Quasi-rational solutions to the seventh equation of the NLS hierarchy

Abstract

The following study is part of a research program of rational solutions of the hierarchy of the nonlinear Schrödinger equation. Here, we are interested in the equation of order 7 and we highlight particular solutions providing the first orders of rogue waves not yet found.

Key Words : NLS hierarchy, quasi-rational solutions.

PACS numbers :
33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd

1 Introduction

Quasi-rational solutions to the seventh equation of the NLS hierarchy are constructed. We give explicit expressions of these solutions for the first orders. They depend on multi-parameters and so patterns of these solutions in the (x,t) plane according the different values of the parameters are studied.
We consider the seventh equation of the NLS hierarchy of order 7 ($NLS7$) which can be written as

$$\begin{aligned}
&iu_t + u_{8x} + 16|u^2|u_{6x} + 2u^2\overline{u}_{6x} + 56\overline{u}u_xu_{5x}\\
&+40u\overline{u}_xu_{5x} + 12uu_x\overline{u}_{5x} + 98|u|^4u_{4x} + 168|u_x|^2u_{4x}\\
&+112\overline{u}u_{2x}u_{4x} + 72u\overline{u}_{2x}u_{4x} + 28u^2|u|^2\overline{u}_{4x} + 42u_x^2\overline{u}_{4x}\\
&+44uu_{2x}\overline{u}_{4x} + 68uu_x\overline{u}_{3x} + 476|u|^2\overline{u}u_xu_{3x} + 252u_x\overline{u}_{2x}u_{3x}\\
&+308u|u|^2\overline{u}_xu_{3x} + 308\overline{u}_xu_{2x}u_{3x} + 70\overline{u}u_{3x}^2 + 196u_xu_{2x}\overline{u}_{3x}\\
&+168u|u|^2u_x\overline{u}_{3x} + 56u^3\overline{u}_x\overline{u}_{3x} + 280|u|^6u_{2x} + 1456|u|^2|u_x|^2u_{2x}\\
&+490\overline{u}^2u_x^2u_{2x} + 238u^2\overline{u}_x^2u_{2x} + 588|u|^2u_x^2\overline{u}_{2x} + 336u^2|u_x|^2\overline{u}_{2x}\\
&+140|u|^4u^2\overline{u}_{2x} + 42u^3\overline{u}_{2x} + 392|u|^2u|u_{2x}|^2 + 322|u|^2\overline{u}u_{2x}^2\\
&+182u_{2x}^2\overline{u}_{2x} + 560|u|^4\overline{u}u_x^2 + 560|u|^4u|u_x|^2 + 420\overline{u}u_x^2|u_x|^2\\
&+140u^3|u|^2\overline{u}_x^2 + 378|u_x|^4u + 70|u|^8u
\end{aligned} \qquad (1)$$

with as usual the subscripts meaning partial derivatives and $\overline{u}$ the complex conjugate of u.
Different classical equations are included in the NLS hierarchy; the first one is the NLS equation [1, 3, 4, 5, 6, 7, 10, 9]; the second one is the mKdV equation [11, 12, 13, 14, 15]; the third one is the LPD equation [16, 19, 20, 21, 22].

Many works has been done for these first three equations of the NLS hierarchy. For example, we can quote the following works, for the NLS equation [2], the mKdV equation [12], the LPD equation [16, 17, 18]. However, very few studies have been carried out for the following orders of this hierarchy. Here we explicitly construct solutions of the order equation seven of this hierarchy.
We construct quasi rational solutions for the first orders. The related patterns of the modulus of these solutions in the plane of coordinates $(x;t)$ are studied.

2 Quasi rational solutions of order 1 to the NLS7 equation

Theorem 2.1 *The function* $v(x,t)$ *defined by*

$$v(x,t) = -\frac{\left(3 - 4\,x^2 - 313600\,t^2 + 2240\,it\right) e^{70\,it}}{1 + 4\,x^2 + 313600\,t^2} \tag{2}$$

is a solution to the $(NLS7)$ *equation* (1).

Proof: We have to replace the expression of the solution given by (2) and check that (1) is verified.

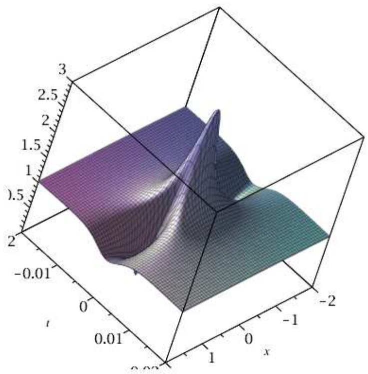

Figure 1. Solution of order 1 to $(NLS7)$.

We get a smooth solution of the equation (1).

