Monthly Rainfall Forecasting Using High Order Singh's Fuzzy Time Series Based on Interval Ratio Methods: Case Study Semarang City, Indonesia

ABSTRACT

Aims: Sample: To determine the effectiveness of the proposed forecasting method, namely Singh's fuzzy time series based on high order (third order) interval ratios. And find out the forecasting results in January 2022.

Study design: Modification of Singh's fuzzy time series based on interval ratios.

Place and Duration of Study: Sample: monthly rainfall data for Semarang City from January 2017 to December 2021.

Methodology: The method proposed by the researcher is the Singh fuzzy time series forecasting method based on high order (third order) interval ratios. This research method uses a combination of Chen and Singh's fuzzy time series. Applying Chen's fuzzy time series in the determining part of the U discussion universe to fuzzification then applying Singh's fuzzy time series in the forecasting part. In the forecasting part, it is obtained through a heuristic approach by building high order forecasting rules to obtain better results and have a very small effect on the Average Forecasting Error Rate (AFER) value. As well as using interval ratios in the interval partition step which aims to reflect variations in historical data. Finally, calculate the Average Forecasting Error Rate (AFER) to test forecasting performance.

Conclusion: Based on the calculation of the AFER value, the AFER for third order is 0.2422%. It can be said that Singh's fuzzy time series forecasting method based on high order (third order) interval ratios on monthly rainfall data for Semarang City from January 2017 to December 2021 is very good. And the rainfall forecast for January 2022 is 196.80 mm^3.

Keywords: Forecasting, rainfall, fuzzy time series, interval rattio

1. INTRODUCTION

One of branch knowledge continued math experience development is fuzzy.Fuzzy has a vague meaning. First time assembling fuzzy discovered by Zadeh in 1965. As time progressed, it emerged method new ones that combine fuzzy with forecasting time series that is fuzzy time series. Principle Work fuzzy time series is fuzzification, formation of FLR and FLRG, as well defuzzification. Fuzzy time series first introduced by Song & Chissom [22]. [22] do forecasting fuzzy time series with develop models using operation min- max composition of the set fuzzy which has a complicated computing process. In the same year, [23] developed method fuzzy time series into a time-variant model fuzzy time series using a 3-layer back propagation neural network for defuzzification. Then [3] proposed method Average-Based length on partitions more intervals efficient compared [22] or average- based interval length. Next, it appears method the new interval partition researched by [6] namely method distribution-based length interval partitioning or based interval length distribution and results his forecast Enough effective compared to method [3]. Afterwards, Kunhuang Huarng propose method new about determination based interval length ratio and proven outperform part big other interval lengths, including based ones distribution and average-based [7]. In

2007, Jilani discussed about modification fuzzy time series based on partition density frequency with determine partition on universe of discourse become the same interval length then the interval defined again based on partition density frequency [9], however results his forecast not yet enough good compared to research [7].

Different with study previously, Singh did study section-focused fuzzy time series forecasting. In 2007, [20] proposed method forecasting fuzzy time series with develop form algorithm computing simple using difference parameters as relation fuzzy applied to the situation moment this for estimate mark circumstances next to use accommodate possibility data ambiguity with way more good and effective. However, in the same year [21] did generalization method previously for accommodate variation big ones that don't regularity and obscurity in the data so make it method strong and proven forecasting results his forecast more Good.

Study related application from forecasting finite fuzzy time series now keep going done for repair shortcomings and weaknesses study previously. Application fuzzy time series does not only used for forecast the alabama data, however for predict production results plantation, foresee price shares, even predict climate something area or specifically rainfall. Rainfall is the amount of water that falls on the surface earth in period certain. Intensity from rainfall tall especially in season raining impact on the environment like floods and land landslide. Because of that's it, rainfall is very interesting for researched because is one of factor biggest influence climate in a region and sector life man so that need exists study for predict rainfall in the future to use anticipate damage environment caused by flooding and soil landslide.

Impact from rainfall enough tall cause study rainfall using fuzzy be center interesting research for researcher. [19] combines three techniques, that is fuzzy sets, entropy, and networks nerves artificial (JST) for face characteristic dynamic Indian summer monsoon rainfall (ISMR) and application to India summer monsoon rainfall monsoon forecasting. Then [25], proposed method a combination of fuzzy logic (fuzzy statistical downscaling) and artificial intelligence (neural statistical downscaling) for predict rainfall. Furthermore [14] did research with combine method Artificial Intelligence (AI) hybrid methods such as Fuzzy Inference System (ANFIS) and use algorithm optimization metaheuristics, specifically Artificial Bee Colony (ABC), Genetic Algorithm (GA), and Simulated Annealing (SA) with the basic Adaptive Neuro model are also proposed for predict rainfall in Hoa Binh province in Vietnam. [8] did study using a system model consisting of experts from rule fuzzy production in the form of linking IF-THEN statements variable input one each other for determine the result as well as logical operators such as the AND operator used for connect based input and output variables system deep fuzzy logic predict rainfall. Another fuzzy logic model was also proposed by [18] for do forecasting rainfall in Nagapattinam with operation mainly is fuzzification and defuzzification as well as offer more results good rather than statistical models. As well as use theorem optimized classical Markov Chains through Weighted Fuzzy Markov Chains for predict rainfall in Guangzhou City [26].

Study related forecasting rainfall use *f*uzzy time series has lots carried out by researchers. Starting from fuzzy time series Hwang and Chen'sfor predict rainfall in Nagapattinam, India [17]. Even in Indonesia it self, this has happened lots study related forecasting rainfall use fuzzy time series. Implement it from fuzzy time series method for predict rainfall in Medan as well do comparison with a number of fuzzy time series methods, namely fuzzy time series Chen, Markov Chain and Cheng, were researched by [1]. In 2016, [4] carried out study about forecasting Chen's fuzzy time series for predict rainfall in Samarinda City. Furthermore is study use average-based fuzzy time series markov chain for determine beginning season rain and seasons dry season in Madura City [11]. Then study about Ruey Chyn Tsaur's fuzzy time series model for predict rainfall at PPKS Bukit Sentang, North Sumatra [16]. Year Next, [5] does study with method forecasting average-based applied fuzzy time series for predict rainfall monthly in Bandung City. Also, research use method fuzzy times series Chen for predict rainfall in Langsa City [12].

