Varied Mistakes Committed by Students in Division

ABSTRACT

Problems in mathematical problems are common across children of all ages and of all places. This study looks at the common problems students generally face during the mathematical operation of Division and attempts to provide solutions to prevent such problems from taking place further. The study surveyed and analyzed the mistakes of 50 respondents in order to isolate the problems students faced during Division. After collecting the data sets, the data was analyzed and the responses separated into different categories. The results showed a majority of the students having a mistake in the base understanding of Division and its processes from among those that committed the mistakes. The mistakes showed a variation in their types, each showing a different problem that the students faced while attempting to solve mathematical problems. Furthermore, solutions to remedy the understanding of students and to prevent any future students from committing the same mistakes were discussed along with future directions, the teachers could take to ensure that the students properly learn the mathematical process of division.

Keyword: Division, Mistake, Model, Process,

I. INTRODUCTION

The method of teaching of mathematics also known as the pedagogy of mathematics is as old as humankind itself. While the systematic curriculum oriented teaching may have developed during the modern times, the history of passing down the knowledge of counting and interacting with numbers goes beyond written history itself. From the most ancient of civilizations, we have evidence that even those civilizations that did not have a written language still kept some sort of methods to record and interpret numbers. The Inca civilization of Mesoamerica famously did not have a written language but recorded information, mostly regarding taxation and crops through the Quipu, knotted strings. The Mayan civilization, an even older civilization which had pictorial script still had advanced numeral and astronomical knowledge systems. It would not be wrong to claim that no civilization has ever existed which did not have some form of Mathematics education in place. The Mesopotamian civilization,The Vedic civilization of the Indian Sub-continent, The Yangtze civilization of the Yellow river valley, The Egyptian civilization of the Nile river etc., all had in place, systems to transmit the knowledge of mathematics through the generations.The modern curriculum driven system where the sequence is Addition followed by Subtraction, Multiplication and finally Division was put in place somewhere during the 1300s in Florence, Italy by bankers and merchants which led to mathematics being considered unchristian(associated with money lending and usury). Despite being considered unchristian, the subject was of paramount importance and thus mathematics was taught through different ways to people during the medieval period in Europe. For the bankers and merchants it was the model of counting money, the scribes and priests learnt it as a form of philosophy and considered numbers to be sacred representations of the astronomical objects and finally the artisans learnt mathematics to count and measure different things all related to their work. (Why We Learn Math Lessons That Date Back 500 Years).

II. REVIEW OF LITERATURE

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Commented [L2]: Provide a fairly detailed explanation of the methodology used in this research, starting from the data collection methods, tools used, and statistical analysis applied in order to provide a clearer picture of this research.

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Commented [L4]: The described solution is still too general. provide more specific solutions that are easy to understand

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Commented [L6]: Provide an explicit explanation regarding the research problem. A clear and unambiguous problem statement is important to provide the specific context and objectives of the research.

Commented [L7]: Provide more recent data to obtain the latest research results according to the current era

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1. The paper studied how the orientation of teachers is towards procedure more than conceptual understanding when it comes to division through a constructivist oriented theoretical framework. The main concept is the study of **Divisibility and its relation to Division, Multiplication, Prime & Composite Numbers, factorization, Divisibility and Prime Decomposition** (Rina Zazkis, Stephen Campbell).
2. The paper provides insights into the significant roles of mathematical concepts such as factorization, multiples, prime and composite numbers and prime factorization in the teaching of Division as a basic concept of elementary number theory. (R. Zazinsky, K. Gadowsky).
3. This paper studied children's comprehension of the concept of Division, by two main methods – Partitive and Quotative, both styles being used under separate circumstances. The study further pressed on the fact that studentscomprehend the concept of Division better if the problem is presented either pictorially or as a story about sharing of objects within a certain number of people. (Sarah Squire, Peter Bryant).
4. This paper studied how the variation in size of numbers in Division(restricted to simple division problems) causes problems across different ages of students and how students of varied age groups, approach division. The study found that younger students usually performed slower and less accurately compared to older students and relied on the strategy of 'Addition' i.e. adding the divisor repeatedly to get the quotient and older students usually used 'Multiplication' i.e. finding which number multiplied with the divisor would provide the dividend. Furthermore the study tested the prevalence of directly retrieving the answer from memory but observed that division is too unique an operation for its answer to be retrieved from memory. (Katherine M. Robinson, Katherine D. Arbuthnott, Danica Rose, Michelle C. McCarron, Carin A. Globa, Sylvia D. Phonexay)

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III. METHODOLOGY

Pedagogy of Division: The teaching of division to children is usually in 2 levels:-Primary Level and Higher level.
Primary Level: Dividing solid numbers and deriving quotients and remainders such that no fractional/ decimals have to be used.
Higher Level: Dividing any number completely and deriving decimal answers if required.