3 Quasi rational solutions of order 2 of the NLS7 equation depending on 2 real parameters

Theorem 3.1 *The function $v(x,t)$ defined by*

$$v(x,t) = \frac{n(x,t)}{d(x,t)} \tag{3}$$

with

$n(x,t) = -(-64\,x^6+2304\,b_1x^5-768\,a_1{}^2x^4+144\,x^4+107520\,itx^4-215040\,a_1tx^4+$
$768\,ia_1x^4-15052800\,t^2x^4-34560\,b_1{}^2x^4-18432\,ia_1x^3b_1-4992\,b_1x^3+276480\,b_1{}^3x^3+$
$18432\,b_1a_1{}^2x^3+361267200\,b_1t^2x^3+5160960\,b_1a_1tx^3-2580480\,itx^3b_1+2580480\,ia_1{}^2tx^2-$
$1244160\,b_1{}^4x^2+2903040\,a_1tx^2-3072\,a_1{}^4x^2+361267200\,ia_1t^2x^2-33718272000\,a_1t^3x^2+$
$23224320\,itb_1{}^2x^2-165888\,b_1{}^2a_1{}^2x^2+293529600\,t^2x^2-1180139520000\,t^4x^2+$
$5760\,a_1{}^2x^2+16859136000\,it^3x^2-1152\,ia_1x^2+58752\,b_1{}^2x^2+6144\,ia_1{}^3x^2-361267200\,a_1{}^2t^2x^2+$
$165888\,ia_1b_1{}^2x^2-1720320\,a_1{}^3tx^2-3251404800\,b_1{}^2t^2x^2-46448640\,b_1{}^2a_1tx^2-$
$806400\,itx^2+180\,x^2-202309632000\,it^3xb_1-3161088000\,b_1t^2x+36864\,b_1a_1{}^4x-$
$29675520\,b_1a_1tx+663552\,b_1{}^3a_1{}^2x-50688\,b_1a_1{}^2x-5616\,b_1x+20643840\,b_1a_1{}^3tx-$
$663552\,ia_1xb_1{}^3-290304\,b_1{}^3x+13005619200\,b_1{}^3t^2x-4608\,ia_1xb_1+4335206400\,b_1a_1{}^2t^2x+$
$14161674240000\,b_1t^4x+7096320\,itb_1x-4335206400\,ia_1t^2xb_1+185794560\,b_1{}^3a_1tx-$
$92897280\,itxb_1{}^3-73728\,ia_1{}^3xb_1-30965760\,ia_1{}^2txb_1+2985984\,b_1{}^5x+404619264000\,b_1a_1t^3x-$
$262080\,it+23602790400000\,ia_1t^4+54792192000\,it^3-1475174400000\,t^4+3225600\,ia_1{}^2t+$
$13005619200\,ia_1t^2b_1{}^2+337182720000\,ia_1{}^2t^3+606928896000\,it^3b_1{}^2+73543680\,b_1{}^2a_1t-$
$30840979456000000\,t^6+139345920\,itb_1{}^4+8601600\,ia_1{}^4t+660878131200000\,it^5+$
$69120\,ia_1b_1{}^2-13547520\,itb_1{}^2-13005619200\,b_1{}^2a_1{}^2t^2-1213857792000\,b_1{}^2a_1t^3-$
$278691840\,b_1{}^4a_1t+995328\,ia_1b_1{}^4+221184\,ia_1{}^3b_1{}^2-45+812851200\,ia_1t^2-98784000\,t^2+$
$92897280\,ia_1{}^2tb_1{}^2+3010560\,a_1{}^3t+270950400\,a_1{}^2t^2-8429568000\,a_1t^3-61931520\,b_1{}^2a_1{}^3t+$
$96768\,b_1{}^2a_1{}^2+8399462400\,b_1{}^2t^2+1872\,a_1{}^2+18000\,b_1{}^2+846720\,a_1t-720\,ia_1+$
$1536\,ia_1{}^3+12288\,ia_1{}^5+2408448000\,ia_1{}^3t^2-42485022720000\,b_1{}^2t^4-995328\,b_1{}^4a_1{}^2-$
$19508428800\,b_1{}^4t^2-110592\,b_1{}^2a_1{}^4-23602790400000\,a_1{}^2t^4-3440640\,a_1{}^5t-1204224000\,a_1{}^4t^2-$
$224788480000\,a_1{}^3t^3-1321756262400000\,a_1t^5-2985984\,b_1{}^6-4096\,a_1{}^6+518400\,b_1{}^4+$
$8448\,a_1{}^4)\mathrm{e}^{2\,i(a_1+35\,t)}$

and

$d(x,t) = 64\,x^6-2304\,b_1x^5+48\,x^4+15052800\,t^2x^4+215040\,a_1tx^4+768\,a_1{}^2x^4+$
$34560\,b_1{}^2x^4-5160960\,b_1a_1tx^3-361267200\,b_1t^2x^3-276480\,b_1{}^3x^3+384\,b_1x^3-$
$18432\,b_1a_1{}^2x^3+1180139520000\,t^4x^2+361267200\,a_1{}^2t^2x^2+46448640\,b_1{}^2a_1tx^2-$
$203212800\,t^2x^2+1720320\,a_1{}^3tx^2-1152\,a_1{}^2x^2-1612800\,a_1tx^2+1244160\,b_1{}^4x^2+$
$3072\,a_1{}^4x^2+165888\,b_1{}^2a_1{}^2x^2+3251404800\,b_1{}^2t^2x^2-17280\,b_1{}^2x^2+108\,x^2+$
$33718272000\,a_1t^3x^2-20643840\,b_1a_1{}^3tx-185794560\,b_1{}^3a_1tx-4335206400\,b_1a_1{}^2t^2x-$
$404619264000\,b_1a_1t^3x-2985984\,b_1{}^5x-13005619200\,b_1{}^3t^2x+2077286400\,b_1t^2x+$
$124416\,b_1{}^3x-14161674240000\,b_1t^4x-663552\,b_1{}^3a_1{}^2x-4608\,b_1a_1{}^2x-2448\,b_1x+$
$14192640\,b_1a_1tx-36864\,b_1a_1{}^4x+9+177020928000\,a_1t^3+19508428800\,b_1{}^4t^2+$
$347155200\,t^2-5148057600\,b_1{}^2t^2+7375872000000\,t^4+30840979456000000\,t^6+$
$69120\,b_1{}^2a_1{}^2+1411200\,a_1t+110592\,b_1{}^2a_1{}^4+995328\,b_1{}^4a_1{}^2-27095040\,b_1{}^2a_1t+$
$23602790400000\,a_1{}^2t^4+3440640\,a_1{}^5t+1204224000\,a_1{}^4t^2+224788480000\,a_1{}^3t^3+$
$1321756262400000\,a_1t^5+2985984\,b_1{}^6+4096\,a_1{}^6-269568\,b_1{}^4+6912\,a_1{}^4+42485022720000\,b_1{}^2t^4+$

$5591040\,a_1{}^3t + 1535385600\,a_1{}^2t^2 + 61931520\,b_1{}^2a_1{}^3t + 13005619200\,b_1{}^2a_1{}^2t^2 + 1213857792000\,b_1{}^2a_1t^3 + 278691840\,b_1{}^4a_1t + 1584\,a_1{}^2 + 20016\,b_1{}^2$

is a solution to the (NLS7) equation (1).

