COVID-19 SIQRV FRACTIONAL-ORDER MATHEMATICAL MODEL WITH VACCINATION AND QUARANTINE CONTROL MEASURES

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Abstract. This study builds a fractional-order mathematical model of epidemic sickness for Omicron, known as B.1.1.529 SARS-Cov-2 Variant. Here, quarantine and COVID-19 vaccinations are taken into account to guarantee the safety of the host population throughout the model. This model's foundations of positivity and boundedness have been examined and proven. To find out if the disease would spread further in Tamilnadu, the reproduction number was computed. The existing infection-free steady-state solutions are locally and globally asymptotically stable for $R_0 < 1$. When $R_0 < 1$, locally stable infection-present steady-state solutions are also found. Ultimately, the data from Tamilnadu, India regarding the Omicron variant pandemic is verified.

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1. INTRODUCTION

As of December 2019, COVID-19 was still the most common strain worldwide and was present in a number of countries. Preliminary study suggests that COVID-19 may induce a milder form of the disease, but some infected individuals still have a chance of becoming really unwell, necessitating hospitalisation, or perhaps dying. Even if only a small percentage of Omicron infection patients need to be hospitalised, you should nevertheless take precautions because the enormous number of cases has the potential to bring down the entire healthcare system. The original COVID-19 virus and the Delta variant were more contagious than the Omicron variety, sometimes referred to as B.1.1.5 29 SARS-Cov-2 Variant. Individuals infected with the Omicron variation may present with symptoms resembling those of previous versions. COVID-19 immunization remains the most effective public health intervention for avoiding COVID-19 and lowering the likelihood that new variations may emerge. This includes the first course, booster shots, and any more dosages that may be required. The current vaccinations are expected to protect against Omicron variant infection-related serious illness, hospital stays, and mortality. Conversely, it is likely that vaccinated individuals will get breakthrough infections.

Many different types of mathematical models have been created to explain how illnesses spread within subpopulations. Mathematicians are being forced to reconsider models that have helped India better understand COVID-19 and respond to the outbreak in light of the highly contagious Omicron strain. The next pandemic wave has changed everyone from those who get tested to those who are most likely to contract the virus which presents new challenges for those who analyse its effects. When a wiped out person recovers from a disease, the vaccination class is remembered for its model definition of an exemplary model, which enables the right antibodies to be delivered to the recovered and helpless individuals in the host population ([7], [32], [37]). The transition from the consistent absence of infection to the consistent presence of disease is characterised by very strong characteristics. It is challenging to investigate the global elements of a pestilence model framework since there are no recognized numerical methods for establishing Lyapunov capabilities for epidemic models ([3], [42]). For COVID-19, multiple mathematical models without the immunization and quarantine compartments have been created ([6],[8],[29],[40]). In this study, we combined the results

of papers ([1],[2],[4],[17], [15], [18],[21],[22],[24],[26],[28],[33]) to create an COVID-19 model with a variable population size.

Over the past few decades, the fractional differential has been used in mathematical modelling of biological phenomena. This is because fractional calculus, as opposed to integer order models, can more accurately process and explain the heritage and retention features of various materials. Consequently, numerous methodologies have been employed to investigate the previously described topic, such as qualitative theory and numerical analysis. A useful tool for studying infectious diseases is a fractional mathematical model. Excellent results were recently obtained when some writers used fractional order derivatives to examine mathematical models of COVID-19. A fractional Omicron mathematical model has been created by referring to ([12], [13], [14], [23], [27], [30], [35], [36], [38], [39], [43])), and the existence, uniqueness, and positivity of the solution are also inferred. The pace at which a specific proportion of vulnerable individuals are placed under quarantine is considered in this model. Additionally, a section is designated for vaccines. At the conclusion of the investigation, computational simulations were performed for Omicron B.1.1.529 SARS-Cov-2 in order to confirm and validate our theoretical results.

2. Model Formulation

In this section some important definitions and lemma has been given. The formulation of the model is discussed.

Definition 1. *Caputo fractional derivative* [34]
Let Ψ be a continuous function on [0, T]. The Caputo fractional derivative of order δ is given by

$$ {}^{C}D^{\delta}\Psi(t) = \frac{1}{\Gamma(n-\delta)}\int_0^t (t-\alpha)^{n-\delta-1}\frac{d^n}{d\alpha^n}\Psi(t)(\alpha)d\alpha, \tag{2.1} $$

where $n = [\delta] + 1$ and $[\delta]$ represents the integer part function and $0 < \delta \leq 1$.

Definition 2. *Riemann-Liouville fractional integral* [34]
The Riemann-Liouville fractional integral of order δ is given by

$$ \mathcal{I}^{\delta}\Psi(t) = \frac{1}{\Gamma(\delta)}\int_0^t (t-\alpha)^{\delta-1}\Psi(\alpha)d\alpha. \tag{2.2} $$

where $0 < \delta \leq 1$.

Definition 3. *Stability for Fractional-order Differential Equations*
Consider the fractional-order system

$$ \begin{aligned} {}^{C}D^{\delta}\mathcal{X}(t) &= \mathcal{J}(t,\mathcal{X}(t)), \quad 0 < \delta < 1, \\ \mathcal{X}(0) &= \mathcal{X}_0. \end{aligned} \tag{2.3} $$

Let $\mathcal{J}(t,\mathcal{X}_0)$ be the unique solution of the system (2.3) satisfying the initial condition $\mathcal{X}(0) = \mathcal{X}_0 \in \mathbb{R}^n$. Then:

1. *the trivial solution of (2.3) is said to be stable if for any $\epsilon > 0$, there exist $\delta = \delta(\epsilon) > 0$ such that, for every $\mathcal{X}_0 \in \mathbb{R}^n$ satisfying $\|\mathcal{X}_0\| < \delta$, we have $\|\mathcal{J}(t,\mathcal{X}_0)\| < \epsilon$ for any $t \geq 0$.*
2. *the trivial solution of (2.3) is said to be asymptotically stable if it is stable and there exists $\xi > 0$ such that $\lim_{t\to\infty}\mathcal{J}(t,\mathcal{X}_0) = 0$ for $\|\mathcal{X}_0\| < \xi$.*

A numerical Omicron mathematical model based on a consistent, non linear first request construction of common differential conditions is examined. The whole population $N(t)$ is subdivided into state factor sub-populations of people who are Susceptible individuals $S(t)$, Quarantined individuals $Q(t)$, Infected individuals $I(t)$, Recovered individuals $R(t)$ and Vaccinated individuals $V(t)$. The other parameters are given in Table 1.