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Eucledian Division Lemma: The Euclidean division lemma states, that for any two integers$a$ and , we have two other positive integers q and $r$such that, $a = bq + r, 0 \leq r < b$. Writing it in numbers, it means that any number a can be written as the sum of the product between two numbers b and q and an indivisible unit left out r. The basis of Euclidean division is Euclid's division algorithm. HCF is the largest number which exactly divides two or more positive integers. That means, on dividing both the integers A and B, the remainder is zero.

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Euclidean Division lemma could be applied in order to find the H.C.F. of 2 numbers :

Let us try to find the H.C.F. of Two numbers

64 and 57

64 = (57 × 1) + 7

57 = (7 × 8) + 1 → (The HCF of the numbers 64 and 57 and the quotient)

7 = (1 × 7) + 0

↓

(The remainder if 64 is divided by 57)

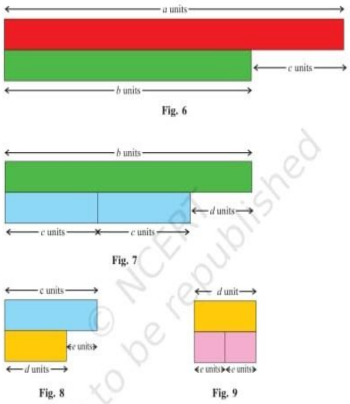


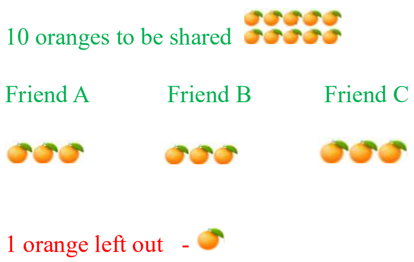
Image 1 : H.C.F. of 2 numbers based on Euclidean Division lemma

IV. PARTITIVE MODEL OF DIVISION

When talking about division, we can approach it as "Sharing" something between a certain number of receivers.

Example - Let 10 oranges be shared equally among 3 friends A, B and C. Thus, each friend shall receive 3 oranges and still there would be a single orange that would remain undivided among them.

Image 2 : A diagrammatic representation would be as follows:-



Thus this can be written in numeral form as, "When 10 is Divided by 3, the quotient is 3 and remainder is 1".When taught, the learners will in the beginning start by giving one orange to each receiver and keep going until they reach a stage where the learner falls short of objects to give out to the receivers (while keeping the distribution equal).

V. QUOTATIVE MODEL OF DIVISION

Another approach to division that can be taken is that of "how many times can one fit the divisor into the dividend". this approach is known as the Quotative approach and makes use of either **Addition** or **Subtraction (also referred to as chunking)** depending on whether one adds the divisor up to the dividend or subtracts the divisor from the dividends in successive Subtractions.

Example - Let the number 45 be divided by 8 i.e., 45 is to be divided into 8 parts, one can proceed as :-

Addition form	**Subtraction form**
8 + 8 = 16	45 - 8 = 37

16+8 = 24 37 - 8 = 29

24+8= 32 29 - 8 = 21

32+8 = 40 21 - 8 = 13

40+8 ≠ 45 13 - 8 = 5

Since 8 had to be added 5 times to reach 40 Since 8 had to be subtracted 5 times from 45 and 5 still remained a non reachable amount to reach the number 5 which cannot be further by addition of 8, the Quotient is equal to 5divided, the quotient is 5 and the remainder too and Remainder is 5.

VI. COLLECTION OF DATA

Using a questionnaire, the data of the mathematical competence in Division of various students were collected using the tool. Given below are the results of the data collection process :

Total no. Of data – 50

Table 1 : Frequency distribution of variates with cumulative frequency

Variate	Frequency	Cumulative frequency
Correct answer(370.33)	22	22
Type I Mistake (37.33)	16	38
Type II Mistake (37)	6	44
Type III Mistake (Miscellaneous)	6	50

Correct (370.33) – 22
Type I Mistake (37.33) – 16
Type II Mistake (37) – 6
Type III Mistake (Miscellaneous) - 6

Chart 1 : Percentage distribution of variates based on frequency

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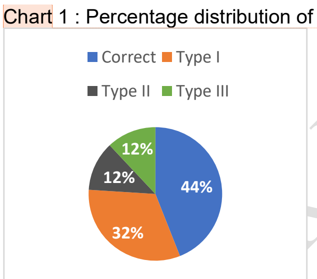