Proof: We have also to replace the expression of the solution given by (3), and we check that the relation (1) is verified.

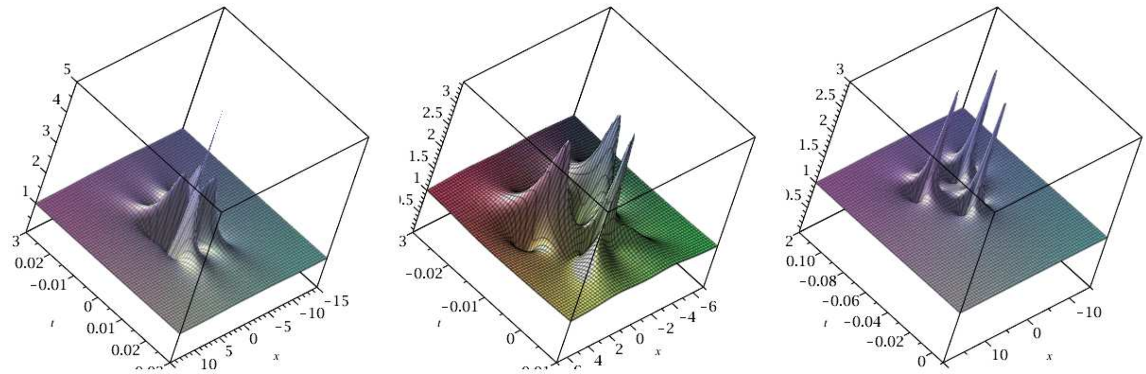

Figure 2. Solution of order 2 to the equation (1); to the left $a_1 = 0$, $b_1 = 0$; in the center $a_1 = 1$, $b_1 =$; to the right $a_1 = 10$, $b_1 = 0$.

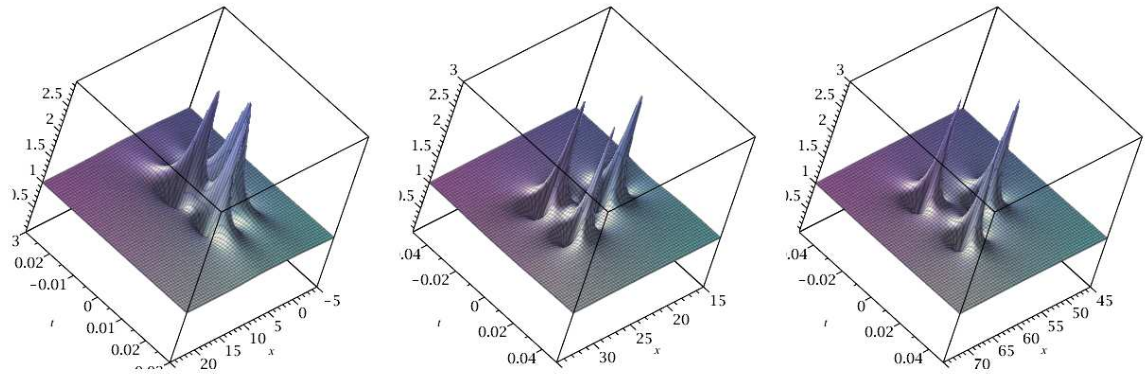

Figure 3. Solution of order 2 to the equation (1); to the left $a_1 = 0$, $b_1 = 1$; in the center $a_1 = 0$, $b_1 = 4$; to the right $a_1 = 0$, $b_1 = 10$.

When one or both parameters increase, three peaks appear. When only one of the parameters increases, the three peaks appear but with different orientations.

4 Quasi rational solutions of order 3 of the NLS7 equation

The solution of order 3, depending on 4 real parameters being too long, we only present in the appendix. Here we give the solution of order 3 without parameters.

Theorem 4.1 *The function $v(x,t)$ defined by*

$$v(x,t) = \frac{n(x,t)}{d(x,t)} \tag{4}$$

with
$n(x,t) = -(-4096\,x^{12}+13762560\,itx^{10}-1926758400\,t^2x^{10}+18432\,x^{10}-377644646400000\,t^4x^8+$
$93929472000\,t^2x^8-258048000\,ix^8t+57600\,x^8+5394923520000\,it^3x^8-167242629120000\,ix^6t^3+$
$20770455552000000\,t^4x^6-1369202688000\,t^2x^6+172800\,x^6+1109606400\,itx^6+$
$845924007936000000\,it^5x^6-3947645370368000000\,t^6x^6-17129961160704000000\,ix^4t^5+$
$716083200\,itx^4+875169792000\,t^2x^4-952878366720000\,ix^4t^3+66320442222182400000000\,it^7x^4+$
$150997435416576000000\,t^6x^4-226800\,x^4+599038820352000000\,t^4x^4-23212154777763840000000000\,t^8x^4-$
$6536320745472000000\,t^4x^2+31336408949981184000000000\,t^8x^2+26789355926323200000\,it^5x^2-$
$7585324185600\,t^2x^2-727933173830674022400000000000\,t^{10}x^2-2024697964894617600000\,t^6x^2-$
$9948066333327360000000\,ix^2t^7+2599761335109550080000000000\,it^9x^2+38266866892800000\,it^3x^2+$
$3969907200\,itx^2-113400\,x^2+407642577345177452544000000000000\,it^{11}+433641600\,it-$
$5496461881344000\,it^3+248452956674850816000000\,it^7-26444377541836800000\,it^5+$
$5546744179200\,t^2+167910924623649177600000000\,t^8+266475536848728883200000000000\,it^9+$
$14175-309371598878036459520000000000\,t^{10}-951166013805414055936000000000000\,t^{12}-$
$704818560983040000\,t^4+5505521933824819200000\,t^6)\mathrm{e}^{70\,it}$
and
$d(x,t) = 4096\,x^{12}+1926758400\,t^2x^{10}+6144\,x^{10}+34560\,x^8-65028096000\,t^2x^8+$
$377644646400000\,t^4x^8+877879296000\,t^2x^6-13217562624000000\,t^4x^6+149760\,x^6+$
$3947645370368000000\,t^6x^6+23212154777763840000000000\,t^8x^4-156203266867200000\,t^4x^4-$
$680968826388480000000\,t^6x^4-3248695296000\,t^2x^4+54000\,x^4+48600\,x^2$
$+2149048794061209600000\,t^6x^2+1886547433881600000\,t^4x^2+2236763289600\,t^2x^2+$
$727933173830674022400000000000\,t^{10}x^2+1044546964999372800000000\,t^8x^2-$
$13357721658064896000000\,t^6+111009809009177788416000000000000\,t^{10}$
$+281244270326081126400000000\,t^8+866898922659840000\,t^4$
$+951166013805414055936000000000000\,t^{12}+2025+3320852774400\,t^2$
is a solution to the (NLS) equation (1).