TABLE 1. Parameters and their descriptions

Parameters	Descriptions
Υ	The rate of recruitment of humans into the population
η_1	The overall compartmental natural death rate
η_2	The rate at which a specific proportion of vulnerable people get immunised
η_3	Rate of effective infectious contact between an infected person and a vulnerable person
η_4	The percentage of vulnerable people under quarantine
η_5	The rate at which treatment-induced immunity is lost in the recovered compartment
η_6	The rate of immunization-induced loss of immunity in the vaccinated compartment
η_7	The diseased class's rate of treatment
η_8	The rates of natural recovery resulting from quarantine
η_9	The frequency of interaction between vaccinated and quarantined individuals
η_{10}	The mortality rate brought on by an illness affected person
η_{11}	The speed at which people who have recovered transition to the vaccinated section

There are related worries in the model development that (i) the birth and passing rates are particular. (ii) When susceptible individuals come into contact with an infectious individual who has not had vaccinations, they become contaminated. (iii) People lose immunity as a result of vaccines losing their efficacy over time. (iv) An afflicted individual gets well with treatment. (v) A sustained recovery is not achievable. Based on the presumptions, the model and network's system of equations is constructed as

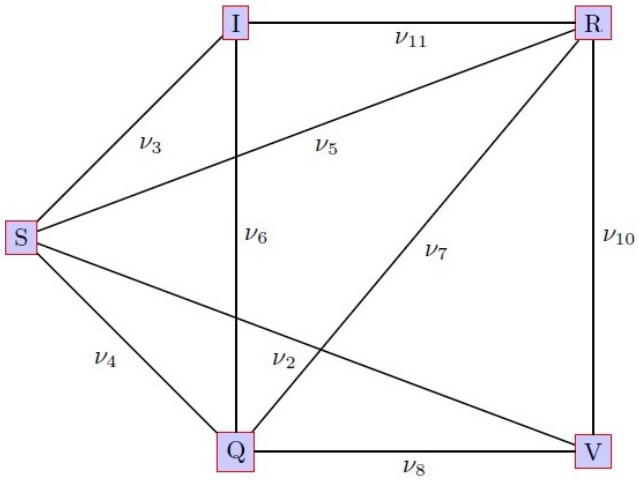

FIGURE 1. Network of the Fractional-order SIQRV Model

$$\begin{aligned}
\frac{dS}{dt} &= \Upsilon - \eta_1 S + \eta_2 SV - \eta_3 SI + \eta_4 Q + \eta_5 R \\
\frac{dI}{dt} &= \eta_3 SI - (\eta_1 + \eta_6 + \eta_9 + \eta_{11})I, \\
\frac{dQ}{dt} &= \eta_6 I - (\eta_1 + \eta_4 + \eta_7)Q - \eta_8 QV, \\
\frac{dR}{dt} &= \eta_{11} I + \eta_7 Q - (\eta_1 + \eta_5 + \eta_{10})R, \\
\frac{dV}{dt} &= \eta_{10} R - \eta_2 SV - \eta_1 V + \eta_8 QV
\end{aligned} \tag{2.4}$$

with the initial conditions: $S(0) = S_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0^0, V(0) = V_0$.

Remark 1. *The system of equations can be written as*

$$\begin{aligned}
\frac{dS}{dt} &= \Upsilon - \eta_1 S + \eta_2 SV - \eta_3 SI + \eta_4 Q + \eta_5 R \\
\frac{dI}{dt} &= \eta_3 SI - (\eta_{22})I, \\
\frac{dQ}{dt} &= \eta_6 I - \eta_{23} Q - \eta_8 QV, \\
\frac{dR}{dt} &= \eta_{11} I + \eta_7 Q - \eta_{24} R, \\
\frac{dV}{dt} &= \eta_{10} R - \eta_2 SV - \eta_{25} V + \eta_8 QV
\end{aligned} \tag{2.5}$$

where $\eta_{21} = \eta_1$, $\eta_{22} = \eta_1 + \eta_6 + \eta_9 + \eta_{11}$, $\eta_{23} = \eta_1 + \eta_4 + \eta_7$, $\eta_{24} = \eta_1 + \eta_5 + \eta_{10}$, *and* $\eta_{25} = \eta_1$
Subject to initial conditions: $S(0) = S_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0^0, V(0) = V_0$.

2.1. **FDE Model Formulation.** The corresponding system of fractional order differential equations model can be written as

$$\begin{aligned}
{}^C D^\delta S &= \Upsilon - \eta_{21} S + \eta_2 SV - \eta_3 SI + \eta_4 Q + \eta_5 R \\
{}^C D^\delta I &= \eta_3 SI - (\eta_{22})I, \\
{}^C D^\delta Q &= \eta_6 I - \eta_{23} Q - \eta_8 QV, \\
{}^C D^\delta R &= \eta_{11} I + \eta_7 Q - \eta_{24} R, \\
{}^C D^\delta V &= \eta_{10} R - \eta_2 SV - \eta_{25} V + \eta_8 QV
\end{aligned} \tag{2.6}$$

Subject to initial conditions: $S(0) = S_0, Q(0) = Q_0, I(0) = I_0, R(0) = R_0^0, V(0) = V_0$.

The system (2.6) can be written in the form:

$$\begin{aligned}
{}^C D^\delta \mathcal{X}(t) &= \mathcal{J}(t, \mathcal{X}(t)), \quad 0 < \delta \leq 1, \\
\mathcal{X}(0) &= \mathcal{X}_0.
\end{aligned} \tag{2.7}$$

To find the existence of solution, a Banach space is defined as $\mathcal{B} = B_1 \times B_2 \times B_3 \times B_4 \times B_5$, where $B_i = C([0, T]), (i = 1, 2, \ldots, 5)$ under the norm

$$\|\mathcal{X}\| = \| (S, I, Q, R, V) \| = \max_{t \in [0,t]} [|S(t)|, |I(t)|, |Q(t)|, |R(t)|, |V(t)|].$$

Let $\mathcal{Y} : \mathcal{A} \to \mathcal{A}$ be an operator defined as follows:

$$\mathcal{Y}(\mathcal{X})(t) = Y_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t - \gamma)^{\delta - 1} \mathcal{J}(\gamma, \mathcal{X}(\gamma)) d\gamma.$$

Using the Riemann Liouville type integral, equation (2.7) solved as follows: $\mathcal{X}(t) = \mathcal{X}_0 + \frac{1}{\Gamma(\delta)}\int_0^t (t-\gamma)^{\delta-1}\mathcal{J}(\gamma,\mathcal{X}(\gamma))d\gamma$ where

$$\begin{cases} S(t) = S_0 + \frac{1}{\Gamma(\delta)}\int_0^t (t-\gamma)^{\delta-1}\mathcal{J}(\gamma,S)d\gamma, \\ I(t) = I_0 + \frac{1}{\Gamma(\delta)}\int_0^t (t-\gamma)^{\delta-1}\mathcal{J}(\gamma,I)d\gamma, \\ Q(t) = Q_0 + \frac{1}{\Gamma(\delta)}\int_0^t (t-\gamma)^{\delta-1}\mathcal{J}(\gamma,Q)d\gamma, \\ R(t) = R_0 + \frac{1}{\Gamma(\delta)}\int_0^t (t-\gamma)^{\delta-1}\mathcal{J}(\gamma,R)d\gamma, \\ V(t) = V_0 + \frac{1}{\Gamma(\delta)}\int_0^t (t-\gamma)^{\delta-1}\mathcal{J}(\gamma,V)d\gamma, \end{cases} \tag{2.8}$$

with

$$\begin{cases} \mathcal{J}(\gamma,S) = \Upsilon - \eta_{21}S - \eta_3 SI + \eta_4 Q + \eta_5 R \\ \mathcal{J}(\gamma,I) = \eta_3 SI - (\eta_{22})I, \\ \mathcal{J}(\gamma,Q) = \eta_6 I - \eta_{23}Q - \eta_8 QV, \\ \mathcal{J}(\gamma,R) = \eta_{11} I + \eta_7 Q - \eta_{24}R, \\ \mathcal{J}(\gamma,V) = \eta_{10} R - \eta_2 SV - \eta_{25}V + \eta_8 QV \end{cases} \tag{2.9}$$