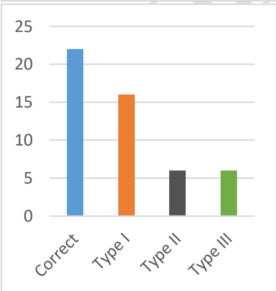

Chart 2 : Frequency distribution of variates with cumulative frequency

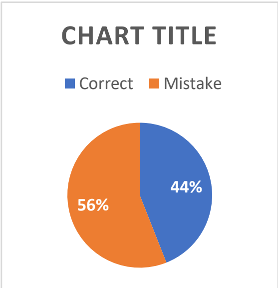

Chart 3 : Percentage distribution of correct and mistake

VII. DATA ANALYSIS

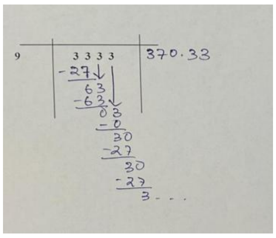

Fig .1 The correct answer of the division

Correct (22 students, 44% of total respondents)

Division is the process of subtracting the divisor step by step from the dividend until the remainder comes out to be zero (0) or a number while cannot be subtracted from further. In the 'Figure 1', respondent went through the entire process accurately to derive the answer of 370.33….

1. The first step is to divide 33 by 9, which yields the quotient 3 and after subtracting 27 from 33, leaves 6 as the remainder.
2. The next step is to divide 63 by 9, which yields the quotient 7 and after subtracting 63 from 63, the remainder left is 0.
3. The next step is to divide 3 by 9, which yields the quotient 0 and after subtracting 0 from 3, the remainder becomes 3 and the division can continue only when we make use of a decimal point (.).
4. After the application of the decimal point, the division continues forward by dividing 30 by 9 which yields the quotient 3 and remainder 3 and the division continues endlessly. Thus, the final correct answer comes out to be 370.33333……

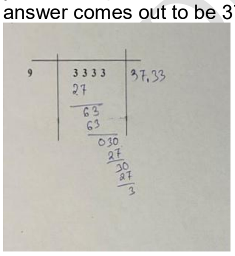

Fig .2 Type – I mistake of the division

Mistake, Type - I (16 students, 32% of total respondents)

In 'Figure 2', the respondent went through half the process correctly (Until step 2) but the mistake was committed in step 3. In this instance, the respondent moved on to dividing by the decimal point without dividing the last remaining 3 by 9 and thus, the answer was half-correct, I.e., 37.3333……. or 37.3 .

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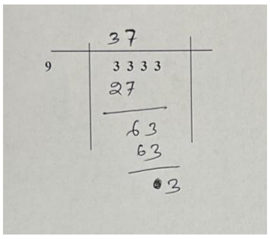

Fig .3 Type – II mistake of the division

Mistake, Type II (6 students, 12% of total respondents)
In 'Figure 3', the respondent went through steps 1 and 2 correctly but the mistake occurred at steps 3 and 4, when the final 3 was left undivided and division by decimal point was not done, thereby deriving the answer of 37.

Miscellaneous, Type III (6 students, 12% of population)
In this case, the respondents made various mistakes ranging from but not limited to :-
1) Going through step 1 correctly and committing a mistake in step 2 and leaving the operation incomplete.
2) Going through step 1 incorrectly and starting the division with either 2 or 4 as quotient.

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VIII. DISCUSSION

After surveying some 50 respondents, the trends of the tests revealed some common findings. The mistakes made were common in that most mistakes were the omission of step of the division by zero and the correct ones were the ones that did not omit the step.

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IX. CONCLUSION

This mistakes committed by students which is indicative of the lack of clarity in the fundamental operation of division could be linked to a lack of understanding in the teachers themselves who too fall prey to such mistakes in the fundamental operations. In order to make sure that students do not commit such mistakes, we must make sure that it is the teacher's who are clear about the concept of Division.

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X. SIGNIFICANCE

The study showed lack of fundamental understanding in one of the basic mathematical operators and thus, it is important that teachers take note of the findings to avoid future problems while teaching division to students. Based on the findings, this study is significant as:-

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1) It will help avoid dependence of students on electronic devices to perform calculations.

2) It will help students learn division in an accurate way and make the concept more appealing to students.

3) Since the mistakes on part of the students reveal lacunae in the teaching methods of the teaches, this will help the teachers clear any misconceptions and increase their understanding of the subject so as to teach better.

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