Proof: It is sufficient to check that the relation (1) is verified when we replace the expression of the solution given by (5).

In the following, we give the patterns of the modules of the solutions according to different values of the parameters.

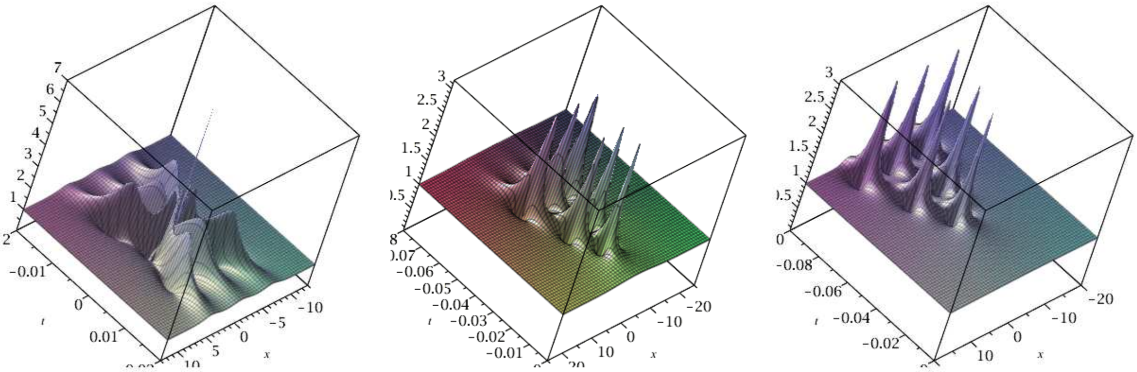

Figure 4. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$; in the center $a_1 = 1$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$; to the right $a_1 = 10$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$.

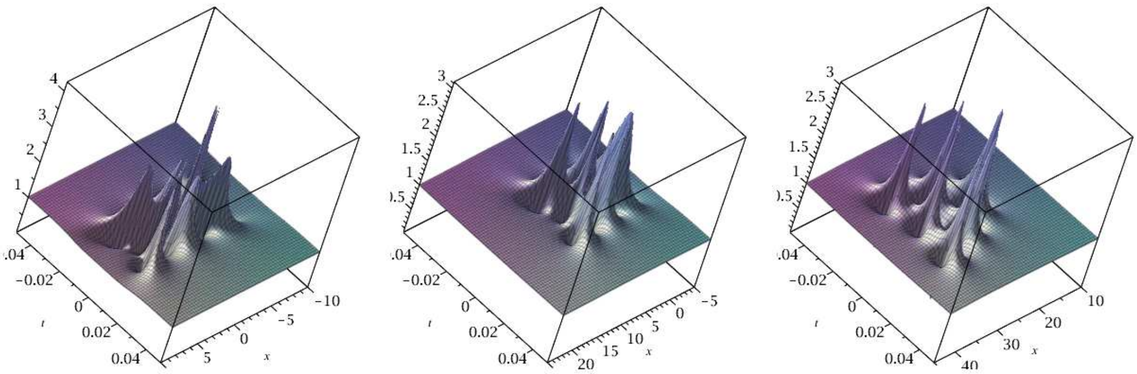

Figure 5. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0,1$, $a_2 = 0$, $b_2 = 0$; in the center $a_1 = 0$, $b_1 = 5$, $a_2 = 0$, $b_2 = 0$; to the right $a_1 = 0$, $b_1 = 5$, $a_2 = 0$, $b_2 = 0$.

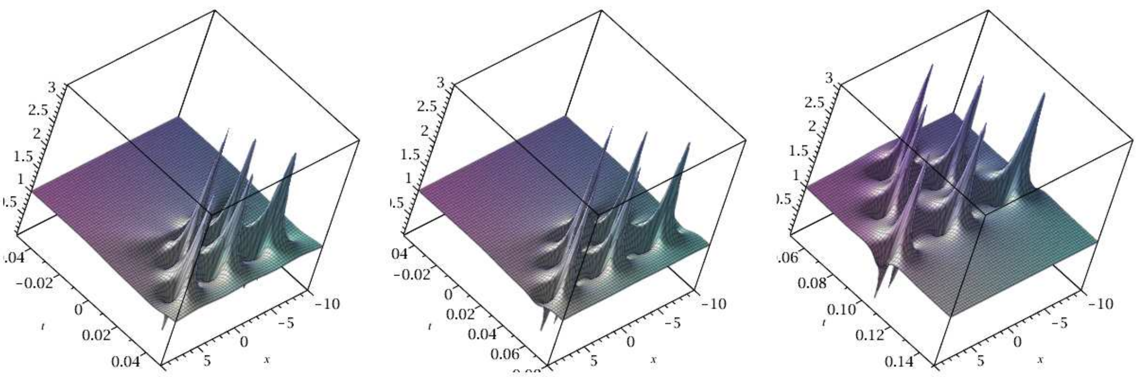

Figure 6. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0$, $a_2 = 0,5$, $b_2 = 0$; in the center $a_1 = 0$, $b_1 = 0$, $a_2 = 1$, $b_2 = 0$; to the right $a_1 = 0$, $b_1 = 5$, $a_2 = 2$, $b_2 = 0$.

