

Quasi rational solutions to the fifth equation of the NLS hierarchy

Abstract

Quasi rational solutions to the fifth equation of the NLS hierarchy are constructed. We give explicit expressions of these solutions for the first orders depending on multi-parameters. We study the patterns of these solutions in the (x, t) plane according the different values of the parameters.

Key Words : NLS hierarchy, rational solutions.

PACS numbers :

33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd

1 Introduction

We consider the fifth equation of the NLS hierarchy of order 5 (*NLS5*) which can be written as

$$\begin{aligned} & iu_y + u_{6x} + 12|u|^2 p_{4x} + 2p^2 \bar{u}_{4x} + 30u_{3x} u_x \bar{u} + 18u_{3x} u \bar{u}_x + 8u_x u \bar{u}_{3x} \\ & + 50u_{2x} |u_x|^2 + 50u_{2x} |u|^4 + 20u_{2x}^2 \bar{u} + 22|u_{2x}|^2 |u| + 20u_x^2 \bar{u}_{2x} + 20|u|^2 u^2 \bar{u}_{2x}, \quad (1) \\ & + 10u^3 \bar{u}_x^2 + 70u_x^2 |u|^2 \bar{u} + 60|u|^2 |u_x|^2 u + 20|u|^6 u \end{aligned}$$

with as usual the subscript meaning the partial derivatives and \bar{u} the complex conjugate of u .

This equation (1) is part of the hierarchy of NLS equations, as the NLS equation [1, 2], the first equation of this hierarchy, the mKdV equation [3, 4] which is the second one, the LPD equation [5, 6, 7] which is the third one.

Here, we construct explicitly rational solutions for the first orders and study the patterns of the modulus of the solutions in the (x, t) plane.

2 Quasi rational solutions of order 1 to the NLS5 equation

Theorem 2.1 *The function $v(x, t)$ defined by*

$$v(x, t) = -\frac{(3 - 4x^2 - 14400t^2 + 480it) e^{20it}}{1 + 4x^2 + 14400t^2} \quad (2)$$

is a solution to the (NLS5) equation (1)

$$\begin{aligned} &iu_y + u_{6x} + 12|u|^2 p_{4x} + 2p^2 \bar{u}_{4x} + 30u_{3x} u_x \bar{u} + 18u_{3x} u \bar{u}_x + 8u_x u \bar{u}_{3x} \\ &+ 50u_{2x} |u_x|^2 + 50u_{2x} |u|^4 + 20u_{2x}^2 \bar{u} + 22|u_{2x}|^2 |u| + 20u_x^2 \bar{u}_{2x} + 20|u|^2 u^2 \bar{u}_{2x}, \\ &+ 10u^3 \bar{u}_x^2 + 70u_x^2 |u|^2 \bar{u} + 60|u|^2 |u_x|^2 u + 20|u|^6 u \end{aligned}$$

Proof: It is sufficient to replace the expression of the solution given by (2) and check that (1) is verified.

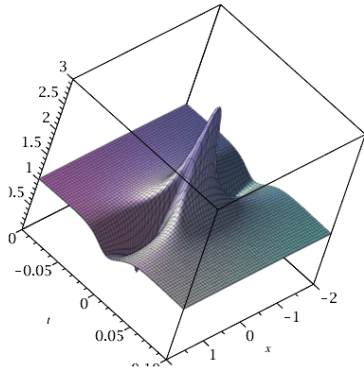


Figure 1. Solution of order 1 to (NLS5).

We get a smooth solution of the equation (1).

3 Quasi rational solutions of order 2 of the NLS5 equation depending on 2 real parameters

Theorem 3.1 *The function $v(x, t)$ defined by*

$$v(x, t) = \frac{n(x, t)}{d(x, t)} \quad (3)$$

with

$$n(x, t) = -(-64x^6 + 2304b_1x^5 + 768ia_1x^4 - 34560b_1^2x^4 - 691200t^2x^4 + 23040itx^4 -$$

$$\begin{aligned}
& 46080 a_1 t x^4 - 768 a_1^2 x^4 + 144 x^4 - 18432 i a_1 x^3 b_1 - 552960 i t x^3 b_1 + 16588800 b_1 t^2 x^3 + \\
& 1105920 b_1 a_1 t x^3 + 18432 b_1 a_1^2 x^3 - 4992 b_1 x^3 + 276480 b_1^3 x^3 + 5760 a_1^2 x^2 - 9953280 b_1^2 a_1 t x^2 + \\
& 180 x^2 + 165888 i a_1 b_1^2 x^2 - 16588800 a_1^2 t^2 x^2 + 552960 i a_1^2 t x^2 - 165888 b_1^2 a_1^2 x^2 - \\
& 368640 a_1^3 t x^2 + 529920 a_1 t x^2 - 1152 i a_1 x^2 - 331776000 a_1 t^3 x^2 + 4976640 i t b_1^2 x^2 + \\
& 16588800 i a_1 t^2 x^2 + 165888000 i t^3 x^2 + 10713600 t^2 x^2 - 2488320000 t^4 x^2 - 1244160 b_1^4 x^2 + \\
& 58752 b_1^2 x^2 - 3072 a_1^4 x^2 - 126720 i t x^2 - 149299200 b_1^2 t^2 x^2 + 6144 i a_1^3 x^2 - 19906560 i t x b_1^3 + \\
& 36864 b_1 a_1^4 x - 111974400 b_1 t^2 x + 4423680 b_1 a_1^3 t x - 290304 b_1^3 x + 39813120 b_1^3 a_1 t x + \\
& 3981312000 b_1 a_1 t^3 x + 967680 i t b_1 x - 73728 i a_1^3 x b_1 - 663552 i a_1 x b_1^3 - 4608 i a_1 x b_1 - \\
& 6635520 i a_1^2 t x b_1 - 199065600 i a_1 t^2 x b_1 - 1990656000 i t^3 x b_1 - 50688 b_1 a_1^2 x + 597196800 b_1^3 t^2 x - \\
& 5616 b_1 x + 2985984 b_1^5 x + 199065600 b_1 a_1^2 t^2 x + 663552 b_1^3 a_1^2 x - 5253120 b_1 a_1 t x + \\
& 29859840000 b_1 t^4 x + 207360000 t^4 - 619200 t^2 + 597196800 i a_1 t^2 b_1^2 - 55296000 a_1^4 t^2 - \\
& 2211840000 a_1^3 t^3 + 19906560 i a_1^2 t b_1^2 + 23500800 a_1^2 t^2 + 248832000 a_1 t^3 - 597196800000 a_1 t^5 + \\
& 1536 i a_1^3 - 2985984000000 t^6 + 298598400000 i t^5 - 110592 b_1^2 a_1^4 + 96768 b_1^2 a_1^2 + \\
& 158400 a_1 t + 12288 i a_1^5 + 373248000 i t^3 - 44640 i t - 45 - 995328 b_1^4 a_1^2 - 895795200 b_1^4 t^2 - \\
& 597196800 b_1^2 a_1^2 t^2 - 49766400000 a_1^2 t^4 - 737280 a_1^5 t + 286156800 b_1^2 t^2 + 995328 i a_1 b_1^4 + \\
& 29859840 i t b_1^4 + 506880 i a_1^2 t + 26265600 i a_1 t^2 - 720 i a_1 + 221184 i a_1^3 b_1^2 + 5971968000 i t^3 b_1^2 + \\
& 768000 a_1^3 t - 13271040 b_1^2 a_1^3 t - 1244160 i t b_1^2 - 11943936000 b_1^2 a_1 t^3 - 59719680 b_1^4 a_1 t + \\
& 3317760000 i a_1^2 t^3 + 49766400000 i a_1 t^4 + 518400 b_1^4 + 8448 a_1^4 - 2985984 b_1^6 - \\
& 4096 a_1^6 + 1872 a_1^2 + 18000 b_1^2 - 89579520000 b_1^2 t^4 + 69120 i a_1 b_1^2 + 12441600 b_1^2 a_1 t + \\
& 1843200 i a_1^4 t + 110592000 i a_1^3 t^2) e^{2i(a_1+10t)}
\end{aligned}$$

