, the finite element method must be utilized to perform the same research in order to produce more precise results.
Yadav et al., (2023) studied Piezo-beam structure in a pipe with turbulent flow as energy harvester: Mathematical modeling and simulation". The study found out that the vortex shedding phenomenon occurs by induced vibrations due to the interaction of the fluid with a piezoelectric beam arrangement of pipe at the frequency range of $10–10^3$ Hz. Consequently, the maximum voltage of $2.95586*10^{-1}$ V has been generated in the structural arrangement at a frequency of 10^3 Hz through simulation. Further, it can be stored in storage devices like batteries for upcoming usage. The simulation and mathematical modeling findings are in good agreement with a percentage error of 2.033.

Mose *et al.,* (2019) modelled fluid flow in an oval cross-section open channel. The results demonstrated that a higher hydraulic radius leads to a deeper fluid level. The accumulation of eroded particles causes the depth of fluid flow along the channel to decrease, which in turn lowers the fluid velocity. Flow velocity is also impacted by variations in friction slope. The flow velocity decreases with increased friction. Shear pressures on the channel bed and walls cause friction, which prevents the water from flowing smoothly. The impact of lateral inflow on the channel's velocity was not covered in this study.
Rotich et al., (2021) investigated open channel flows with a parabolic cross-section.The results demonstrated that higher channel slope and energy coefficients result in higher flow velocities. Conversely, a reduction in top width results in a rise in velocity. The impact of lateral inflow on the channel's velocity has not been examined in this study. In order to solve the governing equations and provide more precise answers, the finite element method must also be used.
Few studies have been conducted on horseshoe channels with lateral inflow, despite the fact that several have been conducted on open channels with a varied cross-sectional area during the past 20 years. Finite element analysis is required for more precise results, even if it is evident from the previously discussed research investigations that finite difference was employed to solve the equations. It is necessary to perform additional research since the impact of lateral inflow on the channel's velocity has not been adequately covered by many previous studies. Flooding is still an issue in the current years, and this research aims to construct a channel that can transfer maximum discharge from flooded areas into agricultural land.

2.0 Geometric Properties of HorseshoeOpen Channel low for Zone 1.

The horseshoe cross section is divided into three zones of flow depth as shown in fugure1. Merkley (2005), calculated the depths h_1,h_2 and h_3 of figure 1using the following formulae.

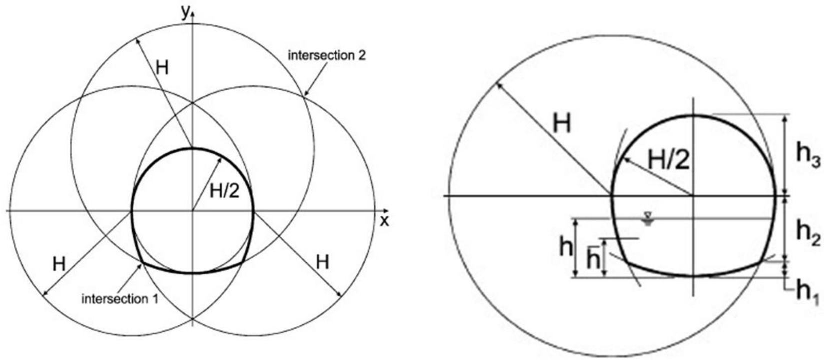

Figure 1: Cross-Section of a Horseshoe Channel.

1. Height

$$h_1 = \mathrm{H}\left[1 - \left(\frac{1+\sqrt{7}}{4}\right)\right] (1)$$

2. Cross-sectional area

$$A_1 = (\mathrm{h} - \mathrm{H})\sqrt{h(2H-h)} + H^2\left[sin^{-1}\left(\frac{h-H}{H}\right) + \frac{\pi}{2}\right] \text{for } 0 \leq \mathrm{h} \leq h_1 \quad (2)$$

3. Wetted perimeter

$$p_1 = 2Hcos^{-1}\left(1 - \frac{h}{H}\right) \text{for } 0 \leq h \leq h_1 \quad (3)$$

4. Top width

$$T_1 = 2H\sqrt{1 - \left(1 - \frac{h}{H}\right)^2} \text{for } 0 \leq h \leq h_1 \quad (4)$$

3.0 Governing Equations

3.1 Continuity equation

Based on the law that mass cannot be created or destroyed, the principle of continuity is established. The continuity equation regulating unsteady flow in open channels is,for any arbitrary shape:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