Forecasting related rainfall especially in the city of Semarang carried out by [24]in 2007, using method Feed Forward Neural Network (FFNN). **Then [10]** predict rainfall use Kalman Filter method. Next [15] also predict amount rainfall and handling General Circulation Model (GCM) data with technique Adaptive Lasso.**Based on study above, not yet there is study forecasting bulk Semarang City rains use method fuzzy time series. Because of that, writer propose** method forecasting Singh based fuzzy time series interval ratio with high order. Research methods this use combination Chen and Singh's fuzzy time series. Apply Chen's fuzzy time series in part determination universe of discourse U to fuzzification then apply Singh's fuzzy time series to the section forecasting. In section forecasting obtained through approach heuristics with build rule forecasting high order for obtain more results good and has a very small effect on the AFER value. As well as using ratio of intervals on steps purposeful interval partitioning capable reflect variations in historical data. The method proposed in the research this applied to monthly raindall data in Semarang City from January 2017 to December 2021. Finally, counting Average Forecasting Error Rate (AFER) for test performance forecasting. Based on calculation AFER value, obtained AFER for third order is 0,2422%. Can it is said, method high order forecasting Singh based fuzzy time series interval ratio order in monthly rainfall data in Semarang City from January 2017 to December 2021 is very good.

2. MATERIAL AND METHODS

2.1 Material and methods

2.1.1 Forecasting

Forecasting is available knowledge predict future events with do studies or analysis to past data for find systematic relationships, patterns, and tendencies. Forecasting alone is vital parts as base taking every very significant decision. Forecasting requires historical data then project it into the future with a number of form a mathematical model. One of method frequent forecasting used is forecasting time series. Forecasting time series is based on values observed in the subsequent past used for predict future data. According to [2], two methods play a role important in forecasting is method stochastic and non-stochastic used by researchers for forecasting deep time series a number of year final. Stochastic method like moving average (MA), autoregressive integrated moving average (ARIMA), vector regression and exponential moving average (EMA) have limitations in handle problem complex real world forecasting and not certain. Therefore that is, non stochastic method more liked than method stochastic. One of non- stochastic method is fuzzy time series forecasting (FTSF) is used in study field because representation linguistics method this more describe real world scenarios and general give more results good compared to method traditional. Four step large fuzzy time series forecasting (FTSF) process as following:

1. Determine universe of discourse (UOD), number of intervals (NOIs), and length of intervals to divide UOD.
2. Determine set fuzzy and fuzzification time series.
3. Building fuzzy logic relationship in fuzzification time series.
4. Defuzzify mark forecasting fuzzy For get firm value (crips).

2.1.2Fuzzy and fuzzy sets

Fuzzy has a vague meaning. The stated set with something function membership and mapping each set domain fuzzy to appropriate one numbers in the codomain in the zero interval until one is fuzzy set. Whereas something curve showing mapping data from the set domain fuzzy to in degrees membership that has intervals is function membership. Something fuzzy set A_h in the universe of discourse U, mathematically it can be stated as set partner sorted following :

$$A_h = \frac{\mu_{A_h}(k_1)}{k_1}, \frac{\mu_{A_h}(k_2)}{k_2}, \frac{\mu_{A_h}(k_3)}{k_3}, \dots, \frac{\mu_{A_h}(k_m)}{k_m},$$

Where $\mu_{A_h}(k_j)$is the degree of membership $k_j$in the fuzzy set A_hAnd $1 \leq j \leq m$. As well asμ_{A_h} is function membership from fuzzy set A_h, with a mapping of the universe of discourse U to the closed interval at [0,1].

2.1.3Time series

Example given$\{X_t, t \in \mathbb{Z}\}$is row stationary time. Function autocovariance(ACVF) and Function autocorrelation (ACF) of $\{X_t\}$in a way mathematical can stated as following:
Function autocovariance (ACVF) is

$$\gamma_X(h) = Cov(X_{t+h}, X_t),$$

Function autocorrelation (ACF) is

$$\rho_X(h) \stackrel{\text{def}}{=} \frac{\gamma_X(h)}{\gamma_X(0)},$$

Where $h$is lag.

Time series is series of data in the form of mark measured observations during period time certain, based on time with the same interval. Meanwhile, another definition of row time is Suite regulated data samples in time and system driven dynamic [13]. With so is time seriesdata can defined as the data is taken based on observation to something events in the period time certain for example daily, monthly, weekly, and yearly.

2.1.4 Forecasting accuracy level

One of technique for test performance forecasting is use Average Forecasting Error Rate(AFER) or more known as Mean Average Percentage Error(MAPE). Average Forecasting Error Rate(AFER) is frequent used in test performance forecasting or mark error because simple interpretation however efficient in handle problem mark error [9].

$$AFER = \frac{1}{n}\sum_{t=1}^{n} \frac{|X_t - F_t|}{X_t} \times 100$$

Where F_t is the result of fuzzy time series forecastingat the time t,X_t is the actual value, and n is the number of time series data, with criteria as following:

Table 1. AFER Criteria (Lewis, 1982).