and

$$\begin{aligned}
d(x, t) = & 64 x^6 - 2304 b_1 x^5 + 46080 a_1 t x^4 + 48 x^4 + 691200 t^2 x^4 + 768 a_1^2 x^4 + \\
& 34560 b_1^2 x^4 - 18432 b_1 a_1^2 x^3 - 1105920 b_1 a_1 t x^3 + 384 b_1 x^3 - 16588800 b_1 t^2 x^3 - \\
& 276480 b_1^3 x^3 + 2488320000 t^4 x^2 + 149299200 b_1^2 t^2 x^2 + 331776000 a_1 t^3 x^2 - 6566400 t^2 x^2 + \\
& 9953280 b_1^2 a_1 t x^2 - 253440 a_1 t x^2 + 108 x^2 + 1244160 b_1^4 x^2 - 17280 b_1^2 x^2 - 1152 a_1^2 x^2 + \\
& 368640 a_1^3 t x^2 + 16588800 a_1^2 t^2 x^2 + 3072 a_1^4 x^2 + 165888 b_1^2 a_1^2 x^2 + 1935360 b_1 a_1 t x - \\
& 4423680 b_1 a_1^3 t x - 199065600 b_1 a_1^2 t^2 x + 124416 b_1^3 x - 663552 b_1^3 a_1^2 x - 2448 b_1 x + \\
& 62208000 b_1 t^2 x - 39813120 b_1^3 a_1 t x - 2985984 b_1^5 x - 29859840000 b_1 t^4 x - 4608 b_1 a_1^2 x - \\
& 3981312000 b_1 a_1 t^3 x - 36864 b_1 a_1^4 x - 597196800 b_1^3 t^2 x + 59443200 a_1^2 t^2 + 1075200 a_1^3 t - \\
& 136857600 b_1^2 t^2 + 1410048000 a_1 t^3 + 69120 b_1^2 a_1^2 + 2985984000000 t^6 + 9259200 t^2 - \\
& 2488320 b_1^2 a_1 t + 89579520000 b_1^2 t^4 + 995328 b_1^4 a_1^2 + 9 + 110592 b_1^2 a_1^4 + 49766400000 a_1^2 t^4 + \\
& 737280 a_1^5 t + 895795200 b_1^4 t^2 + 2211840000 a_1^3 t^3 + 597196800000 a_1 t^5 + 233280 a_1 t + \\
& 55296000 a_1^4 t^2 + 12234240000 t^4 + 13271040 b_1^2 a_1^3 t + 597196800 b_1^2 a_1^2 t^2 + 11943936000 b_1^2 a_1 t^3 + \\
& 59719680 b_1^4 a_1 t - 269568 b_1^4 + 6912 a_1^4 + 2985984 b_1^6 + 4096 a_1^6 + 1584 a_1^2 + \\
& 20016 b_1^2
\end{aligned}$$

is a solution to the (NLS5) equation (1).

Proof: Replacing the expression of the solution given by (3), we check that the relation (1) is verified.

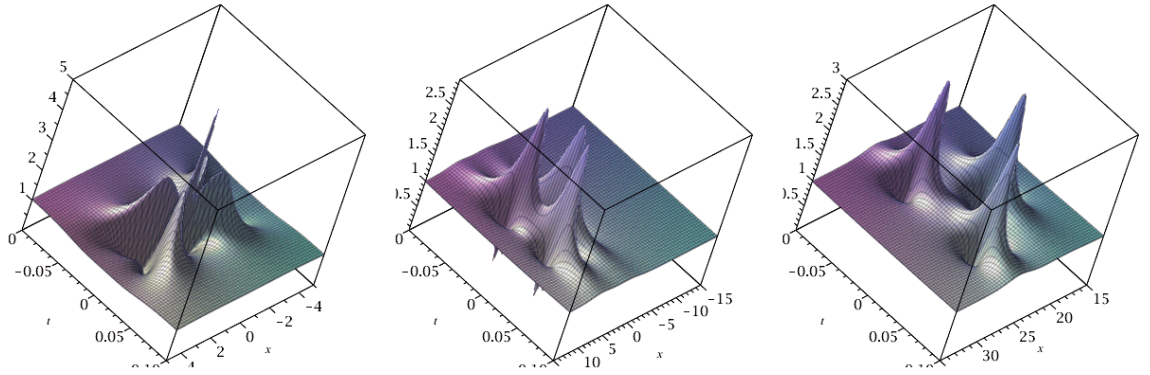


Figure 2. Solution of order 2 to the equation (1); to the left $a_1 = 0, b_1 = 0$; in the center $a_1 = 0, b_1 = 1$; to the right $a_1 = 0, b_1 = 4$.