2.2. **Positivity and Existence of solution.** To investigate the non-negativity of the solution, we define

$R_+^5 = \left\{\mathcal{X} \in R^5 \mid \mathcal{X} \geq 0\right\}$ and $\mathcal{X}(t) = (S(t), Q(t), I(t), R(t), V(t))^T$

Now we remind the generalized mean values theorem [31].

Lemma 1. *Let* $\mathcal{X}(\mathrm{t}) \in \mathrm{C}[\mathrm{c},\mathrm{d}]$ *and* ${}^C D_t^\delta \mathcal{X}(\mathrm{t}) \in (\mathrm{c},\mathrm{d}]$*, then* $\mathcal{X}(t) = \mathcal{X}(c) + \frac{1}{\Gamma(\beta)}\left({}^C D_t^\delta \mathcal{X}\right)(\zeta)(t-c)^\delta$ *with* $c \leq \zeta \leq t, \forall t \in (c,d]$.

Corollary 1. *Let* $\mathcal{X}(\mathrm{t}) \in C[\mathrm{c},\mathrm{d}]$ *and* ${}^C D_t^\delta \mathcal{X}(\mathrm{t}) \in (\mathrm{c},\mathrm{d}]$ *where* $\delta \in (0,1]$. *Then, it is clear from lemma 1 that if* ${}^C D_t^\delta \mathcal{X}(\mathrm{t}) \geq 0, \forall \mathrm{t} \in (\mathrm{c},\mathrm{d}]$*, then the function* $\mathcal{X}(\mathrm{t})$ *is non-decreasing and if* ${}^C D_{\mathrm{t}}^\delta \mathcal{X}(\mathrm{t}) \leq 0, \forall \mathrm{t} \in (\mathrm{c},\mathrm{d}]$*, then the function* $\mathcal{U}(\mathrm{t})$ *is non-increasing for all* $t \in [c,d]$.

In order to demonstrate the non-negativity of the solution, one must examine the fact that the vector field points to R_+^5 on each hyperplane where the solution bounds the positive orthant.

Theorem 1. *If S(0), I(0), Q(0), R(0), V(0) are positive and bounded in* R_+^5*, then S(t), I(t), Q(t), R(t), V(t) are also positive and bounded in* R_+^5 *for all* $t > 0$.

Proof. From model (2.6), we get

$$\begin{aligned} {}^C D_t^\delta S(t)_{S=0} &= \Upsilon + \eta_4 Q + \eta_5 R \geq 0 \\ {}^C D_t^\delta I(t)_{I=0} &= 0 \geq 0 \\ {}^C D_t^\delta Q(t)_{Q=0} &= \eta_6 I \geq 0 \\ {}^C D_t^\delta R(t)_{R=0} &= \eta_{11} I + \eta_7 Q \geq 0 \\ {}^C D_t^\delta V(t)_{V=0} &= \eta_{10} R \geq 0 \end{aligned}$$

As a result, we may determine that our answer is nonnegative and will fall inside the specified feasible region by using the corollary 1.

Adding all the equations of the system in (2.6), we get

$${}^C D_t^\delta N = \Upsilon - \eta_1(S + I + Q + R + V) - \eta_{10} I \tag{2.10}$$

and in the infection free state we have ${}^C D_t^\delta N = \Upsilon - N\eta_1$. Thus by taking Laplace transform we get

$$N(s) = \frac{\Upsilon}{s(s^\delta + \eta_1)} + N(0)\frac{s^{(\delta-1)}}{s^\delta + \eta_1}, \tag{2.11}$$

By taking inverse Laplace transform and solving we get

$$\lim_{t\to\infty} N(t) \leq \frac{\Upsilon}{\eta_1}. \tag{2.12}$$

Then it follows the positivity and bounded for all t>0. □

The following theorems show the existence of a solution.

Theorem 2. *Let $S(t), I(t), Q(t), R(t)$, and $V(t)$ be nonnegative bounded functions. Then the system (2.9) satisfies Lipschitz cindition.*

Proof. Assume that $S(t), I(t), Q(t), R(t)$, and $V(t)$ are nonnegative bounded functions. That is, there are some positive constants $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$, such that

$\|S(t)\| \leq \xi_1, \|I(t)\| \leq \xi_2, \|Q(t)\| \leq \xi_3, \|R(t)\| \leq \xi_4, \|V(t)\| \leq \xi_5$.

Consider the function $\mathcal{J}(\gamma, S)$, for any S and S_1, we can get

$$\begin{aligned}
\|\mathcal{J}(\gamma, S) - \mathcal{J}(\gamma, S_1)\| &= \|\eta_1 (S - S_1) + \eta_2 V (S_1 - S) + \eta_3 I(S - S_1)\| \\
&\leq \|\eta_1 (S - S_1)\| + \|\eta_2 V (S - S_1)\| + \|\eta_3 I (S - S_1)\| \\
&\leq (\eta_1 + \eta_2\|V(t)\| + \eta_3\|I(t)\|) \|S - S_1\| \\
&\leq (\eta_1 + \eta_2\xi_5 + \eta_3\xi_2) \|S - S_1\| \\
&\leq \mathcal{G}_{\mathcal{J}_1} \|S - S_1\|
\end{aligned} \tag{2.13}$$

where $\mathcal{G}_{\mathcal{J}_1} = \eta_1 + \eta_2\xi_5 + \eta_3\xi_2$. Hence, $\mathcal{J}(\gamma, S)$ satisfies the Lipschitz condition. Similarly, we can find $\mathcal{G}_{\mathcal{J}_i}$, for $i = 2, 3, 4, 5$ so that $\mathcal{J}(\gamma, S), \mathcal{J}(\gamma, Q), \mathcal{J}(\gamma, I), \mathcal{J}(\gamma, R)$, and $\mathcal{J}(\gamma, V)$ satisfy the Lipschitz's conditions. □