Criteria	AFER Value
Very good	<10%
Good	0%-20%
Enough Good	20%-50%
Bad	>50%

2.1.5 Fuzzy time series

Three steps important in forecasting fuzzy time series that is fuzzification, determination fuzzy logical relationships, as well defuzzification or forecasting [22]. Fuzzification is the process of changing historical data in the form of non-fuzzyvariables become variable fuzzy. Whereas fuzzy logical relationshipis determine connection between mark linguistic form variable fuzzy based on table fuzzification, with rule$A_a \rightarrow A_b$where A_a is the current state(X_t) and A_b is the next state (X_{t+1}) at time t. The opposite of fuzzification is defuzzification by changing back linguistic variables or fuzzy variables become non-fuzzyvariables.

Algorithm forecasting Chen's fuzzy time seriesis as following:

1. Determine universe of discourse (U). The following is the definition of determining the universe of discourse: **Definition 1.** *Given a discrete data Where X_{min} is the smallest data, is X_{max} the largest data, as well $D_1$and D_2 is any positive number. Defined universe of discourse Uas follows:*
$$U = [X_{min} - D_1, X_{max} + D_2].$$
2. Determine partition universe of discourse (U) into several intervals of the same length. The following is the definition of the length of each interval:
Definition 2. *Given universe of discourse U, where X_{min} is the smallest data, X_{max} is the largest data, as well $D_1$and D_2 is any positive number. Interval length for universe of discourse Udefined as follows :*
$$l = \frac{(X_{max} + D_2) - (X_{min} - D_1)}{m}; m \in \mathbb{N}$$
with $m$is the number of interval partitions.
Furthermore universe of discourse U partitioned using the following definition:
Definition 3. *Example is given universe of discourse U. Intervals in the universe of discourse U defined as $\{k_j | j \in \mathbb{N}\}$with m is amount interval partition, such so that*
$$\bigcup_{j=1}^{m} k_j = K.$$
3. Build fuzzyset on the universe of discourse (U). Following definition fuzzyset:
Definition 4. *For example, given an interval $\{k_j | j \in \mathbb{N}\}$with m is amount interval partition. Fuzzy sets $\{A_h | h \in \mathbb{N}\}$are defined as:*
$$A_1 = \frac{\mu_{A_1}(k_1)}{k_1}, \frac{\mu_{A_1}(k_2)}{k_2}, \frac{\mu_{A_1}(k)}{k_3}, \frac{\mu_{A_1}(k)}{k_4}, \dots, \frac{\mu_{A_1}(k_m)}{k_m}$$
$$A_2 = \frac{\mu_{A_2}(k_1)}{k_1}, \frac{\mu_{A_2}(k_2)}{k_2}, \frac{\mu_{A_2}(k_3)}{k_3}, \frac{\mu_{A_2}(k_4)}{k_4}, \dots, \frac{\mu_{A_2}(k_m)}{k_m}$$
$$\vdots$$
$$A_z = \frac{\mu_{A_z}(k_1)}{k_1}, \frac{\mu_{A_z}(k_2)}{k_2}, \frac{\mu_{A_z}(k_3)}{k_3}, \frac{\mu_{A_z}(k_4)}{k_4}, \dots, \frac{\mu_{A_z}(k_m)}{k_m}$$
where $\mu_{A_h}(k_j) \in [0,1]$, $1 \le h \le z$, and$1 \le j \le m$,with$\mu_{A_h}(k_j)$is degrees membership$k_j$owned byA_h.
4. Fuzzification of historical data with find degrees membership in each fuzzy set. Fuzzyvalues from any historical data based on degrees highest membership follow definition following:
Definition 5. *Example is given fuzzy sets $\{A_h | h \in \mathbb{N}\}$. Fuzzification from historical data is function membership maximum from fuzzy sets $\{A_h | h \in \mathbb{N}\}$.*
Furthermore given definition fuzzy time seriesas following:
Definition 6. *Example is given $U$is universe of discourse consisting from h fuzzy set$\{A_h | h \in \mathbb{N}\}$. Set F=$\{A_h | h \in \mathbb{N}\}$where F called fuzzy time series in U.*
5. Determine fuzzy logical relationshipwhich is connection between every fuzzydata sequence to the next data in form fuzzy set from step 4 are defined as following:
Definition 7. *Given $F(t)$is a fuzzy time series on time certain t. For example, $F(t)$it occurs because of $F(t-1), F(t-2), \dots, F(t-h)$, then stated relation $\{F(t-h), \dots, F(t-2), F(t-1)\} \to F(t)$ so that relation $\{F(t-h), \dots, F(t-2), F(t-1)\} \to F(t)$called fuzzy logical relationship (FLR) order $h$with $h \in \mathbb{R}^+$. Fuzzy logical relationship with $h = 1$and $h \ge 2$each is a fuzzy logical relationship order one and high order fuzzy logical relationship.*
FLR obtained then grouped to in a fuzzy logical relationship group(FLRG) with use definition following:
Definition 8. *Fuzzy logical relationships are arranged repeat into a fuzzy logical relationship group based side left on the same fuzzy set. For example there is*

relation$\{F(t-h),\ldots,F(t-2),\ F(t-1)\} \rightarrow F_1(t)$*and* $\{F(t-h),\ldots,F(t-2),F(t-1)\} \rightarrow F_2(t)$, *so second relation can written down as* $\{F(t-h),\ldots,F(t-2),F(t-1)\} \rightarrow \{F_1(t),F_2(t)\}$.

6. Defuzzification .
For example fuzzy set for time t is $\{A_h|h \in \mathbb{N}\}$. The forecast for the next observation $(F(t+1))$is
If FLRG from $A_a$empty $(A_a \rightarrow)$, then $(F(t+1))$= A_a.
If FLRG from A_a is $(A_a \rightarrow A_b)$, then $(F(t+1))$= A_b.
If FLRG from A_a is $(A_a \rightarrow A_1, A_2, \ldots, A_m)$, then $\big(F(t+1)\big) = A_1, A_2, \ldots, A_m$and the crisp *value* is the average of corresponding fuzzy set with mark the middle or midpoint of the interval.