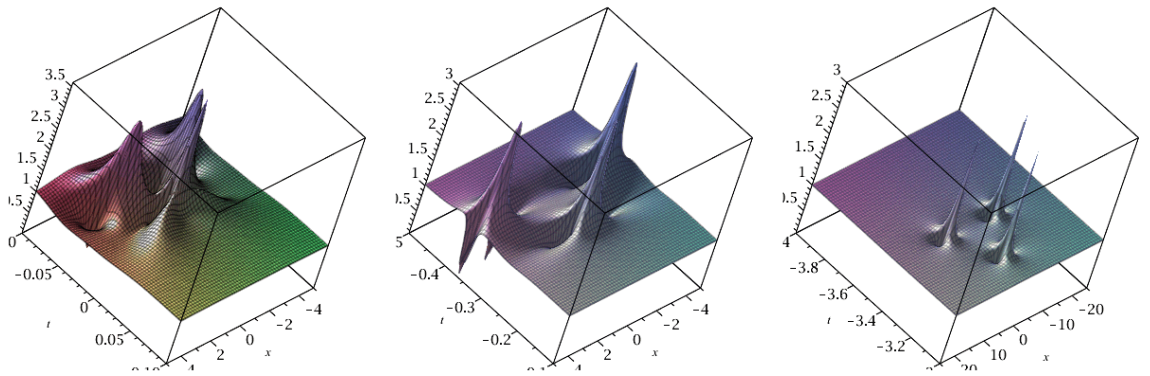


Figure 3. Solution of order 2 to the equation (1); to the left $a_1 = 1, b_1 = 0$; in the center $a_1 = 10, b_1 = 1$; to the right $a_1 = 100, b_1 = 100$.

When one or both parameters increase, three peaks appear. When only one of the parameters increases, the three peaks appear but with different orientations.

4 Quasi rational solutions of order 3 of the NLS5 equation

The solution depending on 4 real parameters being too long, we only present in the appendix. Here we give the solution without parameters.

Theorem 4.1 *The function $v(x, t)$ defined by*

$$v(x, t) = \frac{n(x, t)}{d(x, t)} \tag{4}$$

with

$$n(x, t) = -(-4096 x^{12} + 2949120 itx^{10} + 18432 x^{10} - 88473600 t^2 x^{10} + 57600 x^8 -$$

$$\begin{aligned}
 &40550400 ix^8t + 3428352000 t^2x^8 + 53084160000 it^3x^8 - 796262400000 t^4x^8 + 90316800 itx^6 - \\
 &1220935680000 ix^6t^3 - 34854912000 t^2x^6 - 3822059520000000 t^6x^6 + 172800 x^6 + \\
 &382205952000000 it^5x^6 + 35300966400000 t^4x^6 - 1285632000 t^2x^4 - 5828640768000000 ix^4t^5 + \\
 &1375941427200000000 it^7x^4 - 226800 x^4 + 37125734400000 t^4x^4 + 12326141952000000 t^6x^4 + \\
 &37324800 itx^4 - 4651499520000 ix^4t^3 - 10319560704000000000 t^8x^4 - 229970534400000 t^4x^2 - \\
 &2063912140800000000 ix^2t^7 - 131888217600 t^2x^2 + 485740800 itx^2 + 13931406950400000000 t^8x^2 - \\
 &113400 x^2 + 11588935680000 it^3x^2 - 14860167413760000000000 t^{10}x^2 + 66002190336000000 it^5x^2 + \\
 &2476694568960000000000 it^9x^2 - 1059044917248000000 t^6x^2 + 58190400 it - 61123092480000000 it^5 - \\
 &8916100448256000000000000 t^{12} + 19761958748160000000 it^7 + 14175 + 178322008965120000000000 it^{11} - \\
 &17828771328000 it^3 + 17956035624960000000000 it^9 + 62368963200 t^2 + 729979925299200000000 t^8 - \\
 &645625935360000 t^4 + 2630186090496000000 t^6 - 13621820129280000000000 t^{10})e^{20it}
 \end{aligned}$$

and

$$\begin{aligned}
 d(x, t) = &4096 x^{12} + 6144 x^{10} + 88473600 t^2x^{10} - 2101248000 t^2x^8 + 796262400000 t^4x^8 + \\
 &34560 x^8 + 19372032000 t^2x^6 + 149760 x^6 + 3822059520000000 t^6x^6 - 19375718400000 t^4x^6 + \\
 &10319560704000000000 t^8x^4 + 54000 x^4 - 429981696000000000 t^6x^4 - 51079680000 t^2x^4 - \\
 &176471654400000 t^4x^4 + 1663840051200000 t^4x^2 + 46438023168000000000 t^8x^2 - \\
 &8867750400 t^2x^2 + 14860167413760000000000 t^{10}x^2 + 1179439792128000000 t^6x^2 + \\
 &48600 x^2 + 2025 + 8916100448256000000000000 t^{12} + 51261206400 t^2 + 7715161571328000000000 t^8 + \\
 &704698652160000 t^4 - 423090044928000000 t^6 + 177083661680640000000000 t^{10}
 \end{aligned}$$

is a solution to the (NLS5) equation (1).

Proof: We have to check that the relation (1) is verified when we replace the expression of the solution given by (5).

In the following, we give the patterns of the modules of the solutions according to different values of the parameters.