Consider the equation (2.8), it can be formulated as :

$$X_n(t) = \begin{cases}
S_n(t) = S_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, S_{n-1}) d\gamma, \\
I_n(t) = I_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, I_{n-1}) d\gamma, \\
Q_n(t) = Q_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, Q_{n-1}) d\gamma, \\
R_n(t) = R_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, R_{n-1}) d\gamma, \\
V_n(t) = V_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, V_{n-1}) d\gamma.
\end{cases}$$

The first elements of the following equations are established by the given initial circumstances. Two terms are contrasted by using the following expression:

$$\begin{aligned}
\Psi_{1_n}(t) = S_n(t) - S_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, S_{n-1}) - \mathcal{J}(\gamma, S_{n-2})] d\gamma, \\
\Psi_{2_n}(t) = I_n(t) - I_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, I_{n-1}) - \mathcal{J}(\gamma, I_{n-2})] d\gamma, \\
\Psi_{3_n}(t) = Q_n(t) - Q_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, Q_{n-1}) - \mathcal{J}(\gamma, Q_{n-2})] d\gamma, \\
\Psi_{4_n}(t) = R_n(t) - R_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, R_{n-1}) - \mathcal{J}(\gamma, R_{n-2})] d\gamma, \\
\Psi_{5_n}(t) = V_n(t) - V_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, V_{n-1}) - \mathcal{J}(\gamma, V_{n-2})] d\gamma
\end{aligned}$$

where

$$X_n(t) = \begin{cases} S_n(t) = \sum_{i=0}^{n} \Psi_{1_i}(t), \\ I_n(t) = \sum_{i=0}^{n} \Psi_{2_i}(t), \\ Q_n(t) = \sum_{i=0}^{n} \Psi_{3_i}(t), \\ R_n(t) = \sum_{i=0}^{n} \Psi_{4_i}(t), \\ V_n(t) = \sum_{i=0}^{n} \Psi_{5_i}(t). \end{cases} \tag{2.14}$$

Consider

$$\|\Psi_{1_n}(t)\| = \|S_n(t) - S_{n-1}(t)\| = \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, S_{n-1}) - \mathcal{J}(\gamma, S_{n-1})]\, d\gamma$$

$$= \frac{\xi_1}{\Gamma(\delta)} \int_0^t \|S_{n-1} - S_{n-2}\|\, d\gamma = \frac{\xi_1}{\Gamma(\delta)} \int_0^t \left\|\Psi_{1_{n-1}}(t)\right\| d\gamma$$

Hence, we can get

$$\|\Psi_{i_n}(t)\| = \frac{\xi_i}{\Gamma(\delta)} \int_0^t \left\|\Psi_{i_{n-1}}(t)\right\| d\gamma \ for\ i = 1, 2, \ldots 5. \tag{2.15}$$

Now the functions defined in (2.14) are exist and smooth. For,

We have that the functions $S(t), I(t), Q(t), R(t)$, and $V(t)$ are bounded and all kernels $\mathcal{J}(t, S), \mathcal{J}(t, I), \mathcal{J}(t, Q),$ fulfill Lipschitz's conditions, thus, we obtain the following relations:

$$\begin{cases} \|\Psi_{1_n}(t)\| \leq \|S(0)\| \left\|\frac{\xi_1}{\Gamma(\delta)} t\right\|^n, \\ \|\Psi_{2_n}(t)\| \leq \|I(0)\| \left\|\frac{\xi_2}{\Gamma(\delta)} t\right\|^n, \\ \|\Psi_{3_n}(t)\| \leq \|Q(0)\| \left\|\frac{\xi_3}{\Gamma(\delta)} t\right\|^n, \quad . \\ \|\Psi_{4_n}(t)\| \leq \|R(0)\| \left\|\frac{\xi_4}{\Gamma(\delta)} t\right\|^n, \\ \|\Psi_{5_n}(t)\| \leq \|V(0)\| \left\|\frac{\xi_5}{\Gamma(\delta)} t\right\|^n. \end{cases} \tag{2.16}$$

The system (2.16) shows the existence and smoothness of the function defined in (2.15).

Theorem 3. *Let $\mathcal{Y} : \mathcal{A} \to \mathcal{A}$ be completely continuous and let $\mathcal{J} : [0, T] \times \mathcal{A} \to \mathbb{R}$ is continuous and there exists a constant $\mathcal{G}_{\mathcal{J}} > 0$ such that for $X, X_1 \in \mathcal{A}$,*

$$|\mathcal{J}(t, X) - \mathcal{J}(t, X_1)| \leq \mathcal{G}_{\mathcal{J}} |X - X_1|$$

is hold. Then there is at least one solution for the considered system (2.6).

Proof. To prove the operator $\mathcal{Y}$ is completely continuous. The sequence $\{X_n\}$ converges to $X \in \mathcal{A}$. For, after n-iterations define the remainder terms as $D_{1_n}(t), D_{2_n}(t), D_{3_n}(t), D_{4_n}(t), D_{5_n}(t)$, such that

$$\begin{aligned} S(t) - S(0) &= S_n(t) - D_{1_n}(t), \\ I(t) - I(0) &= I_n(t) - D_{2_n}(t), \\ Q(t) - Q(0) &= Q_n(t) - D_{3_n}(t), \\ R(t) - R(0) &= R_n(t) - D_{4_n}(t), \\ V(t) - V(0) &= V_n(t) - D_{5_n}(t). \end{aligned}$$

Using triangle inequality along with the Lipschitz condition of $\mathcal{J}(t, S)$, we obtain:

$$\|D_{1_n}(t)\| = \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, S) - \mathcal{J}(\gamma, S_{n-1})]\, d\gamma \leq \frac{\xi_1}{\Gamma(\delta)} \|S - S_{n-1}\|\, t.$$

Applying the above process recursively, we get

$$\|D_{1_n}(t)\| \leq \left\| \frac{C_1}{\Gamma(\delta)} t \right\|^{n+1} \xi_1.$$

Then, at t_0

$$\|D_n(t)\| \leq \left\| \frac{C_1}{\Gamma(\delta)} t_0 \right\|^{n+1} \xi_1.$$

Taking limit as n tends to infinity, we get

$$\lim_{n\to\infty} \|D_{1_n}(t)\| \leq \lim_{n\to\infty} \left\| \frac{C_1}{\Gamma(\delta)} t_0 \right\|^{n+1} \xi_1. \tag{2.17}$$

For $\frac{C_i}{\Gamma(\delta)} t_0 < 1$, Equation (2.17) becomes $\lim_{n\to\infty} \|D_{1_n}(t)\| = 0$ Similarly, as n tends to infinity, we can get $\|D_{i_n}(t)\| \to 0$.