(5)Net volume of the fluidis$\frac{\partial Q}{\partial x}dxdt$, lateral inflowis$\frac{q}{L}\sin\theta dxdt$ and discharge for k lateral inflows is $k\frac{q}{L}\sin\theta dxdt$. Increment of the fluid is$\frac{\partial A}{\partial t}dxdt$. Considering that our fluid's density is constant and consistent with the fluid's conservation law then,

$$\frac{\partial Q}{\partial x}dxdt + \frac{\partial A}{\partial t}dxdt = k\frac{q}{L}\sin\theta dxdt. \qquad (6)$$

Dividing equation (6) through out by $dxdt$ equation (7) is obtained

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = k\frac{q}{L}\sin\theta \qquad (7)$$

Discharge is given by the product of cross-sectional area and velocity

$$Q = AV \qquad (8)$$

Substituting equation (8) above into equation (7) and differentiating partially with respect to *x* we get

$$V\frac{\partial A}{\partial x} + A\frac{\partial V}{\partial x} + \frac{\partial A}{\partial t} = k\frac{q}{L}\sin\theta \qquad (9)$$

The flow area can be assumed to be a known function of the depth and therefore the derivatives of A can be expressed in terms of y.

$$\frac{\partial A}{\partial x} = \frac{dA}{dy}\frac{\partial y}{\partial x} = T\frac{\partial y}{\partial x}(10)$$

$$\frac{\partial A}{\partial t} = \frac{dA}{dy}\frac{\partial y}{\partial t} = T\frac{\partial y}{\partial t}$$

Where T is thechannel top width and Franz (1982) assumed that T is determined by

$$T = \frac{dA}{dy}(11)$$

Substituting equations (10) into (9) we get

$$VT\frac{\partial y}{\partial x} + A\frac{\partial V}{\partial x} + T\frac{\partial y}{\partial t} = k\frac{q}{L}\sin\theta(12)$$

Dividing equation (12) throughout by T and rearranging we get,

$$\frac{\partial y}{\partial t} + V\frac{\partial y}{\partial x} + \frac{A}{T}\frac{\partial V}{\partial x} = k\frac{q}{TL}\sin\theta(13)$$

Equation (13) is the general equation of continuity for open channel flow with lateral inflow channel at an angle.

3.2 Momentum equation

This equation is arrived at using Newton's second law of motion and the assumption that the diffusing viscous term plus the pressure term add up to fluid stress. This formula connects an element of fluid's acceleration or rate of change of momentum to the total force acting on it.
According to the conservation law in the momentum equation;

$$\frac{\partial Q}{\partial t}dxdt + \frac{\partial (QV)}{\partial x}dxdt + g\frac{\partial (yA)}{\partial x}dxdt + gA\left(S_f - S_O\right)dxdt = k\frac{q}{L}sin\theta ucos\theta dxdt \quad (14)$$

Dividing equation(14) through out by $dxdt$to obtain equation (15)

$$\frac{\partial Q}{\partial t} + \frac{\partial (QV)}{\partial x} + g\frac{\partial (yA)}{\partial x} + gA\left(S_f - S_O\right) = k\frac{q}{L}sin\theta ucos\theta (15)$$

Substituting equation (8) into equation (15) above and differentiating partially with respect to x considering the area A is a constant to obtain equation (16).

$$A\frac{\partial V}{\partial t} + V\frac{\partial A}{\partial t} + Q\frac{\partial V}{\partial x} + V\frac{\partial Q}{\partial x} + gA\frac{\partial y}{\partial x} + gA\left(S_f - S_O\right) = k\frac{q}{L}sin\theta ucos\theta \quad (16)$$

Rearranging equation (16) to obtain equation (17)

$$V\left(\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x}\right) + A\frac{\partial V}{\partial t} + Q\frac{\partial V}{\partial x} + +gA\frac{\partial y}{\partial x} + gA\left(S_f - S_O\right) = \frac{q}{L}sin\theta ucos\theta \quad (17)$$

Substituting Equation (7) into equation (17) to obtain equation (18)

$$V\left(\frac{q}{L}\sin\theta\right) + A\frac{\partial V}{\partial t} + Q\frac{\partial V}{\partial x} + +gA\frac{\partial y}{\partial x} + gA\left(S_f - S_O\right) = k\frac{q}{L}sin\theta ucos\theta (18)$$

Dividing equation (18) throughout by $A$to obtain equation (19)

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + +g\frac{\partial y}{\partial x} + g\left(S_f - S_O\right) + \frac{V}{A}\left(\frac{q}{L}\sin\theta\right) = k\frac{q}{AL}sin\theta ucos\theta \quad (19)$$

Equation (19) is rearranged to obtain equation (20) we get,

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + +g\frac{\partial y}{\partial x} + g\left(S_f - S_O\right) = k\frac{q}{AL}sin\theta\left(ucos\theta - V\right) \quad (20)$$

Equation (16) is the general momentum equation of an open channel with lateral inflow channel at varying angles.