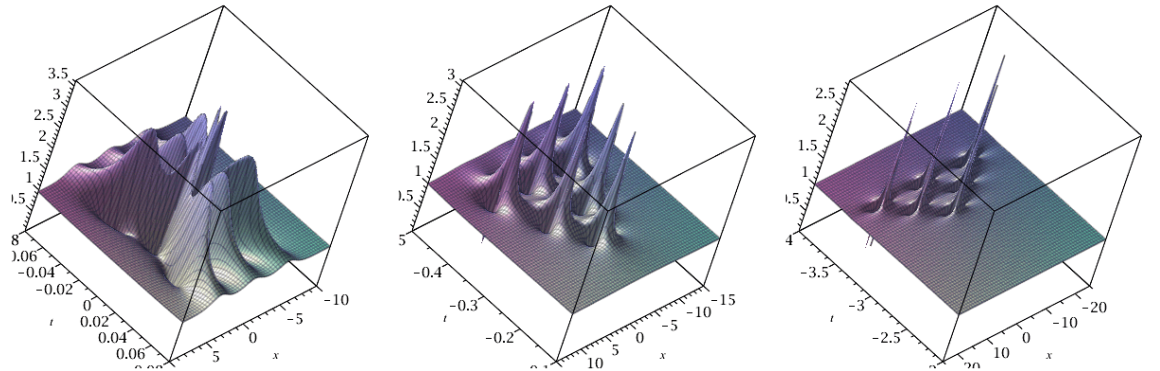


Figure 4. Solution of order 3 to (1); to the left $a_1 = 0, b_1 = 0, a_2 = 0, b_2 = 0$; in the center $a_1 = 10, b_1 = 0, a_2 = 0, b_2 = 0$; to the right $a_1 = 100, b_1 = 0, a_2 = 0, b_2 = 0$.

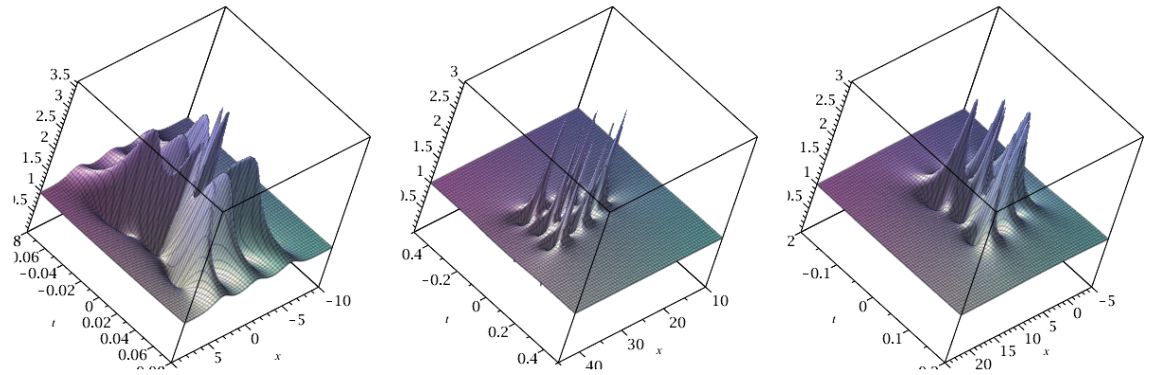


Figure 5. Solution of order 3 to (1); to the left $a_1 = 0, b_1 = 0, 1, a_2 = 0, b_2 = 0$; in the center $a_1 = 0, b_1 = 5, a_2 = 0, b_2 = 0$; to the right $a_1 = 0, b_1 = 10, a_2 = 0, b_2 = 0$.

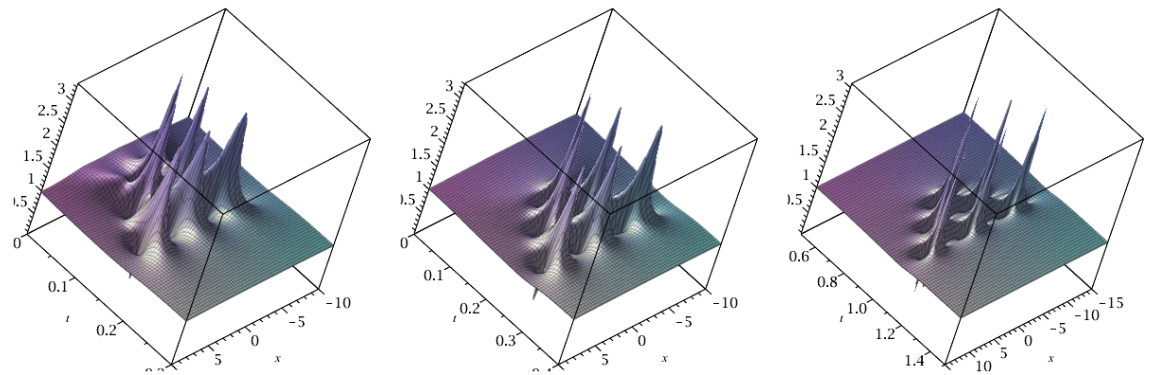


Figure 6. Solution of order 3 to (1); to the left $a_1 = 0, b_1 = 0, a_2 = 0, 5, b_2 = 0$; in the center $a_1 = 0, b_1 = 0, a_2 = 1, b_2 = 0$; to the right $a_1 = 0, b_1 = 5, a_2 = 5, b_2 = 0$.

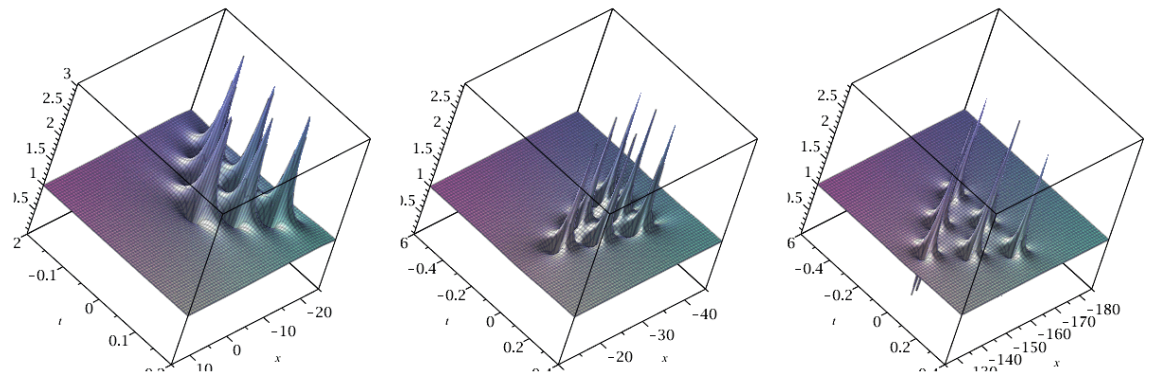


Figure 7. Solution of order 3 to (1); to the left $a_1 = 0, b_1 = 0, a_2 = 0, b_2 = 0, 5$; in the center $a_1 = 0, b_1 = 0, a_2 = 0, b_2 = 1$; to the right $a_1 = 0, b_1 = 5, a_2 = 0, b_2 = 5$.