Hence for $t \in [0, T]$, we have $S_n(t) \to S(t)$ *as* $n \to \infty$

$$\begin{aligned}
\|\mathcal{Y}(S_n) - \mathcal{Y}(S)\| &\leq \frac{1}{\Gamma(\delta)} \max_{t\in[0,T]} \int_0^t (t-\gamma)^{\delta-1} - |\mathcal{J}(\gamma, S_n(\gamma))\mathcal{J}(\gamma, S(\gamma))| d\gamma, \\
&\leq \frac{\mathcal{G}_{\mathcal{J}}}{\Gamma(\delta)} \|S_n - S\|_{t\to[0,T]} \int_0^t (t-\gamma)^{\delta-1} d\gamma \leq \frac{T^\delta \mathcal{G}_{\mathcal{J}}}{\Gamma(\delta+1)} \|S_n - S\|.
\end{aligned}$$

Since, $S_n \to S$, so $\|\mathcal{Y}(S_n) - \mathcal{Y}(S)\| \to 0$ as $n \to \infty$ and hence $\|\mathcal{Y}(X_n) - \mathcal{Y}(X)\| \to 0$ as $n \to \infty$. Thus $\mathcal{Y}$ is continues. Let a bounded set $\mathcal{M} \subset \mathcal{A}$. Then by definition of $\mathcal{A}$, $|\mathcal{J}(\mathrm{t}, X(t))| \leq \mathcal{L}_{\mathcal{J}}, \mathcal{L}_{\mathcal{J}} > 0, \forall X \in \mathcal{M}$. Then for each $X \in \mathcal{M}$, we can obtain

$$\begin{aligned}
\|\mathcal{Y}(X)\| &\leq \frac{1}{\Gamma(\delta)} \max_{t\to[0,T]} \int_0^t (t-\gamma)^{\delta-1} |\mathcal{J}(\gamma, Y(\gamma))| d\gamma \\
&\leq \frac{\mathcal{L}_{\mathcal{J}}}{\Gamma(\delta)} \max_{t\to[0,T]} \int_0^t (t-\gamma)^{\delta-1} d\gamma \leq \frac{T^\delta \mathcal{L}_{\mathcal{J}}}{\Gamma(\delta+1)}
\end{aligned}$$

Thus, $\mathcal{Y}$ is uniformly bounded. Further suppose $0 \leq t_2 \leq t_1 \leq T$. Then

$$\begin{aligned}
\|\mathcal{Y}(X)(t_1) - \mathcal{Y}(X)(t_2)\| &\leq \frac{\mathcal{L}_{\mathcal{J}}}{\Gamma(\delta)} \max_{t\to[0,T]} | \int_0^{t_1} (t_1-\gamma)^{\delta-1} d\gamma - \int_0^{t_2} (t_2-\gamma)^{\delta-1} d\gamma|, \\
&\leq \frac{\mathcal{L}_{\mathcal{J}}}{\Gamma(\delta+1)} max_{t\to[0,T]} |t_1^\delta - t_2^\delta| \to 0 \ as \ t_1 \to t_2.
\end{aligned}$$

Thus, $\mathcal{Y}$ is equicontinuous. $\mathcal{Y}$ is compact and hence it is completely continuous because of the continuousness and boundedness of it . Let $\Psi = \{X \in \mathcal{A} : X = \rho\mathcal{Y}(X), \rho \in [0, 1]\}$, we need to confirm that Ψ is bounded. Suppose $X \in \Psi$, say S then for $t \in$ [0, T] , we have:

$$\begin{aligned}
\|S\| &= max_{t\to[0,T]} \{ \frac{\rho}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, S(\gamma)) d\gamma \} \\
&\leq \frac{\mathcal{L}_{\mathcal{J}}}{\Gamma(\delta)} \max_{T\to[0,T]} \int_0^t (t-\gamma)^{\delta-1} d\gamma \leq \frac{T^\delta \mathcal{L}_{\mathcal{J}}}{\Gamma(\delta+1)}.
\end{aligned}$$

Thus the operator is completely continuous. The set Ψ is also bounded. Therefore, $\mathcal{Y}$ has at least one fixed point[19]. So, the considered system (2.6) has the same number of solutions. □

Theorem 4. *If* $\frac{\xi_i}{\Gamma(\delta)} t < 1$*, for* $i = 1, 2, \cdots, 5$*, then the system (2.6) has a unique solution.*

Proof. . Assume that $\{S_\nu(t), I_\nu(t), Q_\nu(t), R_\nu(t), V_\nu(t)\}$ is another set of solutions of system (2.6) then,

$$\|S(t) - S_\nu(t)\| = \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, S) - \mathcal{J}(\gamma, S_\nu)]\, d\gamma = \frac{\xi_1}{\Gamma(\delta)} t \|S(t) - S_\nu(t)\|$$

Thus

$$(1 - \frac{\xi_1}{\Gamma(\delta)} t)\|S(t) - S_1(t)\| \leq 0. \tag{2.18}$$

Since $\frac{\xi_i}{\Gamma(\delta)} t < 1$ for i = 1, (2.18) becomes $\|S(t) - S_\nu(t)\| = 0$

Hence $S(t) = S_\nu(t)$. Similarly, for $i = 2, 3, 4\ and\ 5$, we can get $I(t) = I_\nu(t);\ Q(t) = Q_\nu(t);\ R(t) = R_\nu(t);\ V_\nu(t) = V_\nu(t)$. Hence the system has unique (2.6) solution. □

2.3. **Existence of Steady State Solutions.** Because R_0 and its steady-state solutions play a major role in the long-term behaviour of the system, research has been done into the steady-state solutions. Two steady-state solutions exist for the model that is the subject of this research. Time-independent solutions are achieved by making the model system (1) static.

The steady-state solutions, or $I = 0$, in the absence of infections are provided by

$$\begin{aligned} E^0 &= (S, I, Q, R, V) \\ &= (\frac{\Gamma}{\eta_1}, 0, 0, 0, 0) \end{aligned} \tag{2.19}$$

Also, the steady-state solutions when infection is persistent i.e., $I \neq 0$ is given by,

$$\begin{aligned} E^* &= (S^*, I^*, Q^*, R^*, V^*) \\ &= (\frac{\eta_{22}}{\eta_3}, \frac{\eta_1\eta_{22}(R_0 - 1)}{\eta_3(\eta_3\eta_{22} - \eta_4 A + \eta_5 B - \eta_2\eta_{22}C)}, \\ &\quad \frac{\eta_6 I^*}{\eta_{23} + \eta_8 V^*}, \frac{(\eta_6\eta_7 + \eta_{11}[\eta_{23} + \eta_8 V^*])I^*}{\eta_{24}(\eta_{23} + \eta_8 V^*)}, \\ &\quad \frac{\eta_{10}\eta_3 B I^*}{\eta_2\eta_{22} + \eta_1\eta_3 + \eta_1\eta_8 Q^*}) \end{aligned} \tag{2.20}$$

where,

$A = \frac{\eta_6}{\eta_{23}+\eta_8 V^*}, B = \frac{(\eta_6\eta_7+\eta_{11}[\eta_{23}+\eta_8 V^*])}{\eta_{24}(\eta_{23}+\eta_8 V^*)}, C = \frac{\eta_{10}\eta_3 B}{\eta_2\eta_{22}+\eta_1\eta_3+\eta_1\eta_8 Q^*}$