2.1.6 Singh,s fuzzy time series

Singh's fuzzy time series uses almost the same idea as Chen's fuzzy time series, the difference is only in the forecasting part using computational methods. The computational method is said to be effective, because it is able to minimize the complexity of calculating complex min-max composition operations by using computations on fuzzy relations, resulting in shorter time consumed in the defuzzification process. Singh's fuzzy time series forecasting algorithm is as follows:

1. Determining the universe of discourse (*U*).
2. Determine the partition of the universe of discourse (*U*) into several intervals of equal length.
3. Building fuzzy sets in the universe of discourse.
4. Fuzzification of historical data, the rule for determining the fuzzy logical relationship is that if A_a is the fuzzy value in year t and A_b is the fuzzy value in year $t+1$, then the fuzzy logical relationship is denoted $A_a \rightarrow A_b$. Where A_a is called the current condition and A_b is the next condition.
5. Determine the fuzzy logical relationship group by grouping the fuzzy logical relationships into one group without repetition of the same relationship.
6. Singh model forecasting.
Some of the notations used for Singh's forecasting are as follows:
$[{}^*K_j]$: The corresponding interval k_j whose membership in A_b,
$L[{}^*K_j]$: lower limit of the interval$[{}^*K_j]$,
$U[{}^*K_j]$: upper limit of the interval$[{}^*K_j]$,
$m[{}^*K_j]$: the middle value of the interval$[{}^*K_j]$.
A_a : fuzzification value in the year to t,
A_b : fuzzification value in the year to $t+1$,
X_t : original value in the year to$t$,
X_{t-1} : original value in the year to$t-1$,
X_{t-2} : original value in the year to$t-2$,
F_j : forecasting results for the next year to $t+1$,
Where$a$ shows the year to$t$and$b$ shows the year to$t+1$.
The third order model utilizes historical data from year to $t-2, t-1,$and t which will be applied to fuzzy logical relationships $(A_a \rightarrow A_b)$, where$A_a$is fuzzification in year$t$andA_b is fuzzification in year to $t+1$. The following is given Rule 1 as a guideline for forecasting:
Rule 1. This rule is used for high order cases or specifically third order, so the forecasting steps are as follows:
   1. Added new difference parameters$D_t = |(|X_t - X_{t-1}| - |X_{t-1} - X_{t-2}|)|$.
   2. Adding other parameters before forecasting, is $E_{t_1} = X_t + (\frac{7}{5})D_t$, $E_{t_2} = X_t - (\frac{7}{5})D_t$, $E_{t_3} = X_t + (\frac{1}{7})D_t$, and$E_{t_4} = X_t - (\frac{1}{7})D_t$.

3. Next if you meet the requirements $E_{t_1} \geq L[{}^*K_j]$ and $E_{t_1} \leq U[{}^*K_j]$, then it is obtained $Y_1 = E_{t_1}$; $n_1 = 1$. Conversely, if it does not meet the requirements $E_{t_1} \geq L[{}^*K_j]$ and $E_{t_1} \leq U[{}^*K_j]$, so Y_1=0; $n_1 = 0$, and do the same steps until obtained Y_4, n_4.
4. Lastly the forecasting part, if $\sum_{p=1}^{4} Y_p = 0$, then the production forecast for the year to $t + 1$ (F_j) is $m[{}^*K_j]$. On the other hand, if $\sum_{p=1}^{4} Y_p \neq 0$, then the production forecast for the next year $t + 1$ (F_j) is $(Y_p + m[{}^*K_j]) / (\sum_{p=1}^{4} n_p + 1)$ where $p \in 1,2,3,4$.

2.1.7Interval partitioning based on ratio

In fuzzy time series forecasting, there are many methods for partitioning the universe of discourse such as seven intervals of equal length, frequency density, frequency distribution, and average-based. However, these methods are not effective enough compared to the ratio-based interval partition method.

In ratio-based interval partitioning, the universe of discourse is partitioned into different lengths based on ratios. An important reason for determining interval length based on ratios is that partitioning intervals with the same length does not reflect data variations accurately because when using large amounts of time series data it has a tendency to fluctuate (up or down). Therefore [7] examined the interval partition method in ratio-based fuzzy time series forecasting. Ratio-based interval partitioning is also considered better than equal length, frequency density, frequency distribution, and mean-based interval partitioning.

Given an algorithm for determining the length of a ratio-based interval as follows [7]:

1. Calculates the difference between two consecutive data $|x_t - x_{t-1}|$ for each x_t and x_{t-1}.
2. Calculating the ratio $r_t = \frac{|x_t - x_{t-1}|}{x_{t-1}}$ for each t.
3. Determining the base by mapping $MIN(r_1, \dots, r_{n-1})$ to Table 2. Then count each r_t to get a base. For example When $MIN(r_1, \dots, r_{n-1})$ = 0,07%, then mapped to 0.1%. Moment $MIN(r_1, \dots, r_{n-1})$ = 0,7% then mapped to 1%.
4. Plot the cumulative distribution for each r_t according to the basis specified in the step (3).
5. Determine the sample percentile ratio (r_α) by setting it at 50%. Reasons for setting the sample percentile ratio (r_α) at 50% is to prevent it from being too small or too large and several experiments have been carried out by Huarng & Yu to ensure that the sample percentile ratio of 50% is the right choice.
6. Determine the interval by following the steps below:
   a. Cutting the minimum observation data to the leftmost two digits and defining as follows:
   $$truncate(MIN(x_t), \forall x_t) = c.d \times 10^e$$
   where truncate is a technique for cutting or making digits into whole numbers, whereas c and d is any digit from 0 to 9, and e is a positive, negative, or 0 integer.
   b. Reduced d by 1
   $$d' = d - 1$$
   c. Determines the initial value
   $$initial = c.d' \times 10^e$$
   d. Determines the interval
   $$interval_j = [lower_j, upper_j]$$
   with
   $$upper_0 = initial;$$

$$lower_j = upper_{j-1};$$
$$upper_j = (1 + r_\alpha)^j \times upper_0$$

where initial is the initial value and r_α is the ratio that has been determined in the step (5) and $j \geq 1$.