As other equations belonging to this NLS hierarchy, for example, the NLS equation [8], the mKdV equation [9], or the Lakshmanan Porsezian Daniel equation [10], we recover the structure of triangles with peaks which appear in function of the different values of the parameters.

5 Conclusion

Rational solutions to the $(NLS5)$ equation have been given for the first orders. In all these N -order solutions we get quotient of a polynomial of degree $N(N+1)$ in x and t for the numerator by a polynomial of degree $N(N+1)$ in x and t for the denominator.

In the case of solutions of order 2, the solutions depend on two real parameters, and the structure of triangles with three peaks is observed for their modules. For the case of solutions of order 3, the solutions depend on four real parameters. In the plane (x, t) of the coordinates, the representation of the modules of the solutions reveals the formation of triangles containing 6 peaks.

References

- [1] V. E. Zakharov, Stability of periodic waves of finite amplitude on a surface of a deep fluid, *J. Appl. Tech. Phys*, V. **9**, 86-94, (1968)
- [2] N. Akhmediev, V. Eleonski, N. Kulagin, Generation of periodic trains of picosecond pulses in an optical fiber : exact solutions, *Sov. Phys. J.E.T.P.*, V. **62**, 894-899, (1985)
- [3] S. Tanaka, Modified Korteweg. de Vries Equation and Scattering Theory, *Proc. Japan Acad.*, V. 48, 466-469, 1972
- [4] M. Wadati, The Exact Solution of the Modified Korteweg-de Vries Equation, *J. Phys. Soc. Jpn.*, V. 32, 1681-1681, 1972
- [5] M. Lakshmanan, K. Porsezian, M. Daniel, Effect of discreteness on the continuum limit of the Heisenberg spin chain, *Phys. Lett. A*, V. 133, N. 9, 483488, 1998
- [6] M. Lakshmanan, M. Daniel, K. Porsezian, On the integrability aspects of the one-dimensional classical continuum isotropic biquadratic Heisenberg spin chain, *J. Math. Phys.*, V. 33, N. 5, 1807-1816, 1992
- [7] M. Daniel, K. Porsezian, M. Lakshmanan, On the integrable models of the higher order water wave equation, *Phys. Lett. A*, V. 174, N. 3, 237-240, 1993

- [8] P. Gaillard, Towards a classification of the quasi rational solutions to the NLS equation, *Theor. And Math. Phys.*, V. **189**, N. **1**, 1440-1449, 2016
- [9] P. Gaillard, Rational solutions to the mKdV equation associated to particular polynomials, *Wave Motion*, V. **107**, 102824-1-11, 2021
- [10] P. Gaillard, Rogue Waves of the Lakshmanan Porsezian Daniel Equation Depending on Multi-parameters, *As. Jour. Of Adv. Res. And Rep.*, V 16, N. 3, 32-40, 2022

Appendix

Solution of order 3 to the (NLS5) equation depending on 4 real parameters :
 The function $v(x, t)$ defined by

$$v(x, t) = \left(1 - 24 \frac{n(x, t)}{d(x, t)} \right) e^{i(2a_1 - 6a_2 + 20t)} \tag{5}$$