4.0 Equations Governing the Fluid Flow in Finite Difference Form for Zone 1.

Subject to their boundary and initial conditions, the governing equations describing the unsteady, incompressible fluid flow through a horseshoe cross-section in finite difference form are solved using the finite difference approach as follows:

$$y_{(i,j+1)} = \Delta t\left\{k\frac{A}{\mathrm{TL}}\sin\theta - v^{*}{}_{(i,j)}\frac{y_{(i+1,j)} - y_{(i-1,j)}}{2\Delta x} - \frac{A}{TL}\frac{v_{(i+1,j)} - v_{(i-1,j)}}{2\Delta x} + y_{(i,j)}\right\}(19)$$

$$v_{(i,j+1)} = \Delta t\left\{\frac{qv}{Fr^2AgL}\sin\theta(u\cos\theta - v_{(i,j)} - \frac{1}{Fr^2}(\frac{n^2v^2}{R^{\frac{4}{3}}} - s_0) - (\frac{1}{Fr^2}\frac{y_{(i+1,j)} - y_{(i-1,j)}}{2\Delta x} - (v_{(i,j)}\frac{v_{(i+1,j)} - v_{(i-1,j)}}{2\Delta x})\right\} - v_{(i,j)}$$

Where

$$A = (\mathrm{h} - \mathrm{H})\sqrt{h(2H - h)} + H^2\left[sin^{-1}\left(\frac{h - H}{H}\right) + \frac{\pi}{2}\right]$$

$$T = 2H\sqrt{1 - \left(1 - \frac{h}{H}\right)^2}$$

Subject to the condition

$H = 5.65, h = 0.5, k = 1,2,3,4,5\ and\ 6. L = 1, \theta =$

$10^0, 20^0, 30^0, 40^0, 50^0, 60^0, 70^0, 80^0 \text{and } 90^0, Fr = 0.05, s_o = 0.004; \mathrm{n} =$

$0.012;\ \mathrm{g} = 9.8; \mathrm{v} = 0.002, \mathrm{u} = 0.002;$

5.0 Results and Discussion

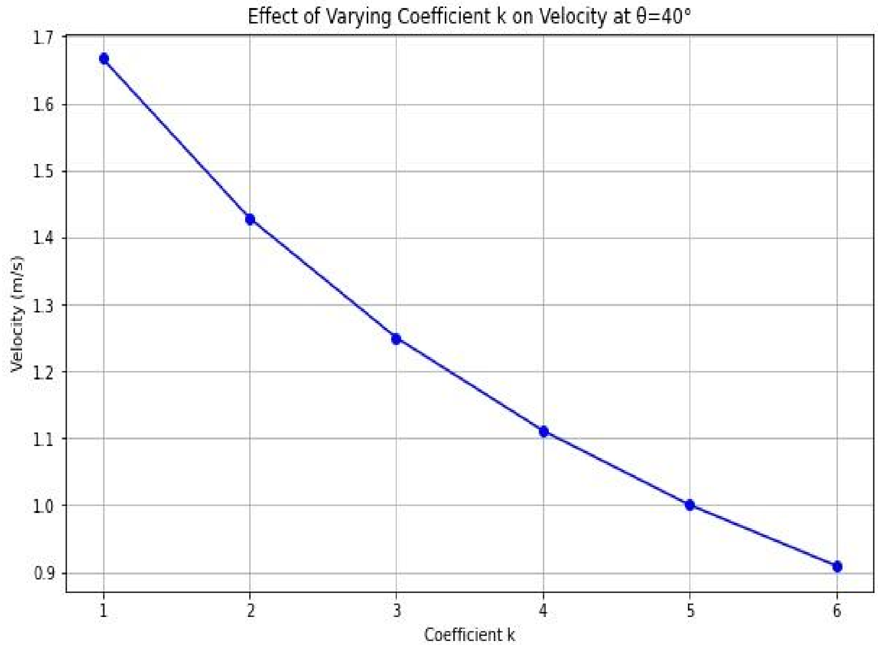

Figure 2: Effects of varying Coefficeint k on velocity at $\theta = 40^0$

Figure 2 shows that the velocity reduces in the main channnel as the lateral inflows increases. The introduction of lateral inflows into the main channel increases the overall cross-sectional area through which the water flows. In a laminar flow, the fluid velocity is inversely proportional to the cross-sectional area based on the principle of continuity. This principle, which states that the mass flow rate must remain constant along a streamline, implies that when the total cross-sectional area increases, the velocity must decrease to maintain the same flow rate.