The following is a base table as a guide in determining base mapping:

Table 2. Base table (Huarng & Yu, 2006).

$MIN(r_1, \dots, r_{n-1})$	*basis*
$MIN(r_1, \dots, r_{n-1}) \leq$ 0,05%	0,01%
0,05% < $MIN(r_1, \dots, r_{n-1}) \leq$ 0,5%	0,1%
0,5% < $MIN(r_1, \dots, r_{n-1}) \leq$ 5%	1%
5% < $MIN(r_1, \dots, r_{n-1}) \leq$ 50%	10%

2.2 Methodology

Below is given Singh's fuzzy time series forecasting algorithm based on interval ratio partitioning. The research steps carried out in this study are described as follows:

1. Determine the universe of discourse$U$, based on a range of available historical time series data, with rules$U = [X_{min} - D_1, X_{max} + D_2]$where$D_1andD_2$ is any positive number.
2. Partitioning the universe of discourse$U$using the interval ratio algorithm.
3. Building fuzzy setsA_h according to the interval in Step 2 and apply the triangle membership rule for each interval in each fuzzy set that has been built.
4. Fuzzify historical data and build fuzzy logical relationships following the rule: IfA_a is fuzzy production in year to $n$andA_b is fuzzy production the year to $t + 1$, then the fuzzy logical relationship is denoted$A_a \rightarrow A_b$. NotationA_a is the current condition andA_b is the next condition.
5. Singh's forecasting uses Rule 1 through a heuristic approach.
6. Testing forecasting performance using AFER.

3. RESULTS AND DISCUSSION

3.1 Result

This chapter contains the results and discussion of Singh's fuzzy time series forecasting based on high-order or third-order interval ratios and applying it to monthly rainfall data for Semarang City from January 2017 to December 2021. The data is presented in Table 3.

Table 3. Monthly rainfall data for Semarang City from January 2017 to December 2021

Month	***Rainfall (mm^3)***				
	2017	***2018***	***2019***	***2020***	***2021***
January	269	348.6	214	301	273
February	404	535.5	224	393	694
March	213	227.3	178	232	122
April	182	212	217	292	131
May	105	17.9	115	270	205
June	190	44.5	1	22	134
July	31	0	1	71.8	15
August	15	0	2	56	65
September	105	20	11	91	199
October	484	134	8	164	119
November	381	271	71	240	349
December	278	249	231	380	173

The following are the steps for implementing Singh's fuzzy time series forecasting method based on high-order or third-order interval ratios:

1. Determine the universe of discourse$\boldsymbol{U}$.

The following are the steps in determining the universe of discourse *U* using the interval ratio algorithm:

1. Calculates the difference between two consecutive data $|x_t - x_{t-1}|$ for each$x_t$andx_{t-1}.
Based on Table 4, e.gx_t is rainfall data at the time$t$ (February 2017) andx_{t-1} is current rainfall data$t-1$ (January 2017) then the absolute difference is 135. And so on until the absolute difference is obtained in the last year. The following results of calculating the absolute difference between two consecutive data are presented in Table 4.:

Table 4. Absolute difference in rainfall data.

Month-Year	***Rainfall (mm^3)***	$\boldsymbol{|x_t - x_{t-1}|}$
January-2017	269	
February-2017	404	135,0
March-2017	213	191,0
April-2017	182	31,0
May-2017	105	77,0
June-2017	190	85,0
July-2017	31	159,0
August-2017	15	16,0
September-2017	105	90,0
…	…	…
December-2021	173	176,0

2. Determine the ratior_t = $\frac{|x_t - x_{t-1}|}{x_{t-1}}$ for each t.
For example, for January 2017 and February 2017 it is obtainedr_t = $\frac{|404-269|}{269}$ = 50,2. And so on until you get an r_t ratio of at least 75% of the data.
The following results of the r_t ratio calculation are presented in Table 5.

Table 5. Ratior_t.

Month-Year	***Rainfall (mm^3)***	$\boldsymbol{|x_t - x_{t-1}|}$	$\boldsymbol{r_t}$
January-2017	269		
February-2017	404	135,0	50,2
March-2017	213	191,0	47,3
April-2017	182	31,0	14,6
May-2017	105	77,0	42,3
June-2017	190	85,0	81,0
July-2017	31	159,0	83,7
August-2017	15	16,0	51,6
September-2017	105	90,0	600,0
…	…	…	…
December-2021	173	176,0	50,4

3. Determining the base by mapping $MIN(r_1, \ldots, r_{n-1})$to table 5.
Based on Table 4, the smallest ratio is taken $(MIN(r_1, \ldots, r_{n-1}))$is 4,7%. Next, determine the basis by mapping the smallest ratio of 4.7% according to Table 5.

The smallest ratio of 4.7% lies at $0{,}5\% < MIN(r_1, \dots, r_{n-1}) \leq 5\%$, so that the base is obtained 1%.

4. Plot the cumulative distribution for each r_t according to the basis determined in Step 3.
   The cumulative distribution plot for each r_t is determined according to a 1% basis. First, when the ratio is smallest $(MIN(r_1, \dots, r_{n-1}))$is 4,7% the cumulative number obtained is 3. When the horizontal axis increases to 5.7% and then 6.7% (with a 1% base), the cumulative number obtained is 3 and 4. The cumulative distribution plot based on a 1% basis is presented in Figure 1, however for simplicity only the horizontal axis plot (ratio) is shown from 41.7% to 53.7%.