The fundamental reproduction number R_0 is from the next generation method ([16], [41]) as follows:

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_3 S & 0 & 0 & 0 \\ 0 & \eta_6 & 0 & 0 & 0 \\ 0 & \eta_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \eta_1 & 0 & \eta_4 & \eta_5 & 0 \\ 0 & \eta_1 + \eta_9 + \eta_{11} + \eta_6 & 0 & 0 & 0 \\ 0 & \eta_6 & \eta_1 + \eta_4 + \eta_7 & 0 & 0 \\ 0 & \eta_{11} & \eta_7 & \eta_1 + \eta_5 + \eta_{10} & 0 \\ 0 & 0 & 0 & 0 & \eta_1 \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\eta_1} & \frac{\eta_5(\eta_{11}\eta_{23}-\eta_6\eta_{24})+\eta_4\eta_6\eta_{24}}{\eta_1\eta_{22}\eta_{23}\eta_{24}} & \frac{\eta_5\eta_7-\eta_4\eta_{24}}{\eta_1\eta_{23}\eta_{24}} & -\frac{\eta_5}{\eta_1\eta_{24}} & 0 \\ 0 & \frac{1}{\eta_{22}} & 0 & 0 & 0 \\ 0 & -\frac{\eta_6}{\eta_{22}\eta_{23}} & \frac{1}{\eta_{23}} & 0 & 0 \\ 0 & \frac{\eta_{11}\eta_{23}+\eta_6\eta_7}{\eta_{22}\eta_{23}\eta_{24}} & -\frac{\eta_7}{\eta_{23}\eta_{24}} & \frac{1}{\eta_{24}} & 0 \\ 0 & \frac{\eta_{10}(\eta_{11}\eta_{23}-\eta_6\eta_7)}{\eta_{22}\eta_{23}\eta_{24}} & \frac{\eta_7\eta_{10}}{\eta_1\eta_{23}\eta_{24}} & \frac{\eta_{10}}{\eta_1\eta_{24}} & \frac{1}{\eta_1} \end{pmatrix}$$

Then, R_0 is the largest eigenvalue of the spectral radius given by

$$R_0 = \rho(FV^{-1}) = \eta_3\Big(\frac{\Upsilon}{\eta_1(\eta_1+\eta_6+\eta_9+\eta_{11})}\Big) \tag{2.21}$$

The infections disappear from the host population at $R_0 < 1$. However, if $R_0 > 1$, the illness spreads widely and becomes endemic, necessitating the use of suitable medical treatments to halt the disease's progression.

3. Stability Analysis

Theorem 5. *The disease-free equilibrium E_0 of the given system (2.6) is stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. To state the stability analysis of the disease-free equilibrium points, we analyse the linearization of the given system (2.6) at any equilibrium point $(S^*, I^*, Q^*, R^*, V^*)$ as follows:

$$\begin{aligned} {}^{C}D^{\delta}S &= \Upsilon - \eta_{21}S + \eta_2 S^*V + \eta_2 SV^* - \eta_3 S^*I - \eta_3 SI^* + \eta_4 Q + \eta_5 R \\ {}^{C}D^{\delta}I &= \eta_3 S^*I + \eta_3 SI^* - \eta_{22}I, \\ {}^{C}D^{\delta}Q &= \eta_6 I - \eta_{23}Q - \eta_8 Q^*V - \eta_8 QV^*, \\ {}^{C}D^{\delta}R &= \eta_{11}I + \eta_7 Q - \eta_{24}R, \\ {}^{C}D^{\delta}V &= \eta_{10}R - \eta_2 S^*V - \eta_2 SV^* - \eta_{25}V + \eta_8 Q^*V + \eta_8 QV^* \end{aligned} \tag{3.1}$$

Applying the Laplace transform on both sides of above system (3.1) gives

$$\begin{aligned} s^{\delta}\mathcal{L}[S(s)] - s^{\delta-1}S(0) &= \Upsilon - \eta_1\mathcal{L}[S(s)] + \eta_2 S^*\mathcal{L}[V(s)] + \eta_2 V^*\mathcal{L}[S(s)] \\ &\quad -\eta_3 S^*\mathcal{L}[I(s)] - \eta_3 I^*\mathcal{L}[S(s)] + \eta_4\mathcal{L}Q + \eta_5 R \\ s^{\delta}\mathcal{L}[I(s)] - s^{\delta-1}I(0) &= \eta_3 S^*\mathcal{L}[I(s)] + \eta_3 I^*\mathcal{L}[S(s)] - \eta_{22}\mathcal{L}[I(s)], \\ s^{\delta}\mathcal{L}[Q(s)] - s^{\delta-1}Q(0) &= \eta_6\mathcal{L}[I(s)] - \eta_{23}\mathcal{L}[Q(s)] - \eta_8 Q^*\mathcal{L}[V(s)] \\ &\quad -\eta_8 V^*\mathcal{L}[Q(s)], \\ s^{\delta}\mathcal{L}[R(s)] - s^{\delta-1}R(0) &= \eta_{11}\mathcal{L}[I(s)] + \eta_7\mathcal{L}[Q(s)] - \eta_{24}\mathcal{L}[R(s)], \\ s^{\delta}\mathcal{L}[V(s)] - s^{\delta-1}V(0) &= \eta_{10}\mathcal{L}[R(s)] - \eta_2 S^*\mathcal{L}[V(s)] - \eta_2 V^*\mathcal{L}[S(s)] \\ &\quad -\eta_{25}\mathcal{L}[V(s)] + \eta_8 Q^*\mathcal{L}[V(s)] + \eta_8 V^*\mathcal{L}[Q(s)] \end{aligned} \tag{3.2}$$

where $\mathcal{L}[S(s)], \mathcal{L}[I(s)], \mathcal{L}[Q(s)], \mathcal{L}[R(s)]$, and $\mathcal{L}[I_(s)]$ are the Laplace transformations of $S(t), Q(t), I(t), R(t)$, and $V(t)$. The proposed system (3.2) can be rewritten by

$$\Delta(s)\cdot[L[S(s)]L[I(s)]L[Q(s)]L[R(s)]L[V(s)]] = [\nu_1(s)\nu_2(s)\nu_3(s)\nu_4(s)\nu_5(s)]$$

where

$$\begin{pmatrix} \nu_1(s) = s^{\delta-1}S(0) \\ \nu_2(s) = s^{\delta-1}I(0) \\ \nu_3(s) = s^{\delta-1}Q(0) \\ \nu_4(s) = s^{\delta-1}R(0) \\ \nu_5(s) = s^{\delta-1}V(0) \end{pmatrix}$$