As more lateral inflows join the main channel, the increased cross-sectional area distributes the flow over a larger area, leading to a reduction in the average velocity. This redistribution of flow and reduction in velocity are key characteristics of a laminar flow environment, where the smooth, orderly movement of water is maintained. This decrease in velocity helps preserve the laminar flow pattern and avoid the onset of turbulence, which is more likely to occur at higher velocities.

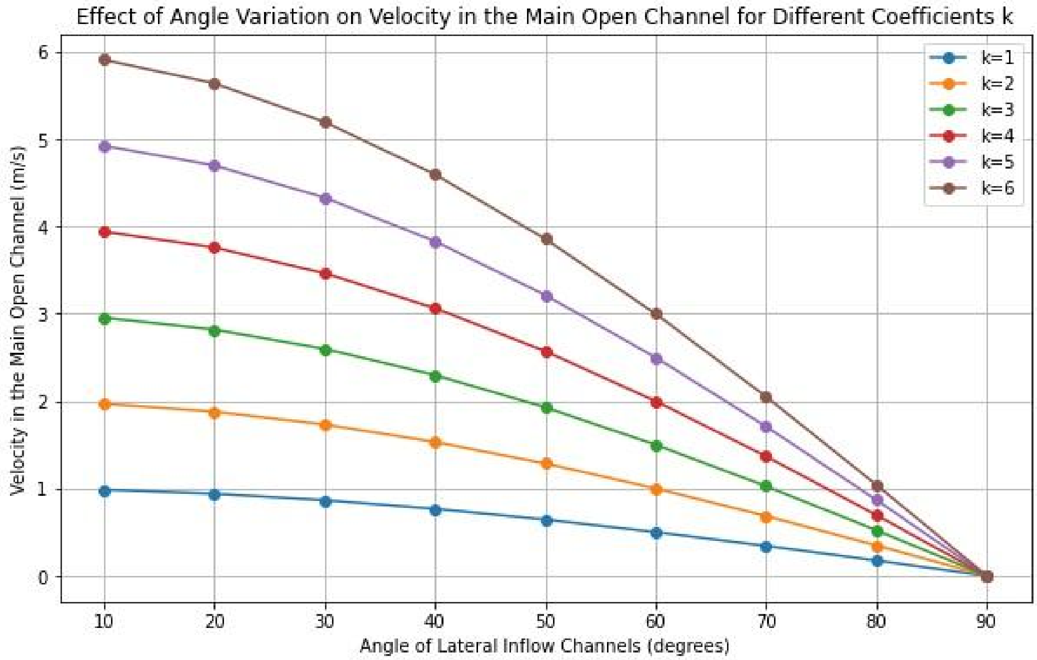

Figure 3: Effect of angle variation on velocity in the main open channel for different coefficients k.

Figure 3 shows that the velocity reduces in the main channnel as the angles of the lateral inflows increases.The direction of the influx into the main channel gets increasingly oblique as the angle of lateral inflows rises. The fluid's motion in the main channel is more severely disrupted by this shift in direction. The smooth, linear motion of the laminar flow in the main channel is disrupted when the lateral flow enters it at a greater angle and collides with other flows. Because of this disturbance, momentum is transferred from the main channel flow to the lateral flow, which may result in a decrease in the main channel's velocity.

A fluid in laminar flow moves in parallel layers with little to no mixing in between. Shear stress between the fluid layers is increased when the angle of lateral inflows increases due to the interaction between the inflow and the main channel flow. The main channel's flow velocity may be decreased as a result of this rise in shear stress due to an increase in internal resistance and friction. The interruption to the smooth flow pattern and the impact of increased shear stress are more noticeable at higher inflow angles.

A larger angle of lateral inflow can also cause flow separation within the main channel. This phenomenon occurs when the inflow angle causes the fluid streams to diverge from each other, leading to areas of recirculation or eddies. Flow separation not only disrupts the laminar flow pattern but also results in energy dissipation, further reducing the overall velocity in the main channel.

6.0 Conclusion

A horseshoe cross- section channel for zone one has been developed with the resulting partial differential equations solved using python software to obtain velocity profiles. The research found out that the velocity reduces in the main channnel as the lateral inflows and angles increases. It is recommended that future research should be carried out on

1. The effect of lateral outflow channel on discharge on zone 1
2. The effect of two or more lateral inflow channels at various

   locations on discharge in the main channel on zone 1

Disclaimer (Artificial intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

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