with

$$\begin{aligned} n(x, t) = & 675 + 353894400 t^2 + 91800 (16 a_2 - 160 t)^2 + 2190 (4 a_1 - 24 a_2 + 120 t)^6 + \\ & 495 (4 a_1 - 24 a_2 + 120 t)^8 + 11 (4 a_1 - 24 a_2 + 120 t)^{10} + 88473600 b_2^2 + (2 x - \\ & 12 b_1 + 60 b_2)^{10} + 27000 (8 b_1 - 80 b_2)^2 - 11059200 (16 a_2 - 160 t)t + i(1857600 t + \\ & 64800 (16 a_2 - 160 t)^3 - 870 (4 a_1 - 24 a_2 + 120 t)^7 + 25 (4 a_1 - 24 a_2 + 120 t)^9 + \\ & (4 a_1 - 24 a_2 + 120 t)^{11} - 151200 a_2 - 5529600 (16 a_2 - 160 t)^2 t - 90 (4 a_1 - 24 a_2 + \\ & 120 t)^8 (16 a_2 - 160 t) - 120 (4 a_1 - 24 a_2 + 120 t)^6 (80 a_2 - 1248 t) + 900 (4 a_1 - \\ & 24 a_2 + 120 t)^4 (464 a_2 - 4000 t) + 5529600 (8 b_1 - 80 b_2)^2 t + (-240 (4 a_1 - 24 a_2 + \\ & 120 t)^7 (8 b_1 - 80 b_2) - 7200 (4 a_1 - 24 a_2 + 120 t)^4 (8 b_1 - 80 b_2) (16 a_2 - 160 t) + \\ & 10800 (4 a_1 - 24 a_2 + 120 t) (24 b_1 - 400 b_2 + 4 (8 b_1 - 80 b_2)^3 + 4 (8 b_1 - 80 b_2) (16 a_2 - \\ & 160 t)^2) + 3600 (4 a_1 - 24 a_2 + 120 t)^3 (24 b_1 - 176 b_2) + 720 (4 a_1 - 24 a_2 + 120 t)^5 (56 b_1 - \\ & 400 b_2) + 21600 (8 b_1 - 80 b_2) (16 a_2 - 160 t) + 1382400 (16 a_2 - 160 t) b_2 - 2764800 (8 b_1 - \\ & 80 b_2) t - 43200 (4 a_1 - 24 a_2 + 120 t)^2 ((8 b_1 - 80 b_2) (16 a_2 - 160 t) + 32 (16 a_2 - \\ & 160 t) b_2 - 64 (8 b_1 - 80 b_2) t)) (2 x - 12 b_1 + 60 b_2) + 90 (4 a_1 - 24 a_2 + 120 t)^5 (-107 + \\ & 28 (8 b_1 - 80 b_2)^2 + 12 (16 a_2 - 160 t)^2) - 21600 (8 b_1 - 80 b_2)^2 (16 a_2 - 160 t) + \\ & 5400 (4 a_1 - 24 a_2 + 120 t)^2 (176 a_2 - 2464 t + 4 (8 b_1 - 80 b_2)^2 (16 a_2 - 160 t) + \\ & 4 (16 a_2 - 160 t)^3) - 225 (4 a_1 - 24 a_2 + 120 t)^3 (11 + 80 (8 b_1 - 80 b_2)^2 + 80 (16 a_2 - \\ & 160 t)^2 + 4096 (8 b_1 - 80 b_2) b_2 + 8192 (16 a_2 - 160 t) t) - 675 (4 a_1 - 24 a_2 + 120 t) (-7 + \\ & 56 (8 b_1 - 80 b_2)^2 + 88 (16 a_2 - 160 t)^2 - 4096 (8 b_1 - 80 b_2) b_2 - 131072 b_2^2 - \\ & 524288 t^2) + (4 a_1 - 24 a_2 + 120 t) (2 x - 12 b_1 + 60 b_2)^{10} + (-60 a_1 + 840 a_2 - \\ & 6600 t + 5 (4 a_1 - 24 a_2 + 120 t)^3) (2 x - 12 b_1 + 60 b_2)^8 + (-600 a_1 - 240 a_2 + \\ & 58800 t - 140 (4 a_1 - 24 a_2 + 120 t)^3 + 10 (4 a_1 - 24 a_2 + 120 t)^5 + 240 (4 a_1 - 24 a_2 + \\ & 120 t)^2 (16 a_2 - 160 t)) (2 x - 12 b_1 + 60 b_2)^6 + (-240 (4 a_1 - 24 a_2 + 120 t)^3 (8 b_1 - \\ & 80 b_2) - 1440 (8 b_1 - 80 b_2) (16 a_2 - 160 t) + 720 (4 a_1 - 24 a_2 + 120 t) (8 b_1 - 176 b_2)) (2 x - \\ & 12 b_1 + 60 b_2)^5 + (-450 (4 a_1 - 24 a_2 + 120 t)^3 - 210 (4 a_1 - 24 a_2 + 120 t)^5 + \\ & 10 (4 a_1 - 24 a_2 + 120 t)^7 + 300 (4 a_1 - 24 a_2 + 120 t)^4 (16 a_2 - 160 t) + 450 (4 a_1 - \\ & 24 a_2 + 120 t) (-3 + 12 (8 b_1 - 80 b_2)^2 - 4 (16 a_2 - 160 t)^2) - 14400 a_2 + 259200 t + \\ & 1800 (4 a_1 - 24 a_2 + 120 t)^2 (16 a_2 - 224 t)) (2 x - 12 b_1 + 60 b_2)^4 + (-480 (4 a_1 - \\ & 24 a_2 + 120 t)^5 (8 b_1 - 80 b_2) + 14400 (4 a_1 - 24 a_2 + 120 t)^2 (8 b_1 - 80 b_2) (16 a_2 - \\ & 160 t) + 7200 (4 a_1 - 24 a_2 + 120 t) (8 b_1 - 48 b_2) - 2400 (4 a_1 - 24 a_2 + 120 t)^3 (16 b_1 - \end{aligned}$$