Hence $\Delta(s) =$

$$\begin{bmatrix} A_{11} & -\eta_3 S^* & \eta_4 & \eta_5 & \eta_2 S^* \\ \eta_3 I^* & A_{22} & 0 & 0 & 0 \\ 0 & \eta_6 & s^\delta + \eta_{23} & 0 & -\eta_8 Q^* \\ 0 & \eta_{11} & \eta_7 & s^\delta + \eta_{24} & 0 \\ -\eta_2 V^* & 0 & \eta_8 V^* & \eta_{10} & A_{55} \end{bmatrix}$$

where $A_{11} = s^\delta + (\eta_1 + \eta_3 I^* + \eta_2 V^*)$, $A_{22} = s^\delta + \eta_{22} - \eta_3 S^*$ and $A_{55} = s^\delta + \eta_1 + \eta_2 S^* - \eta_8 Q^*$,

which is a characteristic matrix of system (3.2). Now, the characteristic matrix of the proposed system at disease-free equilibrium (DFE) (2.19) is given by $\Delta(s) =$

$$\begin{bmatrix} s^\delta + \eta_1 & -\eta_3 S & \eta_4 & \eta_5 & \eta_2 S \\ 0 & B_{22} & 0 & 0 & 0 \\ 0 & \eta_6 & s^\delta + \eta_{23} & 0 & 0 \\ 0 & \eta_{11} & \eta_7 & s^\delta + \eta_{24} & 0 \\ 0 & 0 & 0 & \eta_{10} & B_{55} \end{bmatrix}$$

where $B_{22} = s^\delta - \eta_3 S + \eta_{22}$ and $B_{55} = s^\delta + \eta_1 + \eta_2 S$.

Then from the Jacobian matrix, the characteristic polynomial is $(s^\delta + \eta_1)(s^\delta + (\eta_1 + \eta_4 + \eta_6 + \eta_7))(s^\delta + (\eta_1 + \eta_4 + \eta_{10}))(s^\delta + (-\eta_3 S - (\eta_1 + \eta_6 + \eta_9 + \eta_{11})))(s^\delta + (\eta_1 + \eta_5 + \eta_{10}))(s^\delta + (\eta_1 + \eta_2 S))$.

Now, the System (2.4) is stable iff $\eta_3 S - (\eta_1 + \eta_6 + \eta_9 + \eta_{11}) < 0$.

Hence $\eta_3\big(\frac{\Gamma}{\eta_1(\eta_1+\eta_6+\eta_9+\eta_{11})}\big) < 1$.

Then clearly the infection free steady state of (2.19) is locally asymptotically stable if $R_0 < 1$. □

4. Numerical Analysis

India had a substantial number of infections during the second Coronavirus wave. For this article, we have collected data from Tamilnadu, India ([10]). The numerical solution is simulated using Matlab and Mathematica. The Tables 2 and 3 below contain the values of the variables and parameters.

Table 2. Variables in Model and their values

Variable	Descriptions	Values
$S(0)$	Susceptible individuals	30095
$Q(0)$	Quarantine Individuals	322
$I(0)$	Infected individuals	35
$R(0)$	Recovered individuals	51
$V\ (0)$	Vaccinated Individuals	42846

As can be seen in the figures, the solution for (2.6) shows that it is unstable locally and will never become stable when $R_0 > 1$. When the vaccination class is raised and the contact rate is regulated at $R_0 < 1$, the steady state solution becomes stable. Based on all of the information, we may conclude that the host community will be protected from the Omicron variation if there is an increase in the number of separated, recovered, and vaccinated individuals. We also found that the spread of the second wave of the SARS Cov-2 Omicron variant is inhibited if the intercessions are faithfully adhered to.

The impact of the Omicron variant in the susceptible and infected individuals is shown against time t for the entire state of Tamilnadu, as shown in Figure 2. Increased immunisation and quarantine are making people less likely to contract infections and less susceptible to infection.

In the state of Tamilnadu, Figure 3 shows the population's recovered and quarantined individuals, respectively, over time t. Figures 2 and 3 illustrate how individuals in Tamilnadu were initially

TABLE 3. Parameters in Model and their values

Parameters	Values
Υ	5
η_1	0.065
η_2	0.0109
η_3	0.0012
η_4	0.0098
η_5	0.0017
η_6	0.1087
η_7	0.0146
η_8	0.0098
η_9	0.0006
η_{10}	0.92
η_{11}	0.045

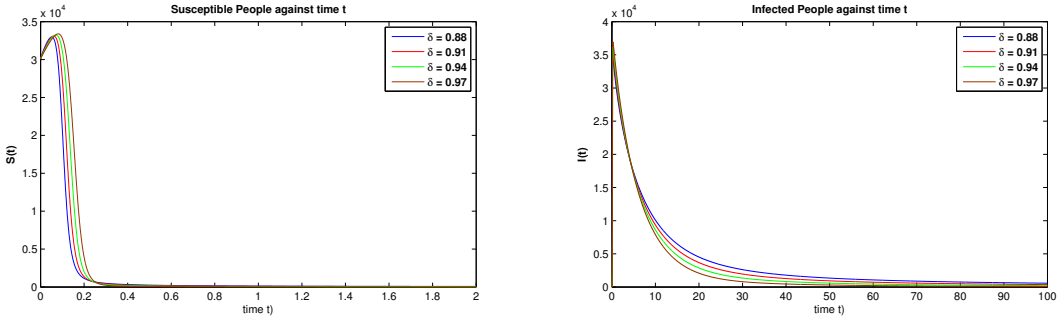

FIGURE 2. Susceptible E(t) and Infected I(t) people against time t in the data of Tamilnadu

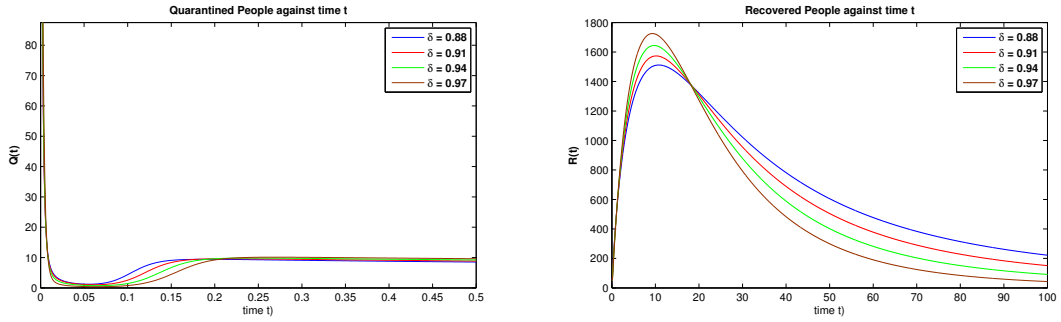

FIGURE 3. Quarantined and recovered people against time t

infected and verified to have the Omicron variant, and how they recovered by March 31st, 2022. Figures 2 and 3 make it clear that as the diseased population grows, so do all other compartments.

Based on RT PCR sample tests, as of March 31, Table 3 shows that the number of infected individuals in Tamilnadu districts has dropped to a low level and there have been no deaths.