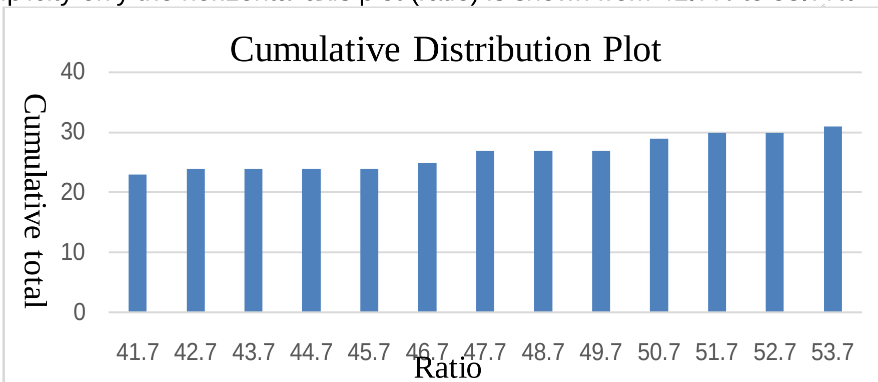

Figure 1. Cumulative distribution plot on a 1% basis.

   Based on Figure 1, when the horizontal axis is 42.7% and 43.7% (with a base of 1%), the cumulative number is 24. And so on until a cumulative distribution plot is obtained for eachr_t.

5. Determine the sample percentile ratio (α) dengan mengatur pada 50%.
   Determine α by setting the 50th percentile to prevent obtaining intervals that are too large or too small.Based on Table 4, the number of ratios$(r_t)$from January 2017 to December 2021 is 60. Then we get 30, which is 50% of the total ratio (r_t). From Figure 1, the ratios 51.7% and 52.7% have a cumulative total greater than or equal to 30, but choose the smallest one. Therefore, an alpha ratio is obtained(r_α)= 51,7%.

6. Determine the interval. Following are the steps in determining the interval:
   a. Cutting the minimum or smallest observation data to the leftmost two digits and defining as follows:

$$truncate(MIN(x_t), \forall x_t) = c, d \times 10^e$$
$$truncate(11) = 1{,}1 \times 10^1.$$

   b. Reduce$b$by 1

$$d' = d - 1$$
$$d' = 1 - 1 = 0.$$

   c. Determines the initial value

$$initial = c, d' \times 10^e$$
$$initial = 1{,}0 \times 10^1 = 10.$$

   d. Determine the interval
      Start by determining the initial value enentukan interval (*initial*),

$$upper_0 = initial$$
$$upper_0 = 10;$$

      when $j = 1$,

$$lower_j = upper_{j-1}$$
$$lower_1 = upper_0 = 10;$$
$$upper_j = (1 + r_\alpha)^j \times upper_0$$

$$upper_1 = (1 + 51{,}7\%)^1 \times 10 = 15{,}17;$$

Obtained

$$interval_j = [lower_j, upper_j]$$
$$interval_1 = [lower_1, upper_1]$$
$$interval_1 = [10, 15{,}17].$$

When $j = 2$,

$$lower_j = upper_{j-1}$$
$$lower_2 = upper_1 = 15{,}17;$$
$$upper_j = (1 + r_\alpha)^j \times upper_0$$
$$upper_2 = (1 + 51{,}7\%)^2 \times 10 = 23{,}01;$$

Obtained

$$interval_j = [lower_j, upper_j]$$
$$interval_2 = [lower_2, upper_2]$$
$$interval_2 = [15{,}17, 23{,}01].$$

And so on until all intervals are obtained with the last interval containing maximum observation data. Thus the interval obtained is presented in Table 6.

Based on the range of available time series data, it is obtained $X_{min} = 0$ and $X_{max} = 694$. Then from Table 6, choose D_1 and D_2 based on the following interval ratio algorithm:

$$X_{min} - D_1 = 0$$
$$D_1 = 0,$$

and

$$X_{max} + D_2 = 979{,}13$$
$$D_2 = 285{,}13.$$

Next, the universe of discourse ***U*** is obtained as follows:

$$U = [X_{min} - D_1, X_{max} + D_2]$$
$$U = [0 - 0, 694 + 285{,}13] = [0, 979{,}13].$$

The following is a detailed table of intervals and midpoint values obtained:

Table 6. Interval range and midpoint.

Index	*Intervals*	*Index*	*Middle Value (MidPoint)*
k_1	[0, 10]	m_1	5
k_2	[10, 15,17]	m_2	12.59
k_3	[15.17, 23.01]	m_3	19.09
k_4	[23.01, 34.91]	m_4	28.96
k_5	[34.91, 52.96]	m_5	43.94
k_6	[52.96, 80.34]	m_6	66.65
k_7	[80.34, 121.87]	m_7	101.11
k_8	[121.87, 184.88]	m_8	153.38
k_9	[184.88, 280.47]	m_9	232.68
k_{10}	[280.47, 425.47]	m_{10}	352.97
k_{11}	[425.47, 645.44]	m_{11}	535.46
k_{12}	[645.44, 979.13]	m_{12}	812.29

2. **Partitioning the universe od discourse using the interval ratio algorithm.**
   After get universe talks U in Step 4.1, then universe of discourse U partitioned become $k_1, k_2, k_3, \ldots, k_{12}$ based on interval ratio algorithm . Furthermore midpoint from each interval is also obtained , as presented in Table 6.
3. **Building fuzzy set A_h.**

After get the interval $k_1, k_2, k_3, \dots, k_{12}$, the next step is to define the *fuzzy set* A_h as follows:

$$A_1 = \frac{1}{k_1} + \frac{0{,}5}{k_2} + \frac{0}{k_3} + \frac{0}{k_4} + \dots + \frac{0}{k_{12}}$$
$$A_2 = \frac{0{,}5}{k_1} + \frac{1}{k_2} + \frac{0{,}5}{k_3} + \frac{0}{k_4} + \dots + \frac{0}{k_{12}}$$
$$\vdots$$
$$A_{12} = \frac{0}{k_1} + \frac{0}{k_2} + \dots + \frac{0}{k_{10}} + \frac{0{,}5}{k_{11}} + \frac{1}{k_{12}}.$$