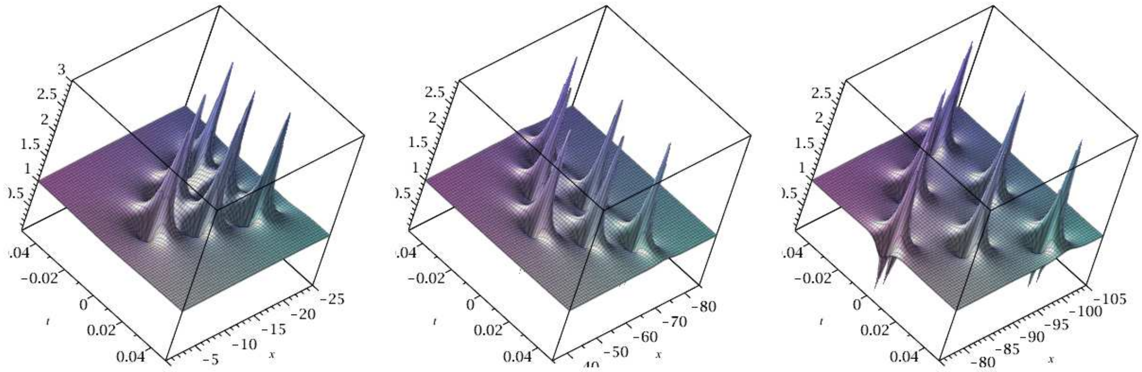

Figure 7. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0,5$; in the center $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 2$; to the right $a_1 = 0$, $b_1 = 5$, $a_2 = 0$, $b_2 = 3$.

We remark the similarity with these solutions and those relative to other equations belonging to this NLS hierarchy. For example, we recover the same types of patterns like in the NLS equation [23], the mKdV equation [24], or the Lakshmanan Porsezian Daniel equation [25]. We get the structure of triangles with peaks which appear in function of the different values of the parameters.

5 Conclusion

Quasi-rational solutions to the $(NLS7)$ equation have been constructed for the first orders. These N-order solutions appear as the quotient of a polynomial of degree $N(N+1)$ in x and t for the numerator by a polynomial of degree $N(N+1)$ in x and t for the denominator.
The solutions of order 2 depend on two real parameters, and the structure of triangles with three peaks is observed for their modules.
The solutions of order 3 depend on four real parameters. In the plane (x,t) of the coordinates, the representation of the modules of the solutions reveals the formation of triangles containing 6 peaks.
It will be relevant to study other solutions of this equations and study the patterns of their modulus.

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Appendix

Solution of order 3 to the (NLS5) equation depending on 4 real parameters :
The function $v(x,t)$ defined by

$$v(x,t) = \left(1 - 24\frac{n(x,t)}{d(x,t)}\right) \mathrm{e}^{i(2\,a_1-6\,a_2+70\,t)} \tag{5}$$

with
$n(x,t) = (675+69363302400\,t^2+88473600\,{b_2}^2+(2\,x-12\,b_1+60\,b_2)^{10}+27000\,(8\,b_1-80\,b_2)^2+91800\,(16\,a_2-1120\,t)^2+2190\,(4\,a_1-24\,a_2+560\,t)^6+495\,(4\,a_1-24\,a_2+560\,t)^8+11\,(4\,a_1-24\,a_2+560\,t)^{10}-720\,(4\,a_1-24\,a_2+560\,t)^7(16\,a_2-1120\,t)-3600\,(4\,a_1-24\,a_2+560\,t)^3(48\,a_2-15904\,t)-720\,(4\,a_1-24\,a_2+560\,t)^5(272\,a_2-25312\,t)-154828800\,(16\,a_2-1120\,t)t+i(15422400\,t+64800\,(16\,a_2-1120\,t)^3-$