$$\begin{aligned}
 &128 b_2) - 14400 (8 b_1 - 80 b_2)(16 a_2 - 160 t) - 460800 (16 a_2 - 160 t)b_2 + 921600 (8 b_1 - \\
 &80 b_2)t(2x - 12 b_1 + 60 b_2)^3 + (1710 (4 a_1 - 24 a_2 + 120 t)^5 - 60 (4 a_1 - 24 a_2 + \\
 &120 t)^7 + 5 (4 a_1 - 24 a_2 + 120 t)^9 - 900 (4 a_1 - 24 a_2 + 120 t)^3(7 + 4 (8 b_1 - 80 b_2)^2 - \\
 &12 (16 a_2 - 160 t)^2) + 675 (4 a_1 - 24 a_2 + 120 t)(7 + 16 (8 b_1 - 80 b_2)^2 + 16 (16 a_2 - \\
 &160 t)^2) - 345600 a_2 + 4492800 t - 21600 (8 b_1 - 80 b_2)^2(16 a_2 - 160 t) - 21600 (16 a_2 - \\
 &160 t)^3 + 691200 (4 a_1 - 24 a_2 + 120 t)^2 t - 1800 (4 a_1 - 24 a_2 + 120 t)^4(64 a_2 - \\
 &448 t))(2x - 12 b_1 + 60 b_2)^2 - 5529600 (8 b_1 - 80 b_2)(16 a_2 - 160 t)b_2 + 15 (1 + \\
 &(4 a_1 - 24 a_2 + 120 t)^2)(2x - 12 b_1 + 60 b_2)^8 + (210 - 60 (4 a_1 - 24 a_2 + 120 t)^2 + \\
 &50 (4 a_1 - 24 a_2 + 120 t)^4 + 480 (4 a_1 - 24 a_2 + 120 t)(16 a_2 - 160 t))(2x - 12 b_1 + \\
 &60 b_2)^6 + (-720 (4 a_1 - 24 a_2 + 120 t)^2(8 b_1 - 80 b_2) - 5760 b_1 - 11520 b_2)(2x - \\
 &12 b_1 + 60 b_2)^5 + (450 (4 a_1 - 24 a_2 + 120 t)^2 - 150 (4 a_1 - 24 a_2 + 120 t)^4 + 70 (4 a_1 - \\
 &24 a_2 + 120 t)^6 + 1200 (4 a_1 - 24 a_2 + 120 t)^3(16 a_2 - 160 t) - 450 + 5400 (8 b_1 - \\
 &80 b_2)^2 - 1800 (16 a_2 - 160 t)^2 + 3600 (4 a_1 - 24 a_2 + 120 t)(16 a_2 - 224 t))(2x - \\
 &12 b_1 + 60 b_2)^4 + (-2400 (4 a_1 - 24 a_2 + 120 t)^4(8 b_1 - 80 b_2) + 28800 (4 a_1 - 24 a_2 + \\
 &120 t)(8 b_1 - 80 b_2)(16 a_2 - 160 t) + 57600 b_1 - 806400 b_2 - 7200 (4 a_1 - 24 a_2 + \\
 &120 t)^2(16 b_1 - 128 b_2))(2x - 12 b_1 + 60 b_2)^3 + (6750 (4 a_1 - 24 a_2 + 120 t)^4 + \\
 &420 (4 a_1 - 24 a_2 + 120 t)^6 + 45 (4 a_1 - 24 a_2 + 120 t)^8 - 2700 (4 a_1 - 24 a_2 + \\
 &120 t)^2(5 + 4 (8 b_1 - 80 b_2)^2 - 12 (16 a_2 - 160 t)^2) - 675 - 10800 (8 b_1 - 80 b_2)^2 - \\
 &10800 (16 a_2 - 160 t)^2 + 21600 (4 a_1 - 24 a_2 + 120 t)(32 a_2 - 384 t) - 7200 (4 a_1 - \\
 &24 a_2 + 120 t)^3(32 a_2 - 128 t))(2x - 12 b_1 + 60 b_2)^2 + (-1680 (4 a_1 - 24 a_2 + \\
 &120 t)^6(8 b_1 - 80 b_2) - 28800 (4 a_1 - 24 a_2 + 120 t)^3(8 b_1 - 80 b_2)(16 a_2 - 160 t) - \\
 &10800 (4 a_1 - 24 a_2 + 120 t)^2(8 b_1 - 272 b_2) + 86400 b_1 - 1209600 b_2 + 43200 (8 b_1 - \\
 &80 b_2)^3 + 43200 (8 b_1 - 80 b_2)(16 a_2 - 160 t)^2 + 3600 (4 a_1 - 24 a_2 + 120 t)^4(8 b_1 + \\
 &80 b_2) - 86400 (4 a_1 - 24 a_2 + 120 t)((8 b_1 - 80 b_2)(16 a_2 - 160 t) + 32 (16 a_2 - \\
 &160 t)b_2 - 64 (8 b_1 - 80 b_2)t))(2x - 12 b_1 + 60 b_2) + 450 (4 a_1 - 24 a_2 + 120 t)^4(-17 + \\
 &28 (8 b_1 - 80 b_2)^2 + 12 (16 a_2 - 160 t)^2) + 10800 (4 a_1 - 24 a_2 + 120 t)(-16 a_2 + 224 t + \\
 &4 (8 b_1 - 80 b_2)^2(16 a_2 - 160 t) + 4 (16 a_2 - 160 t)^3) + 675 (4 a_1 - 24 a_2 + 120 t)^2(-3 + \\
 &16 (8 b_1 - 80 b_2)^2 + 16 (16 a_2 - 160 t)^2 - 4096 (8 b_1 - 80 b_2)b_2 - 8192 (16 a_2 - 160 t)t) - \\
 &720 (4 a_1 - 24 a_2 + 120 t)^7(16 a_2 - 160 t) - 3600 (4 a_1 - 24 a_2 + 120 t)^3(48 a_2 - \\
 &1376 t) - 720 (4 a_1 - 24 a_2 + 120 t)^5(272 a_2 - 3168 t) - 2764800 (8 b_1 - 80 b_2)b_2 \\
 &\text{and}
 \end{aligned}$$

$$\begin{aligned}
 d(x, t) = &2024 + 2123366400 t^2 + 874800 (16 a_2 - 160 t)^2 + 3720 (4 a_1 - 24 a_2 + \\
 &120 t)^8 + 120 (4 a_1 - 24 a_2 + 120 t)^{10} + 518400 (16 a_2 - 160 t)^4 + (1 + (2x - 12 b_1 + \\
 &60 b_2)^2 + (4 a_1 - 24 a_2 + 120 t)^2)^6 + 530841600 b_2^2 + 356400 (8 b_1 - 80 b_2)^2 + \\
 &518400 (8 b_1 - 80 b_2)^4 + 120 (8 b_1 - 80 b_2)(2x - 12 b_1 + 60 b_2)^9 + 46080 b_2(2x - 12 b_1 + \\
 &60 b_2)^7 - 82944000 (16 a_2 - 160 t)t + (-1440 (4 a_1 - 24 a_2 + 120 t)^4 + 720 (4 a_1 - \\
 &24 a_2 + 120 t)^5(16 a_2 - 160 t) + 240 (4 a_1 - 24 a_2 + 120 t)^2(56 + 135 (8 b_1 - 80 b_2)^2 - \\
 &45 (16 a_2 - 160 t)^2) + 32400 (4 a_1 - 24 a_2 + 120 t)(16 a_2 - 288 t) + 7200 (4 a_1 - \\
 &24 a_2 + 120 t)^3(48 a_2 - 544 t) + 3360 + 32400 (8 b_1 - 80 b_2)^2 - 54000 (16 a_2 - \\
 &160 t)^2 + 2764800 (8 b_1 - 80 b_2)b_2 + 5529600 (16 a_2 - 160 t)t(2x - 12 b_1 + 60 b_2)^4 + \\
 &(-960 (4 a_1 - 24 a_2 + 120 t)^6(8 b_1 - 80 b_2) + 57600 (4 a_1 - 24 a_2 + 120 t)^3(8 b_1 - \\
 &80 b_2)(16 a_2 - 160 t) - 43200 (4 a_1 - 24 a_2 + 120 t)^2(24 b_1 - 272 b_2) - 7200 (4 a_1 - \\
 &24 a_2 + 120 t)^4(48 b_1 - 448 b_2) + 345600 b_1 - 5529600 b_2 - 86400 (8 b_1 - 80 b_2)^3 - \\
 &86400 (8 b_1 - 80 b_2)(16 a_2 - 160 t)^2 + 172800 (4 a_1 - 24 a_2 + 120 t)((8 b_1 - 80 b_2)(16 a_2 - \\
 &160 t) - 32 (16 a_2 - 160 t)b_2 + 64 (8 b_1 - 80 b_2)t))(2x - 12 b_1 + 60 b_2)^3 + (13440 (4 a_1 - \\
 &24 a_2 + 120 t)^6 + 240 (4 a_1 - 24 a_2 + 120 t)^8 - 240 (4 a_1 - 24 a_2 + 120 t)^4(-326 +
 \end{aligned}$$