Simply put ,

$$A_h = \sum_{j=1}^{m} \frac{\mu_{ij}}{k_j},$$

Where A_h is a fuzzy set with $1 \le i \le z = 12$ and k_j is the interval with $1 \le j \le m = 12$. Then the degree of membership (μ_{ij}) is a closed interval with members 0 to 1.For example, when it is a fuzzy set A_1, membership degree (μ_{ij}) for k_1 equal to 1 because $i = j$, while the degree of membership (μ_{ij}) for k_2 of 0,5 because $i = j - 1$, and degree of membership (μ_{ij}) equal to 0 for others. And so on until a fuzzy set is obtained A_{12}.

4. **Fuzzify historical data and build fuzzy logical relationships.**

Fuzzification of historical data is determined by fuzzifying rainfall data based on the fuzzy set obtained in Step 3. For example, rainfall data in January 2017 is 269. Therefore 269 is in the interval $k_9 = [184{,}88, 280{,}47]$, which results in the maximum membership degree being found at k_4. Furthermore, fuzzification was obtained in January 2017 which was represented by a fuzzy set with a maximum membership degree, namely $A_9 = \frac{0}{k_1} + \frac{0}{k_2} + \dots + \frac{0{,}5}{k_8} + \frac{1}{k_9} + \frac{0{,}5}{k_{10}} + \frac{0}{k_{11}} + \frac{0}{k_{12}}$. All historical data fuzzifications are presented in Table 7.

Next is to build a third order fuzzy logical relationship which is composed of 4 sequences of fuzzy sets. For example, if the fuzzy logical relationship is third order, then the fuzzy logical relationship composed by fuzzification in January 2017, February 2017, March 2017, and May 2017 respectively is A_9, A_{10}, A_9, and A_8. Therefore, the fuzzy logical relationship can be expressed as $A_9, A_{10}, A_9 \rightarrow A_8$. The complete results of the third order fuzzy logical relationship are presented in Table 7.

Table 7. Fuzzification

Month-Year	***Rainfall (mm^3)***	***Fuzzification***	***Fuzzy logical relationship third order***
January-2017	269	A_9	
February-2017	404	A_{10}	
March-2017	213	A_9	
April-2017	182	A_8	$A_9, A_{10}, A_9 \rightarrow A_8$
May-2017	105	A_7	$A_{10}, A_9, A_8 \rightarrow A_7$
June-2017	190	A_9	$A_9, A_8, A_7 \rightarrow A_9$
July-2017	31	A_4	$A_8, A_7, A_9 \rightarrow A_4$
August-2017	15	A_2	$A_7, A_9, A_4 \rightarrow A_9$
September-2017	105	A_7	$A_9, A_4, A_2 \rightarrow A_7$
…	…	…	…
December-2021	173	A_8	$A_9, A_7, A_{10} \rightarrow A_8$

After obtaining the fuzzy logical relationship, the next step is to determine the fuzzy logical relationship group. From the fuzzy logical relationship, the following fuzzy logical relationship groups are obtained $A_9, A_{10}, A_9 \rightarrow A_8, A_{12}$ and $A_9, A_9, A_9 \rightarrow A_8, A_9$.

5. **Singh's forecasting uses Rule 1 through a heuristic approach.**

After performing fuzzification, the next step is Singh's forecasting using a heuristic approach. Forecasting results using a heuristic approach are obtained based on Rule 1. For example, for a third-order fuzzy logical relationship in May 2017, $A_{10}, A_9, A_8 \rightarrow A_7$. Fuzzy logical relationship groups are arranged based on the same left part. Based on Table 7, it can be seen that the left side in May 2017 is not the same as the others, so the fuzzy logical relationship group is equivalent to the fuzzy logical relationship. Therefore, the production forecast for the next year$F(t+1) = F(\text{mei } 2017) = A_7$. Based on Table 4, new difference parameters are obtained$D_{\text{april } 2017} = \left|\left(|X_{\text{april } 2017} - X_{\text{March } 2017}| - |X_{\text{March } 2017} - X_{\text{February } 2017}|\right)\right| = |31 - 191| = 160$. Then determine the parameters$E_{t_1} = X_{\text{april } 2017} + (\frac{7}{5}) D_{\text{april } 2017} = 182 + (\frac{7}{5})160 = 291{,}4$, $E_{t_2} = X_{\text{april } 2017} - \left(\frac{7}{5}\right) D_{\text{april } 2017} = 182 - \left(\frac{7}{5}\right)160 = -42$, $E_{t_3} = X_{\text{april } 2017} + \left(\frac{1}{7}\right) D_{\text{april } 2017} = 182 - \left(\frac{1}{7}\right)160 = 204{,}86$, and$E_{t_4} = X_{\text{april } 2017} + \left(\frac{1}{7}\right) D_{\text{april } 2017} = 182 - \left(\frac{1}{7}\right)160 = 159{,}14$. Next determine$Y_p$; n_p based on the lower limit of the interval$(L[{}^*K_j])$ and the upper limit of the interval $(U[{}^*K_j])$. Based on Table 6, because291,41 > 80,34and291,4 > 121,87 does not meet both conditions$E_{t_1} \geq L[k_7]$and$E_{t_1} \leq U[k_7]$, then it is obtainedY_1= 0; n_1 = 0. Then because –42 < 80,34and -42<121,87 does not meet both conditions$E_{t_2} \geq L[k_7]$and$E_{t_2} \leq U[k_7]$, then because Y_2= 0; n_2 = 0. Next because 204,86 > 80,34and204,86 > 121,87 does not meet both conditions$E_{t_3} \geq L[k_7]$and$E_{t_3} \leq U[k_7]$, then it is obtainedY_3= 0; n_3 = 0. And because 159,14 > 80,34and159,14 > 121,87 does not meet both conditions$E_{t_4} \geq L[k_7]$and$E_{t_4} \leq U[k_7]$, then it is obtainedY_4= 0; n_4 = 0. Finally, the forecasting part, because $\sum_{p=1}^{4} Y_p = Y_1 + Y_2 + Y_3 + Y_4 = 0soF(t+1) = F(\text{may } 2017) = m[k_7] = 101{,}12$. The complete forecasting results for third order are presented in Table 8.