$$
870\,(4\,a_1-24\,a_2+560\,t)^7+25\,(4\,a_1-24\,a_2+560\,t)^9+(4\,a_1-24\,a_2+560\,t)^{11}-
151200\,a_2-5529600\,(8\,b_1-80\,b_2)(16\,a_2-1120\,t)b_2-77414400\,(16\,a_2-1120\,t)^2t-
90\,(4\,a_1-24\,a_2+560\,t)^8(16\,a_2-1120\,t)-120\,(4\,a_1-24\,a_2+560\,t)^6(80\,a_2-
11872\,t)+900\,(4\,a_1-24\,a_2+560\,t)^4(464\,a_2-23520\,t)+(-450\,(4\,a_1-24\,a_2+
560\,t)^3-210\,(4\,a_1-24\,a_2+560\,t)^5+10\,(4\,a_1-24\,a_2+560\,t)^7+300\,(4\,a_1-24\,a_2+
560\,t)^4(16\,a_2-1120\,t)+450\,(4\,a_1-24\,a_2+560\,t)(-3+12\,(8\,b_1-80\,b_2)^2-4\,(16\,a_2-
1120\,t)^2)-14400\,a_2+2620800\,t+1800\,(4\,a_1-24\,a_2+560\,t)^2(16\,a_2-2016\,t))(2\,x-
12\,b_1+60\,b_2)^4+(-480\,(4\,a_1-24\,a_2+560\,t)^5(8\,b_1-80\,b_2)+14400\,(4\,a_1-24\,a_2+
560\,t)^2(8\,b_1-80\,b_2)(16\,a_2-1120\,t)+7200\,(4\,a_1-24\,a_2+560\,t)(8\,b_1-48\,b_2)-
2400\,(4\,a_1-24\,a_2+560\,t)^3(16\,b_1-128\,b_2)-14400\,(8\,b_1-80\,b_2)(16\,a_2-1120\,t)-
460800\,(16\,a_2-1120\,t)b_2+12902400\,(8\,b_1-80\,b_2)t)(2\,x-12\,b_1+60\,b_2)^3-21600\,(8\,b_1-
80\,b_2)^2(16\,a_2-1120\,t)+(1710\,(4\,a_1-24\,a_2+560\,t)^5-60\,(4\,a_1-24\,a_2+560\,t)^7+
5\,(4\,a_1-24\,a_2+560\,t)^9-900\,(4\,a_1-24\,a_2+560\,t)^3(7+4\,(8\,b_1-80\,b_2)^2-12\,(16\,a_2-
1120\,t)^2)+675\,(4\,a_1-24\,a_2+560\,t)(7+16\,(8\,b_1-80\,b_2)^2+16\,(16\,a_2-1120\,t)^2)-
345600\,a_2+38707200\,t-21600\,(8\,b_1-80\,b_2)^2(16\,a_2-1120\,t)-21600\,(16\,a_2-
1120\,t)^3+9676800\,(4\,a_1-24\,a_2+560\,t)^2t-1800\,(4\,a_1-24\,a_2+560\,t)^4(64\,a_2-
1792\,t))(2\,x-12\,b_1+60\,b_2)^2+(-240\,(4\,a_1-24\,a_2+560\,t)^7(8\,b_1-80\,b_2)-7200\,(4\,a_1-
24\,a_2+560\,t)^4(8\,b_1-80\,b_2)(16\,a_2-1120\,t)+10800\,(4\,a_1-24\,a_2+560\,t)(24\,b_1-
400\,b_2+4\,(8\,b_1-80\,b_2)^3+4\,(8\,b_1-80\,b_2)(16\,a_2-1120\,t)^2)+3600\,(4\,a_1-24\,a_2+
560\,t)^3(24\,b_1-176\,b_2)+720\,(4\,a_1-24\,a_2+560\,t)^5(56\,b_1-400\,b_2)+21600\,(8\,b_1-
80\,b_2)(16\,a_2-1120\,t)+1382400\,(16\,a_2-1120\,t)b_2-38707200\,(8\,b_1-80\,b_2)t-
43200\,(4\,a_1-24\,a_2+560\,t)^2((8\,b_1-80\,b_2)(16\,a_2-1120\,t)+32\,(16\,a_2-1120\,t)b_2-
896\,(8\,b_1-80\,b_2)t))(2\,x-12\,b_1+60\,b_2)+90\,(4\,a_1-24\,a_2+560\,t)^5(-107+28\,(8\,b_1-
80\,b_2)^2+12\,(16\,a_2-1120\,t)^2)+5400\,(4\,a_1-24\,a_2+560\,t)^2(176\,a_2-22176\,t+
4\,(8\,b_1-80\,b_2)^2(16\,a_2-1120\,t)+4\,(16\,a_2-1120\,t)^3)-225\,(4\,a_1-24\,a_2+560\,t)^3(11+
80\,(8\,b_1-80\,b_2)^2+80\,(16\,a_2-1120\,t)^2+4096\,(8\,b_1-80\,b_2)b_2+114688\,(16\,a_2-
1120\,t)t)-675\,(4\,a_1-24\,a_2+560\,t)(-7+56\,(8\,b_1-80\,b_2)^2+88\,(16\,a_2-1120\,t)^2-
4096\,(8\,b_1-80\,b_2)b_2-131072\,{b_2}^2-102760448\,t^2)+77414400\,(8\,b_1-80\,b_2)^2t+
(4\,a_1-24\,a_2+560\,t)(2\,x-12\,b_1+60\,b_2)^{10}+(-60\,a_1+840\,a_2-42000\,t+5\,(4\,a_1-
24\,a_2+560\,t)^3)(2\,x-12\,b_1+60\,b_2)^8+(-600\,a_1-240\,a_2+722400\,t-140\,(4\,a_1-
24\,a_2+560\,t)^3+10\,(4\,a_1-24\,a_2+560\,t)^5+240\,(4\,a_1-24\,a_2+560\,t)^2(16\,a_2-
1120\,t))(2\,x-12\,b_1+60\,b_2)^6+(-240\,(4\,a_1-24\,a_2+560\,t)^3(8\,b_1-80\,b_2)-1440\,(8\,b_1-
80\,b_2)(16\,a_2-1120\,t)+720\,(4\,a_1-24\,a_2+560\,t)(8\,b_1-176\,b_2))(2\,x-12\,b_1+
60\,b_2)^5)+15\,(1+(4\,a_1-24\,a_2+560\,t)^2)(2\,x-12\,b_1+60\,b_2)^8+(210-60\,(4\,a_1-
24\,a_2+560\,t)^2+50\,(4\,a_1-24\,a_2+560\,t)^4+480\,(4\,a_1-24\,a_2+560\,t)(16\,a_2-
1120\,t))(2\,x-12\,b_1+60\,b_2)^6+(-720\,(4\,a_1-24\,a_2+560\,t)^2(8\,b_1-80\,b_2)-5760\,b_1-
11520\,b_2)(2\,x-12\,b_1+60\,b_2)^5+(450\,(4\,a_1-24\,a_2+560\,t)^2-150\,(4\,a_1-24\,a_2+
560\,t)^4+70\,(4\,a_1-24\,a_2+560\,t)^6+1200\,(4\,a_1-24\,a_2+560\,t)^3(16\,a_2-1120\,t)-
450+5400\,(8\,b_1-80\,b_2)^2-1800\,(16\,a_2-1120\,t)^2+3600\,(4\,a_1-24\,a_2+560\,t)(16\,a_2-
2016\,t))(2\,x-12\,b_1+60\,b_2)^4+(-2400\,(4\,a_1-24\,a_2+560\,t)^4(8\,b_1-80\,b_2)+
28800\,(4\,a_1-24\,a_2+560\,t)(8\,b_1-80\,b_2)(16\,a_2-1120\,t)+57600\,b_1-806400\,b_2-
7200\,(4\,a_1-24\,a_2+560\,t)^2(16\,b_1-128\,b_2))(2\,x-12\,b_1+60\,b_2)^3+(6750\,(4\,a_1-
24\,a_2+560\,t)^4+420\,(4\,a_1-24\,a_2+560\,t)^6+45\,(4\,a_1-24\,a_2+560\,t)^8-2700\,(4\,a_1-
24\,a_2+560\,t)^2(5+4\,(8\,b_1-80\,b_2)^2-12\,(16\,a_2-1120\,t)^2)-675-10800\,(8\,b_1-
80\,b_2)^2-10800\,(16\,a_2-1120\,t)^2+21600\,(4\,a_1-24\,a_2+560\,t)(32\,a_2-3136\,t)-
7200\,(4\,a_1-24\,a_2+560\,t)^3(32\,a_2+448\,t))(2\,x-12\,b_1+60\,b_2)^2+(-1680\,(4\,a_1-
$$