$$\begin{aligned}
 &45(8b_1 - 80b_2)^2 - 135(16a_2 - 160t)^2 + 480(4a_1 - 24a_2 + 120t)^2(-76 + 135(8b_1 - 80b_2)^2 + 1215(16a_2 - 160t)^2) - 129600(4a_1 - 24a_2 + 120t)^3(32a_2 - 256t) - \\
 &12960(4a_1 - 24a_2 + 120t)^5(32a_2 - 256t) - 64800(4a_1 - 24a_2 + 120t)(-96a_2 + 1280t + 4(8b_1 - 80b_2)^2(16a_2 - 160t) + 4(16a_2 - 160t)^3) + 12144 - 97200(8b_1 - 80b_2)^2 + \\
 &32400(16a_2 - 160t)^2 + 530841600b_2^2 - 33177600(16a_2 - 160t)t + 2123366400t^2)(2x - 12b_1 + 60b_2)^2 + (-360(4a_1 - 24a_2 + 120t)^8(8b_1 - 80b_2) - \\
 &17280(4a_1 - 24a_2 + 120t)^5(8b_1 - 80b_2)(16a_2 - 160t) - 1440(4a_1 - 24a_2 + 120t)^6(8b_1 - 240b_2) + 32400(4a_1 - 24a_2 + 120t)^4(8b_1 + 112b_2) + 64800(4a_1 - 24a_2 + 120t)^2(-40b_1 + 752b_2 + 4(8b_1 - 80b_2)^3 + 4(8b_1 - 80b_2)(16a_2 - 160t)^2) - \\
 &777600(4a_1 - 24a_2 + 120t)((8b_1 - 80b_2)(16a_2 - 160t) + 64(16a_2 - 160t)b_2 - 128(8b_1 - 80b_2)t) - 172800(4a_1 - 24a_2 + 120t)^3(3(8b_1 - 80b_2)(16a_2 - 160t) + \\
 &32(16a_2 - 160t)b_2 - 64(8b_1 - 80b_2)t) - 648000b_1 + 8553600b_2 + 259200(8b_1 - 80b_2)^3 + 1296000(8b_1 - 80b_2)(16a_2 - 160t)^2 - 33177600(8b_1 - 80b_2)^2b_2 + \\
 &33177600(16a_2 - 160t)^2b_2 - 132710400(8b_1 - 80b_2)(16a_2 - 160t)t)(2x - 12b_1 + 60b_2) + 80(4a_1 - 24a_2 + 120t)^6(191 + 63(8b_1 - 80b_2)^2 + 27(16a_2 - 160t)^2) + \\
 &21600(4a_1 - 24a_2 + 120t)^3(-368a_2 + 3488t + 4(8b_1 - 80b_2)^2(16a_2 - 160t) + 4(16a_2 - 160t)^3) + 240(4a_1 - 24a_2 + 120t)^4(599 + 135(8b_1 - 80b_2)^2 - 225(16a_2 - 160t)^2 - 11520(8b_1 - 80b_2)b_2 - 23040(16a_2 - 160t)t) - 16200(4a_1 - 24a_2 + 120t)(496a_2 - 6240t + 80(8b_1 - 80b_2)^2(16a_2 - 160t) + 16(16a_2 - 160t)^3 + 4096(8b_1 - 80b_2)(16a_2 - 160t)b_2 - 4096(8b_1 - 80b_2)^2t + 4096(16a_2 - 160t)^2t) + 24(4a_1 - 24a_2 + 120t)^2(3881 + 12150(8b_1 - 80b_2)^2 + 28350(16a_2 - 160t)^2 + 691200(8b_1 - 80b_2)b_2 + 22118400b_2^2 + 88473600t^2) - 120(4a_1 - 24a_2 + 120t)^9(16a_2 - 160t) - 2160(4a_1 - 24a_2 + 120t)^5(240a_2 - 4576t) - 1440(4a_1 - 24a_2 + 120t)^7(80a_2 - 864t) + 1036800(8b_1 - 80b_2)^2(16a_2 - 160t)^2 + (-120(4a_1 - 24a_2 + 120t)^2 + 360(4a_1 - 24a_2 + 120t)(16a_2 - 160t) + 120)(2x - 12b_1 + 60b_2)^8 + (480(4a_1 - 24a_2 + 120t)^2 - 240(4a_1 - 24a_2 + 120t)^4 + 960(4a_1 - 24a_2 + 120t)^3(16a_2 - 160t) + 2320 + 2160(8b_1 - 80b_2)^2 + 5040(16a_2 - 160t)^2 - 1440(4a_1 - 24a_2 + 120t)(64a_2 - 960t))(2x - 12b_1 + 60b_2)^6 + (-720(4a_1 - 24a_2 + 120t)^4(8b_1 - 80b_2) - 17280(4a_1 - 24a_2 + 120t)(8b_1 - 80b_2)(16a_2 - 160t) + 4320(4a_1 - 24a_2 + 120t)^2(8b_1 - 176b_2) - 51840b_1 + 103680b_2)(2x - 12b_1 + 60b_2)^5 - 24883200(8b_1 - 80b_2)b_2
 \end{aligned}$$

is a solution to the (NLS5) equation (1).