6. **Testing forecasting performance using AFER.**

After obtaining the forecasting results, the next step is to evaluate the performance of the forecasting results using the average forecasting error rate. The following is an example of calculating the average forecasting error rate for third order:

$$AFER = \frac{1}{n}\sum_{t=1}^{n} \frac{|X_t - F_t|}{X_t} \times 100$$

$$AFER = \frac{0{,}20885 + 0{,}03705 + 0{,}22463 + \cdots + 0{,}15497}{19} \times 100$$

$$AFER = 24{,}22$$

$$AFER = 0{,}2422\,\%.$$

The detailed average forecasting error rate (AFER) calculation is presented in Table 8. The following table evaluates forecasting results using the average forecasting error rate (AFER) for order three:

Table 8. Forecasting results.

Month-Year	*Rainfall (mm^3)*	*Fuzzification*	*Fuzzy logical relationship third order*	*Forecasting results*	*AFER*
January-2017	269	A_9			
February-2017	404	A_{10}			
March-2017	213	A_9			
April-2017	182	A_8	$A_9, A_{10}, A_9 \rightarrow A_8$	143,99	0.20885
May-2017	105	A_7	$A_{10}, A_9, A_8 \rightarrow A_7$	101,11	0.03705
June-2017	190	A_9	$A_9, A_8, A_7 \rightarrow A_9$	232,68	0.22463

July-2017	31	A_4	$A_8, A_7, A_9 \rightarrow A_4$	28,96	0.06581
August-2017	15	A_2	$A_7, A_9, A_4 \rightarrow A_9$	12,59	0.16067
September-2017	105	A_7	$A_9, A_4, A_2 \rightarrow A_7$	101,11	0.03705
…	…	…	…	…	…
December-2021	173	A_8	$A_9, A_7, A_{10} \rightarrow A_8$	146,19	0,15497
Average forecasting error rate (AFER) %					0,2422 %

Furthermore, based on Table 8, because the average forecasting error rate value is less than 10%, it can be concluded that the forecasting results have very good criteria.

3.2Singh's fuzzy time series based on interval ratio forecasting for January 2022

Singh's fuzzy time series forecasting method based on interval ratios is used to predict rainfall data for the city of Semarang in January 2022. Based on Table 5, the fuzzy logical relationship obtained in January 2022 is $A_7, A_{10}, A_8 \rightarrow$. Because of the fuzzy logical relationship $(A_7, A_{10}, A_8 \rightarrow.)$ is not the same as a fuzzy logical relationship $(A_a \rightarrow A_b)$ so that the forecasting results in January 2022 do not follow Rule 2, but are in the form of an average forecasting result that corresponds to A_7, A_{10} and A_8 is $\frac{91,25+352,97+146,19}{3}$ = 196,80 mm^3.

3.3Discussion

The method proposed by the researcher is the fuzzy time series Singh based on interval ratio forecasting method. An important step in the determination process r_t At a minimum, 4.7 is taken instead of 0 because the interval ratio algorithm requires a minimum of two digit numbers to get the correct interval. Apart from that, because it is an outlier, the base is too small, which results in the process of getting r_α being ineffective or taking a long time. Next, the interval from 0 to the minimum ratio obtained is entered into a new additional interval. Another important point that influences the forecasting results apart from determining the interval partition is in the forecasting section. In Singh's fuzzy time series forecasting method based on interval ratios, the forecast is based on the upper and lower limits and midpoint. Therefore, if there is a fuzzy logical relationship group of A_a is $(A_b \rightarrow A_1, A_2, \dots, A_m)$ then it does not affect the forecasting results and there is no need to determine the average of the corresponding fuzzy set. Thus the forecasting results $\left(F(t+1)\right)$obtained by forecasting according to Rule 1. However, if there is a fuzzy logical relationship group of A_a blank $(A_a \rightarrow)$, then the forecasting results $\left(F(t+1)\right) = A_a$. The effectiveness of Singh's fuzzy time series forecasting method based on interval ratios lies in the part of determining the interval partition which better reflects the variance of the data and the forecasting part because it is able to minimize the complexity of calculating complex min-max composition operations using Rule 1 through a heuristic approach, as well as the time consumed in the process defuzzification becomes shorter. Finally, it was proven through testing forecasting performance using a very small average forecasting error rate.

4. CONCLUSION

Based on the results of research and discussions that have been carried out, it can be concluded that:

1. Singh's research on fuzzy time series forecasting methods based on interval ratios is a combination of fuzzy time series forecasting methods [3] and Singh [20]. As well as making modifications to determining the partition of the universe of discourse

using the interval ratio algorithm [7] and in the forecasting section building Rule 1 through a heuristic approach. Then apply it to Semarang City rainfall data from January 2017 to December 2021.

2. The effectiveness of Singh's fuzzy time series forecasting method based on interval ratios lies in the part of determining the interval partition which better reflects the variance of the data and the forecasting part because it is able to minimize the complexity of calculating complex min-max composition operations, and the time consumed in the defuzzification process is shorter. Finally, it was proven through testing forecasting performance using the average forecasting error rate which was obtained at 0.2422%, therefore it can be said to be very good.
3. Semarang City rainfall forecasting results for January 2022 are 196.80 mm^3.

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