$24\,a_2 + 560\,t)^6(8\,b_1 - 80\,b_2) - 28800\,(4\,a_1 - 24\,a_2 + 560\,t)^3(8\,b_1 - 80\,b_2)(16\,a_2 -$
$1120\,t) - 10800\,(4\,a_1 - 24\,a_2 + 560\,t)^2(8\,b_1 - 272\,b_2) + 86400\,b_1 - 1209600\,b_2 +$
$43200\,(8\,b_1 - 80\,b_2)^3 + 43200\,(8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t)^2 + 3600\,(4\,a_1 - 24\,a_2 +$
$560\,t)^4(8\,b_1 + 80\,b_2) - 86400\,(4\,a_1 - 24\,a_2 + 560\,t)((8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t) +$
$32\,(16\,a_2 - 1120\,t)b_2 - 896\,(8\,b_1 - 80\,b_2)t))(2\,x - 12\,b_1 + 60\,b_2) + 450\,(4\,a_1 - 24\,a_2 +$
$560\,t)^4(-17 + 28\,(8\,b_1 - 80\,b_2)^2 + 12\,(16\,a_2 - 1120\,t)^2) + 10800\,(4\,a_1 - 24\,a_2 +$
$560\,t)(-16\,a_2 + 2016\,t + 4\,(8\,b_1 - 80\,b_2)^2(16\,a_2 - 1120\,t) + 4\,(16\,a_2 - 1120\,t)^3) +$
$675\,(4\,a_1 - 24\,a_2 + 560\,t)^2(-3 + 16\,(8\,b_1 - 80\,b_2)^2 + 16\,(16\,a_2 - 1120\,t)^2 - 4096\,(8\,b_1 -$
$80\,b_2)b_2 - 114688\,(16\,a_2 - 1120\,t)t) - 2764800\,(8\,b_1 - 80\,b_2)b_2)$
and
$d(x,t) = 2024 + 416179814400\,t^2 + 530841600\,{b_2}^2 + 356400\,(8\,b_1 - 80\,b_2)^2 +$
$518400\,(8\,b_1 - 80\,b_2)^4 + 874800\,(16\,a_2 - 1120\,t)^2 + 3720\,(4\,a_1 - 24\,a_2 + 560\,t)^8 +$
$120\,(4\,a_1 - 24\,a_2 + 560\,t)^{10} + 518400\,(16\,a_2 - 1120\,t)^4 + (1 + (2\,x - 12\,b_1 + 60\,b_2)^2 +$
$(4\,a_1 - 24\,a_2 + 560\,t)^2)^6 + (-360\,(4\,a_1 - 24\,a_2 + 560\,t)^8(8\,b_1 - 80\,b_2) - 17280\,(4\,a_1 -$
$24\,a_2 + 560\,t)^5(8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t) - 1440\,(4\,a_1 - 24\,a_2 + 560\,t)^6(8\,b_1 -$
$240\,b_2) + 32400\,(4\,a_1 - 24\,a_2 + 560\,t)^4(8\,b_1 + 112\,b_2) + 64800\,(4\,a_1 - 24\,a_2 +$
$560\,t)^2(-40\,b_1 + 752\,b_2 + 4\,(8\,b_1 - 80\,b_2)^3 + 4\,(8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t)^2) -$
$777600\,(4\,a_1 - 24\,a_2 + 560\,t)((8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t) + 64\,(16\,a_2 - 1120\,t)b_2 -$
$1792\,(8\,b_1 - 80\,b_2)t) - 172800\,(4\,a_1 - 24\,a_2 + 560\,t)^3(3\,(8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t) +$
$32\,(16\,a_2 - 1120\,t)b_2 - 896\,(8\,b_1 - 80\,b_2)t) - 648000\,b_1 + 8553600\,b_2 + 259200\,(8\,b_1 -$
$80\,b_2)^3 + 1296000\,(8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t)^2 - 33177600\,(8\,b_1 - 80\,b_2)^2 b_2 +$
$33177600\,(16\,a_2 - 1120\,t)^2 b_2 - 1857945600\,(8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t)t)(2\,x -$
$12\,b_1 + 60\,b_2) + 80\,(4\,a_1 - 24\,a_2 + 560\,t)^6(191 + 63\,(8\,b_1 - 80\,b_2)^2 + 27\,(16\,a_2 -$
$1120\,t)^2) + 21600\,(4\,a_1 - 24\,a_2 + 560\,t)^3(-368\,a_2 + 23072\,t + 4\,(8\,b_1 - 80\,b_2)^2(16\,a_2 -$
$1120\,t) + 4\,(16\,a_2 - 1120\,t)^3) + 120\,(8\,b_1 - 80\,b_2)(2\,x - 12\,b_1 + 60\,b_2)^9 + 46080\,b_2(2\,x -$
$12\,b_1 + 60\,b_2)^7 - 1161216000\,(16\,a_2 - 1120\,t)t + 240\,(4\,a_1 - 24\,a_2 + 560\,t)^4(599 +$
$135\,(8\,b_1 - 80\,b_2)^2 - 225\,(16\,a_2 - 1120\,t)^2 - 11520\,(8\,b_1 - 80\,b_2)b_2 - 322560\,(16\,a_2 -$
$1120\,t)t) - 16200\,(4\,a_1 - 24\,a_2 + 560\,t)(496\,a_2 - 52640\,t + 80\,(8\,b_1 - 80\,b_2)^2(16\,a_2 -$
$1120\,t) + 16\,(16\,a_2 - 1120\,t)^3 + 4096\,(8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t)b_2 - 57344\,(8\,b_1 -$
$80\,b_2)^2 t + 57344\,(16\,a_2 - 1120\,t)^2 t) + 24\,(4\,a_1 - 24\,a_2 + 560\,t)^2(3881 + 12150\,(8\,b_1 -$
$80\,b_2)^2 + 28350\,(16\,a_2 - 1120\,t)^2 + 691200\,(8\,b_1 - 80\,b_2)b_2 + 22118400\,{b_2}^2 + 17340825600\,t^2) +$
$(-120\,(4\,a_1 - 24\,a_2 + 560\,t)^2 + 360\,(4\,a_1 - 24\,a_2 + 560\,t)(16\,a_2 - 1120\,t) + 120)(2\,x -$
$12\,b_1 + 60\,b_2)^8 + (480\,(4\,a_1 - 24\,a_2 + 560\,t)^2 - 240\,(4\,a_1 - 24\,a_2 + 560\,t)^4 + 960\,(4\,a_1 -$
$24\,a_2 + 560\,t)^3(16\,a_2 - 1120\,t) + 2320 + 2160\,(8\,b_1 - 80\,b_2)^2 + 5040\,(16\,a_2 - 1120\,t)^2 -$
$1440\,(4\,a_1 - 24\,a_2 + 560\,t)(64\,a_2 - 8960\,t))(2\,x - 12\,b_1 + 60\,b_2)^6 + (-720\,(4\,a_1 -$
$24\,a_2 + 560\,t)^4(8\,b_1 - 80\,b_2) - 17280\,(4\,a_1 - 24\,a_2 + 560\,t)(8\,b_1 - 80\,b_2)(16\,a_2 -$
$1120\,t) + 4320\,(4\,a_1 - 24\,a_2 + 560\,t)^2(8\,b_1 - 176\,b_2) - 51840\,b_1 + 103680\,b_2)(2\,x -$
$12\,b_1 + 60\,b_2)^5 + (-1440\,(4\,a_1 - 24\,a_2 + 560\,t)^4 + 720\,(4\,a_1 - 24\,a_2 + 560\,t)^5(16\,a_2 -$
$1120\,t) + 240\,(4\,a_1 - 24\,a_2 + 560\,t)^2(56 + 135\,(8\,b_1 - 80\,b_2)^2 - 45\,(16\,a_2 - 1120\,t)^2) +$
$32400\,(4\,a_1 - 24\,a_2 + 560\,t)(16\,a_2 - 2912\,t) + 7200\,(4\,a_1 - 24\,a_2 + 560\,t)^3(48\,a_2 -$
$4256\,t) + 3360 + 32400\,(8\,b_1 - 80\,b_2)^2 - 54000\,(16\,a_2 - 1120\,t)^2 + 2764800\,(8\,b_1 -$
$80\,b_2)b_2 + 77414400\,(16\,a_2 - 1120\,t)t)(2\,x - 12\,b_1 + 60\,b_2)^4 + (-960\,(4\,a_1 - 24\,a_2 +$
$560\,t)^6(8\,b_1 - 80\,b_2) + 57600\,(4\,a_1 - 24\,a_2 + 560\,t)^3(8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t) -$
$43200\,(4\,a_1 - 24\,a_2 + 560\,t)^2(24\,b_1 - 272\,b_2) - 7200\,(4\,a_1 - 24\,a_2 + 560\,t)^4(48\,b_1 -$
$448\,b_2) + 345600\,b_1 - 5529600\,b_2 - 86400\,(8\,b_1 - 80\,b_2)^3 - 86400\,(8\,b_1 - 80\,b_2)(16\,a_2 -$
$1120\,t)^2 + 172800\,(4\,a_1 - 24\,a_2 + 560\,t)((8\,b_1 - 80\,b_2)(16\,a_2 - 1120\,t) - 32\,(16\,a_2 -$