The mathematical model for Tamilnadu's Omicron stability is shown in Figure (4). There is a significant rate of disease among the residents of these four districts from December 25 to March 11,

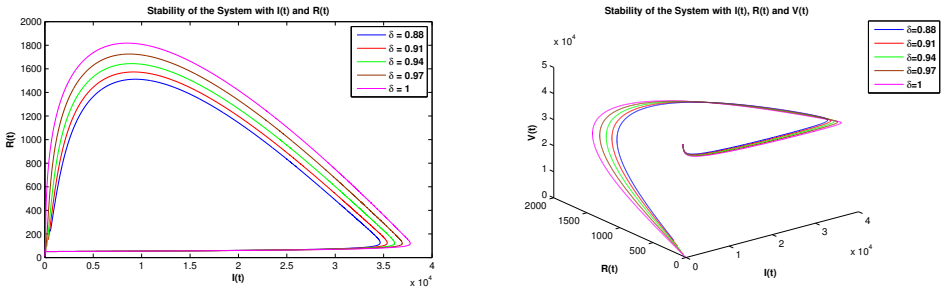

FIGURE 4. Stability condition of the fractionsl model against time t with I(t) and R(t) and Stability condition against time t with I(t), R(t) and V(t) -3D Plot

2022, which is known as the Omicron period. Following government vaccination guidelines resulted in a steady decline in the illness rate until it reached a low point.

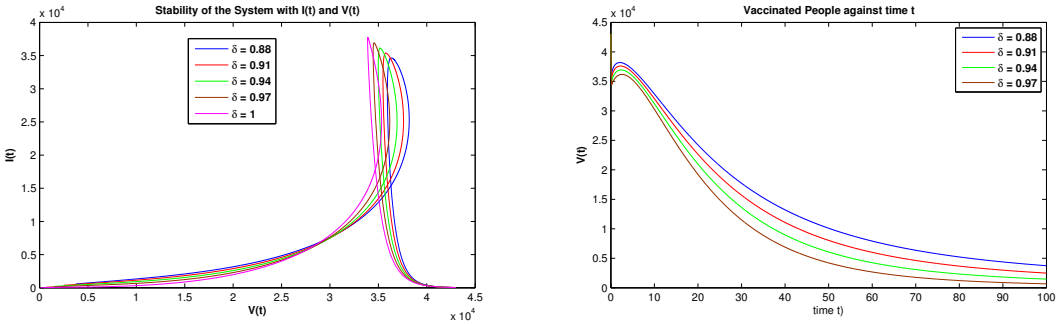

FIGURE 5. Infected and Vaccinated against time t in Tamilnadu

The relationship between the rates of vaccinated, isolated, and infected individuals during the Omicron infection phase in Tamilnadu state is depicted in Figure 5.

The stability graph representation of the built model in the Tamilnadu host population with different order of δ is demonstrated in Figure 6. The Omicron variation spread quickly when it was initially found, as Figure 6 illustrates. However, the variant's spread was slowed to a safe level when the government imposed quarantine and mass vaccination programs. The state of Tamilnadu learnt on March 31, 2022, that Omicron's death was not the result of human error. Vaccination against the Covid-19 form of SARS CoV-2 Omicron helped patients prevent infection.

Figure 6 and Table 3 show that, following a quick spread over a brief period of time, the infection rate declines with a reproduction number of $R_0 < 1$. As a result, the four districts' systems and Tamilnadu's overall system are stable.

5. Conclusions

An SIQRV fractional order mathematical model for COVID-19 was created in this work. According to our mathematical model and the data gathered from Tamil Nadu, the Coronavirus strain of COVID-19 infection appears to have steadied after a few months. By accounting for the nonlinear forces of quarantine, vaccination, infection, and care, as well as the appropriate inclusion of important parameters, this model performs better than previous mathematical models. In Tamilnadu, where $R_0 < 1$, the principles of reproduction number computed with this model serve as an outbreak threshold that determines whether or not the disease spreads. The essential elements of positivity and boundedness of this model have been investigated and confirmed. When $R_0 < 1$,

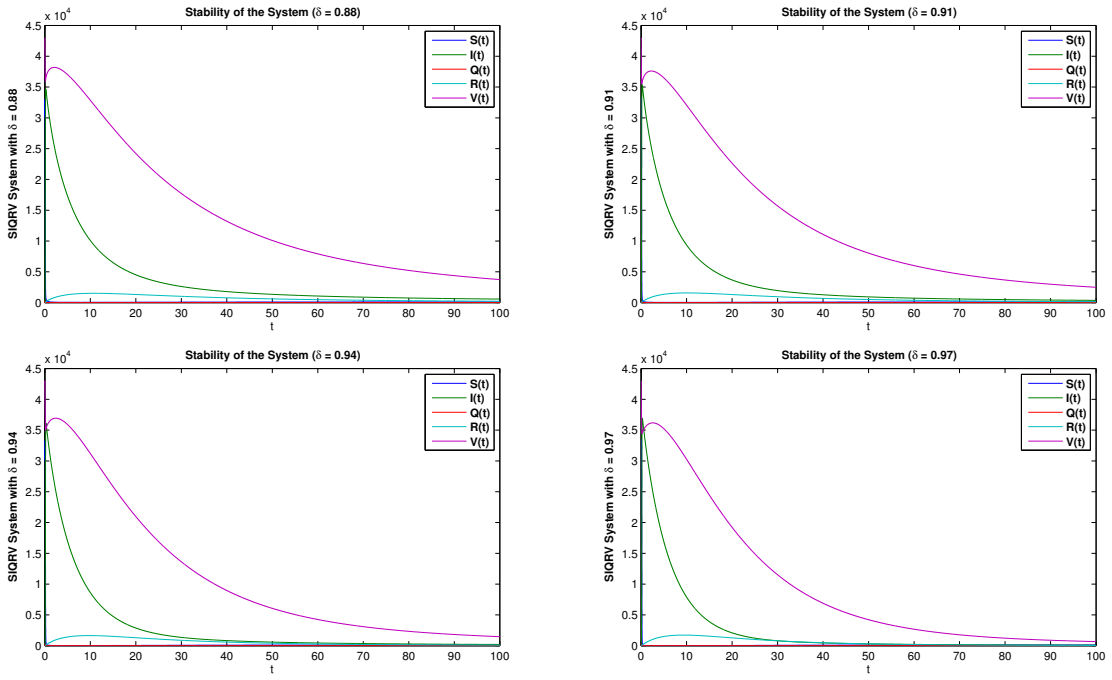

FIGURE 6. Stability of the given Omicron System against time t with repect of four different $\delta's$

there exist infection-free steady-state solutions that are both locally and globally asymptotically stable. Furthermore Stable locally infection-present steady-state solutions are found when $R_0 < 1$ is present. Lastly, validation of the Omicron variant pandemic data from Tamilnadu, India, has been completed.

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Availability of data and materials. Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Authors' contributions. All authors contributed equally to this work. The authors declare that they have read and approved the final manuscript.

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