$1120\,t)b_2 + 896\,(8\,b_1 - 80\,b_2)t))(2\,x - 12\,b_1 + 60\,b_2)^3 + (13440\,(4\,a_1 - 24\,a_2 + 560\,t)^6 + 240\,(4\,a_1 - 24\,a_2 + 560\,t)^8 - 240\,(4\,a_1 - 24\,a_2 + 560\,t)^4(-326 + 45\,(8\,b_1 - 80\,b_2)^2 - 135\,(16\,a_2 - 1120\,t)^2) + 480\,(4\,a_1 - 24\,a_2 + 560\,t)^2(-76 + 135\,(8\,b_1 - 80\,b_2)^2 + 1215\,(16\,a_2 - 1120\,t)^2) - 129600\,(4\,a_1 - 24\,a_2 + 560\,t)^3(32\,a_2 - 1344\,t) - 12960\,(4\,a_1 - 24\,a_2 + 560\,t)^5(32\,a_2 - 1344\,t) - 64800\,(4\,a_1 - 24\,a_2 + 560\,t)(-96\,a_2 + 11200\,t + 4\,(8\,b_1 - 80\,b_2)^2(16\,a_2 - 1120\,t) + 4\,(16\,a_2 - 1120\,t)^3) + 12144 - 97200\,(8\,b_1 - 80\,b_2)^2 + 32400\,(16\,a_2 - 1120\,t)^2 + 530841600\,{b_2}^2 - 464486400\,(16\,a_2 - 1120\,t)t + 416179814400\,t^2)(2\,x - 12\,b_1 + 60\,b_2)^2 - 2160\,(4\,a_1 - 24\,a_2 + 560\,t)^5(240\,a_2 - 47264\,t) - 1440\,(4\,a_1 - 24\,a_2 + 560\,t)^7(80\,a_2 - 6496\,t) - 120\,(4\,a_1 - 24\,a_2 + 560\,t)^9(16\,a_2 - 1120\,t) + 1036800\,(8\,b_1 - 80\,b_2)^2(16\,a_2 - 1120\,t)^2 - 24883200\,(8\,b_1 - 80\,b_2)b_2$
is a solution to the $(NLS5)$ equation (1).